CHAPTER 4

LPV MODELING OF THE BOILER FURNACE, BOILER DRUM AND COAL FIRED BOILER

4.1 INTRODUCTION

For many processes in a chemical process industry, it is necessary to operate at various working points (scheduling variables). Such processes often show smooth, non-chaotic and non-linear behavior as a function of certain scheduling variables. Often an approximation in the form of Linear Parameter Varying (LPV) systems is sufficient to describe such systems. The LPV model represents the non-linear behavior of the process as a function of scheduling variables. This chapter deals with the Linear Parameter Varying modeling of the Boiler furnace, Boiler drum and coal fired boiler, represented as an Integrated Boiler system.

4.2 LPV MODELING OF THE BOILER FURNACE

The LPV model is adopted by considering the fact that the boiler furnace in the thermal power plant has several operating conditions. By assuming that in every operating condition there are parameter changes, the LPV model is suitable for covering all operating conditions.

4.2.1 Development of Linear Transfer Function Models

The furnace is one of the most important parts of the boiler. It is used to produce an enormous amount of heat, which is the result of the
combustion of the air and fuel in the furnace. Effective combustion results in good quality steam that is used in power production. The major inputs for the furnace are air flow and fuel flow. The above mentioned two parameters play a vital role in effective combustion. The output parameters that are used to pass the heat to the other parts, are the heat transferred by radiation to the riser and the furnace temperature. Therefore, for the development of the LPV model of furnace, the air flow and fuel flow are considered as inputs. The heat transferred by radiation to the riser and the furnace temperature are considered as outputs. The heat transferred by radiation to the riser is considered as the working point variable or scheduling variable. Figure 4.1 represents the inputs, outputs and scheduling variable of the Boiler furnace.

![Figure 4.1 Input-Output representation of the Boiler furnace](image)

As the first step in LPV modeling, linear transfer function models are developed, and used to capture the dynamics of different operating regions along the entire trajectory of the process. Therefore, the idea is to only test and identify the process models along their operating trajectories. The entire operating trajectory of the Boiler furnace considered in this work, is obtained by perturbing the input variable by 15% from its nominal value. In the entire operating trajectory, consecutive 5% increases in step input are given to the first principle model of the boiler furnace developed in section 3.2. Therefore,
the entire operating trajectory is divided into three regions, and the corresponding input, and output data are collected.

For individual regions, linear transfer function models are developed using the System Identification Toolbox for every input output combination. The linear transfer function models for the first operating region are denoted as follows

$$\begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} G^1_{11} & G^1_{12} \\ G^1_{21} & G^1_{22} \end{bmatrix} \begin{bmatrix} u_1'(t) \\ u_2'(t) \end{bmatrix}$$

$$w(t) = w_1(t) \quad (4.1)$$

The linear transfer function models for the second operating region are denoted as follows

$$\begin{bmatrix} y_1''(t) \\ y_2''(t) \end{bmatrix} = \begin{bmatrix} G^2_{11} & G^2_{12} \\ G^2_{21} & G^2_{22} \end{bmatrix} \begin{bmatrix} u_1''(t) \\ u_2''(t) \end{bmatrix}$$

$$w(t) = w_2(t) \quad (4.2)$$

The linear transfer function models for the third operating region are denoted as follows

$$\begin{bmatrix} y_1'''(t) \\ y_2'''(t) \end{bmatrix} = \begin{bmatrix} G^3_{11} & G^3_{12} \\ G^3_{21} & G^3_{22} \end{bmatrix} \begin{bmatrix} u_1'''(t) \\ u_2'''(t) \end{bmatrix}$$

$$w(t) = w_3(t) \quad (4.3)$$

where $y(t)$ is the output, $u(t)$ is the input, $w(t)$ is the working point variable (scheduling variable) and $G$ is the linear transfer function. The subscripts of $y$ and $u$ represent the number of outputs and inputs. The superscript of $y$, $G$ and $u$ represent the operating region number.
The input-output data collected from the first principle model of the boiler furnace in the three regions, are used to identify the linear transfer function models using the system identification toolbox. The linear transfer function models for the three operating regions of the boiler furnace are denoted as follows:

First operating region

\[
\begin{bmatrix}
q^1_g \\
T^1_g
\end{bmatrix} = \begin{bmatrix}
671.9291 & 345.2581 \\
9.9206s + 1 & 8.1171s + 1
\end{bmatrix} \begin{bmatrix}
F^1_a \\
F^1_f
\end{bmatrix}
\]

(4.4)

Second operating region

\[
\begin{bmatrix}
q^2_g \\
T^2_g
\end{bmatrix} = \begin{bmatrix}
633.0208 & 401.2568 \\
4.3668s + 1 & 7.4582s + 1
\end{bmatrix} \begin{bmatrix}
F^2_a \\
F^2_f
\end{bmatrix}
\]

(4.5)

Third operating region

\[
\begin{bmatrix}
q^3_g \\
T^3_g
\end{bmatrix} = \begin{bmatrix}
634.38347 & 365.2458 \\
9.6032s + 1 & 7.2564s + 1
\end{bmatrix} \begin{bmatrix}
F^3_a \\
F^3_f
\end{bmatrix}
\]

(4.6)

where,

\( q^1_g, q^2_g, q^3_g \) – Heat transferred by radiation to the riser in the first, second and third operating regions.

\( T^1_g, T^2_g, T^3_g \) – Furnace temperature in the first, second and third operating regions.
F_1^f, F_2^f, F_3^f – Fuel flow rate in the first, second and third operating regions.

F_1^a, F_2^a, F_3^a – Air flow rate in the first, second and third operating regions.

4.2.2 Determination of Weighting Function

The working point variable (scheduling variable) determines the working point of the process operation. It is a measured variable from the process or can be calculated from measurable process variables; it can be an input, output, or an independent variable. Examples of the working point variables are: the load of a power plant (independent variable), air feed rate of an air separation process (input variable) and product viscosity of a lubricant oil unit (output variable), where

\[ w(t) \in [w_1, w_h] \]

\( w_1 \) and \( w_h \) are the low and high limits of \( w(t) \), which projects the operating trajectory of the process.

The heat transferred by radiation to the riser of the boiler furnace is considered as the scheduling variable (working point variable). In the three different operating regions of the boiler furnace the scheduling variable values are chosen as

\[ w_1 = 168321.65 \text{ Kcal/sec} \]

\[ w_2 = 174121.99 \text{ Kcal/sec} \]

\[ w_3 = 188471.10 \text{ Kcal/sec} \]
Using these scheduling variable values, the weights in the three operating regions are estimated. These weights can be determined using the triangular weighting function which is pre-assigned and needs no estimation.

\[
\begin{align*}
\alpha_1(q_r) &= \begin{cases} 
1 & q_r(t) \leq 168321.65 \\
\frac{174121.99 - q_r(t)}{174121.99 - 168321.65} & 168321.65 < q_r(t) \leq 174121.99 \\
0 & q_r(t) > 174121.99
\end{cases} \\
\alpha_2(q_r) &= \begin{cases} 
0 & q_r(t) \leq 168321.65 \\
\frac{q_r(t) - 168321.65}{174121.99 - 168321.65} & 168321.65 < q_r(t) \leq 174121.99 \\
\frac{188471.10 - q_r(t)}{188471.10 - 174121.99} & 174121.99 < q_r(t) \leq 188471.10 \\
0 & q_r(t) > 188471.10
\end{cases} \\
\alpha_3(q_r) &= \begin{cases} 
0 & q_r(t) \leq 174121.99 \\
\frac{q_r(t) - 174121.99}{188471.10 - 174121.99} & 174121.99 < q_r(t) < 188471.10 \\
1 & q_r(t) \geq 188471.10
\end{cases}
\end{align*}
\]

where \(\alpha_1(q_r), \alpha_2(q_r), \alpha_3(q_r)\) are weights which are the functions of the scheduling point variable (heat transferred by radiation to the riser). The weights associated with each and every operating point of the scheduling variable are shown in Figure 4.2. In the figure, the blue, green and red lines represent the weights associated with the first, second and third operating regions respectively.
4.2.3 Determination of the LPV model by interpolation

The identified linear transfer function models in section 4.2.1 are multiplied by their associated weights obtained in section 4.2.2, and then interpolated to obtain the Linear Parameter Varying model.

The output responses, such as the heat transferred by radiation to the riser and furnace temperature of the boiler furnace are captured in three different operating regions, and multiplied by weights that are the function of the scheduling variable, viz, the heat transferred by radiation to the riser. Then they are interpolated to obtain the LPV model.

The LPV model for the boiler furnace is represented by the following equation:

\[
\begin{bmatrix}
q_r(t) \\
T_g(t)
\end{bmatrix}
= a_1(q_r) \begin{bmatrix}
q_r^1 \\
T_g^1
\end{bmatrix} + a_2(q_r) \begin{bmatrix}
q_r^2 \\
T_g^2
\end{bmatrix} + a_3(q_r) \begin{bmatrix}
q_r^3 \\
T_g^3
\end{bmatrix}
\]  

(4.7)

The designed LPV model response and the first principle model response of the boiler furnace are compared, and shown in Figures 4.3(a)
(furnace temperature) and 4.4(a) (heat transferred by radiation to the riser). The error response between first principle and LPV model for furnace temperature is shown in Figure 4.3(b) and for heat transferred by radiation to the riser is shown in Figure 4.4(b).

![Figure 4.3](image)

(a) Model response

(b) Error response

Figure 4.3  (a) Model response (b) Error response of the furnace temperature
Figure 4.4  (a) Model response (b) Error response of the heat transferred by radiation to the riser
4.2.4 LPV Model Validation

From Figures 4.3 and 4.4 it is inferred that the LPV model response tracks the first principle model response accurately. The LPV model is proposed for its merit of tracking both static and dynamic non linearities. The developed LPV model represents the boiler furnace in the entire operating trajectory, as linear models interpolated by the weighting function.

4.3 LPV MODELING OF THE BOILER DRUM

The Boiler drum is a crucial part of the boiler process, in view of modeling and control system design. The Boiler drum mimics the non-linearity and time varying nature of many industrial applications. Hence, the Boiler drum is chosen to be the process for implementing the concept of LPV.

4.3.1 Selection of Linear Transfer Function Models

The First principle model of the Boiler drum is modeled using the fundamental mass and energy balance equations, and the data collected from the real time plant as discussed in section 3.3. The LPV model of the Boiler drum is obtained by considering the feedwater flow rate as the input and the drum level as the output and scheduling variable. Figure 4.5 represents the input, output and scheduling variable of the boiler drum. The parameters which contribute to the time varying nature of the system are called as scheduling variables. The Boiler drum level is selected as the scheduling variable.

![Figure 4.5 Input-output representation of the Boiler Drum](image)
The entire operating trajectory of the Boiler drum considered in this work, is obtained by perturbing the input variable by 25% from its nominal value. The feed water flow rate of the boiler drum is excited in steps in the entire operating trajectory, and the changes in the drum level as four distinct slope regions are shown in Figure 4.6(a,b).

![Figure 4.6](image.png)

**Figure 4.6** Open loop response of the boiler drum system showing variation of (a) the feed water flow rate (input) with (b) drum level (output)

The level of the boiler obtained by the excitation of the first principle model is non linear, as shown in Figure 4.6. This non linear response is divided into four linear regions. The four operating points are chosen at a height of 92cm, 97.4cm, 104.2cm and 112.3cm. For individual regions the linear transfer function models are developed, using the System Identification Toolbox for every input output combination. The linear transfer function models are denoted as \( \frac{k_p}{\tau_p s + 1} \), where \( k_p \) is the process gain and \( \tau_p \) is the time constant. The linear transfer function model obtained for the four linear regions is reported as the process model parameters at different operating points, for the boiler drum process in Table 4.1.
Table 4.1  Process model parameters at different operating points for the boiler drum process

<table>
<thead>
<tr>
<th>Operating Region</th>
<th>Feed water Flow rate (kg/sec)</th>
<th>Drum Level (cm)</th>
<th>Process gain (k_p)</th>
<th>Time constant(τ_p) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>195.6</td>
<td>92</td>
<td>0.63227</td>
<td>135.31</td>
</tr>
<tr>
<td>Region 2</td>
<td>211.9</td>
<td>97.4</td>
<td>1.2384</td>
<td>160.61</td>
</tr>
<tr>
<td>Region 3</td>
<td>228.2</td>
<td>104.2</td>
<td>1.8097</td>
<td>186.88</td>
</tr>
<tr>
<td>Region 4</td>
<td>244.5</td>
<td>112.3</td>
<td>2.5774</td>
<td>232.10</td>
</tr>
</tbody>
</table>

Each linear transfer function model is identified using the data sets at each working point. The developed linear transfer function models are validated with the help of the first principle model output. Figures 4.7, 4.8, 4.9 and 4.10 show the change in the drum level in each linear region of the linear transfer function model and the first principle model.

![Figure 4.7 Validation of the transfer function model in the first linear region](image)

Figure 4.7  Validation of the transfer function model in the first linear region
Figure 4.8  Validation of the transfer function model in the second linear region

Figure 4.9  Validation of the transfer function model in the third linear region
4.3.2 LPV Model Identification

The working point variable (scheduling variable) determines the working point of the process operation. Due to the time varying nature of the level of the boiler drum, it is considered as the scheduling variable. The entire operating trajectory of the boiler drum is divided into four regions, and the corresponding scheduling variable values are $w_1 = 92\text{cm}$, $w_2 = 97.4\text{cm}$, $w_3 = 104.2\text{cm}$ and $w_4 = 112.3\text{cm}$.

The weights which are functions of the scheduling variable, drum level, are estimated using the triangular weighting function method and are listed below.
\[
\alpha_i(l_d) = \begin{cases}
1 & l_d(t) \leq 92 \\
\frac{97.4 - l_d(t)}{97.4 - 92} & 152.2 < l_d(t) \leq 97.4 \\
0 & l_d(t) > 97.4
\end{cases}
\]

\[
\alpha_2(l_d) = \begin{cases}
0 & l_d(t) \leq 92 \\
\frac{l_d(t) - 92}{97.4 - 92} & 92 < l_d(t) \leq 97.4 \\
\frac{104.2 - l_d(t)}{104.2 - 97.4} & 97.4 < l_d(t) \leq 104.2 \\
0 & l_d(t) > 104.2
\end{cases}
\]

\[
\alpha_3(l_d) = \begin{cases}
0 & l_d(t) \leq 97.4 \\
\frac{l_d(t) - 97.4}{104.2 - 97.4} & 97.4 < l_d(t) \leq 104.2 \\
\frac{112.3 - l_d(t)}{112.3 - 104.2} & 104.2 < l_d(t) < 112.3 \\
0 & l_d(t) \geq 112.3
\end{cases}
\]

\[
\alpha_4(l_d) = \begin{cases}
0 & l_d(t) \leq 104.2 \\
\frac{l_d(t) - 104.2}{112.3 - 104.2} & 104.2 < l_d(t) < 112.3 \\
1 & l_d(t) \geq 112.3
\end{cases}
\]

where \(\alpha_i(l_d)\), \(\alpha_2(l_d)\), \(\alpha_3(l_d)\), \(\alpha_4(l_d)\) are the weights, which are the functions of the scheduling point variable (boiler drum level). The weights associated
with each and every operating point of the scheduling variable are shown in Figure 4.11. In the figure, \( R_1, R_2, R_3, R_4 \) represent the weights associated with the first, second, third, and fourth operating regions respectively.

\[
y(t) = a_1(l_d)y_1^1(t) + a_2(l_d)y_2^2(t) + a_3(l_d)y_3^3(t) + a_4(l_d)y_4^4(t)
\]  \hspace{1cm} (4.8)

where \( y_1^1(t) \), \( y_2^2(t) \), \( y_3^3(t) \) and \( y_4^4(t) \) represent the output of the first, second, third, and fourth linear regions of the boiler drum. Figure 4.12 shows the comparison of the open loop responses of the LPV model with the first principle model of boiler drum, for the feedwater flow rate variation, given in Figure 4.6.
4.3.3 LPV Model Validation

From Figure 4.12, it is inferred that the LPV model response tracks the first principle model response accurately. The developed LPV model represents the boiler drum in the entire operating trajectory as a linear model interpolated by the weighting function.

4.4 LPV MODELING OF THE COAL FIRED BOILER

In the previous sections, the implementation of the LPV modeling of the two main subsystems of the Boiler was discussed. In this section, the entire coal fired boiler comprising of six components, is considered for LPV modeling. The Industrial Coal Fired Boiler in the thermal power plant has several operating conditions due to the fluctuations in the steam flow based on demands. By assuming that in every operating condition, there are changes in the parameters, the LPV model is suitable for covering all the operating conditions.
4.4.1 Identification of the Linear Transfer Function Models

The identification of the LPV model is carried out using inputs, such as the fuel flow, air flow, feed water flow and attemperator spray flow, and output variables such as the steam flow, steam temperature, furnace temperature, and furnace pressure. The steam flow, which is the measured variable from the process, is considered as the scheduling variable that determines the operating condition of the process. Figure 4.13 represents the input, output and scheduling variable of the integrated Boiler model.

![Input-output representation of the Coal Fired Boiler](image)

**Figure 4.13 Input-output representation of the Coal Fired Boiler**

The first principle model of a coal fired boiler is developed, by integrating the subsystems of the boiler, as discussed in section 3.8.2. The entire operating trajectory is considered from 5% decrease to 10% increase in the nominal value of inputs. To cover the entire operating trajectory of the integrated boiler model, the four inputs viz, the, fuel flow, air flow, feed water flow and attemperator spray flow of the first principle model, are excited and for these excitations, the steam flow, steam temperature, furnace temperature and furnace pressure change as three distinct slope regions, as shown in Figure 4.14(a-d).
Figure 4.14(a-d) Open loop response of an integrated boiler model
The input-output data collected from the first principle model of the integrated boiler model along the entire operating trajectory divided into three regions, are used to identify the linear transfer function models, using the system identification toolbox. The linear transfer function models are denoted as $\frac{k_p}{\tau_p s + 1}$, where $k_p$ is the process gain and $\tau_p$ is the time constant. The linear transfer function model obtained for the three linear regions for various outputs, by exciting the four inputs of the integrated boiler model, is reported in Table 4.2.

**Table 4.2 Linear transfer function at different operating regions for various outputs by exciting the four inputs of the Integrated Boiler model**

<table>
<thead>
<tr>
<th>Operating region</th>
<th>Output 1 Steam flow</th>
<th>Output 2 Steam Temperature</th>
<th>Output 3 Furnace Pressure</th>
<th>Output 4 Furnace Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>0.40335 26.134s + 1</td>
<td>0.3127 19.033s + 1</td>
<td>27.751 5.828s + 1</td>
<td>0.63564 0.767s + 1</td>
</tr>
<tr>
<td>Region 2</td>
<td>0.36209 28.46s + 1</td>
<td>0.1771 23.892s + 1</td>
<td>22.296 5.8136s + 1</td>
<td>0.6111 0.7691s + 1</td>
</tr>
<tr>
<td>Region 3</td>
<td>0.36097 29.371s + 1</td>
<td>0.1668 24.884s + 1</td>
<td>22.301 5.760s + 1</td>
<td>0.5888 0.7725s + 1</td>
</tr>
</tbody>
</table>

The responses of the linear transfer function models are compared with those of the first principle model in all the operating regions. Figure 4.15(a-d) shows the various output responses of the integrated boiler model in the first operating region. Figure 4.16(a-d) shows the various output responses of the integrated boiler model in the second operating region. Figure 4.17(a-d) shows the various output responses of the integrated boiler model in the third operating region.
Figure 4.15  Output responses of the integrated boiler model in the first operating region
Figure 4.16 Output responses of the integrated boiler model in the second operating region
Figure 4.17  Output responses of the integrated boiler model in the third operating region
4.4.2 LPV Model Identification

Steam flow is considered as the scheduling variable (working point variable). In the three different operating regions of the coal fired boiler, the scheduling variable values are chosen as

\[ w_1 = 191.3 \text{ Kg/sec} \]
\[ w_2 = 197.3 \text{ Kg/sec} \]
\[ w_3 = 202.8 \text{ Kg/sec} \]

Using these scheduling variable values, the weights in the three operating regions are estimated. These weights can be determined using the Triangular Weighting function.

\[
\alpha_1(F_s) = \begin{cases} 
1 & F_s(t) \leq 191.3 \\
\frac{197.3 - F_s(t)}{197.3 - 191.3} & 191.3 < F_s(t) \leq 197.3 \\
0 & F_s(t) > 197.3 
\end{cases}
\]

\[
\alpha_2(F_s) = \begin{cases} 
0 & F_s(t) \leq 191.3 \\
\frac{F_s(t) - 191.3}{197.3 - 191.3} & 191.3 < F_s(t) \leq 197.3 \\
\frac{202.8 - F_s(t)}{202.8 - 197.3} & 197.3 < F_s(t) \leq 202.8 \\
0 & F_s(t) > 202.8 
\end{cases}
\]

\[
\alpha_3(F_s) = \begin{cases} 
0 & F_s(t) \leq 197.3 \\
\frac{F_s(t) - 197.3}{202.8 - 197.3} & 197.3 < F_s(t) \leq 202.8 \\
1 & F_s(t) \geq 202.8 
\end{cases}
\]
where \( \alpha_1(F_s), \alpha_2(F_s), \alpha_3(F_s) \) are the weights which are the functions of the scheduling point variable (i.e.) steam flow. The weights associated with each and every operating point of the scheduling variable are shown in Figure 4.18. The blue, green and red lines represent the weights associated with the first, second and third operating regions respectively.

\[
y(t) = \alpha_1(F_s)y^1(t) + \alpha_2(F_s)y^2(t) + \alpha_3(F_s)y^3(t)
\]

where \( y^1(t), y^2(t) \) and \( y^3(t) \) represent the output of the first, second and third linear regions of the integrated boiler model. The LPV model is identified by interpolating the linear models developed in the three operating regions using the weights, which are the functions of the scheduling point variable, as given in the above equation.
The comparison of the first principle model and LPV model outputs is illustrated in Figures 4.19, 4.20, 4.21 and 4.22 for the steam flow, steam temperature, furnace temperature and furnace pressure, respectively.

**Figure 4.19** Simulated responses of the LPV model and first principle model for steam flow

**Figure 4.20** Simulated responses of the LPV model and first principle model for steam temperature
Figure 4.21 Simulated responses of the LPV model and first principle model for furnace temperature

Figure 4.22 Simulated responses of the LPV model and first principle model for furnace pressure
4.4.3 LPV Model Validation

From Figures 4.19, 4.20, 4.21 and 4.22, it is inferred that the LPV model response tracks the nonlinearities present in the first principle model response in an accurate way. The developed LPV model represents the coal fired boiler(or) integrated boiler model in the entire operating trajectory, as linear models interpolated by the weighting function. Any variable can be considered as the scheduling variable in LPV modeling. In the proposed work, the steam flow, steam temperature, furnace temperature and furnace pressure are tracked accurately, using steam flow as the scheduling variable.

4.5 SUMMARY

The LPV model of the boiler furnace, boiler drum and coal fired boiler is designed in this chapter, and the output response of each process is compared with the first principle model output. It is established that the LPV model output follows the first principle model output.