CHAPTER 2

MODELING OF PROCESSES USING THE LINEAR PARAMETER VARYING METHOD

2.1 LINEAR PARAMETER VARYING MODEL

Industrial processes are designed to perform certain processing tasks, that convert raw materials into certain products. Hence, they are operated in certain “orderly” ways. The orderly way of an industrial process can be expressed by the so-called operating-trajectory. This concept can be used for both batch and continuous processes. For a batch process, its operating trajectory is its recipe or operation curve, carried out to produce a product. For a continuous process, its operating trajectory consists of its typical working points and related transition periods. Examples of continuous nonlinear processes are as follows: lubricant oil units that produce products of different viscosities, polymer plants that produce different product grades, and electrical power plants that operate on different loads.

For the identification of control, it is sufficient to have a model that can approximately represent the process behavior in a thin envelop covering its operating trajectory. Therefore, the idea is only to test and identify the process models along with their operating trajectories. It can be shown that this will result in a low cost testing method and simple and reliable identification computations. A simple linear parameter varying model structure is used in model identification. Linear models are identified using
data sets of the process at corresponding working-points; then the LPV model is obtained by interpolating the linear models using the scheduling variable. The scheduling variable is a measured variable from the process or can be calculated from measurable process variables.

2.2 ADVANTAGES OF USING LPV MODELING

LPV modeling is simple to identify. It can model both static and dynamic nonlinearities and can take into account the process operation knowledge in selecting the working-point variables (scheduling variables). Further LPV modeling is suitable for both continuous and batch processes. Also, LPV modeling can track the process dynamics along the operating trajectory accurately.

2.3 DIFFERENT APPROACHES TO LPV MODELING

When the identification of linear time-invariant (LTI) systems based on measured input-output (IO) data is known, the estimation of the LPV models remains a difficult problem, that is still in a state of development. In the literature, there exist two main approaches for the identification of LPV models: global and local approach.

The global approach is based on the assumption that it is possible to perform a global identification experiment by exciting the system, when the scheduling variables are persistently changing the system dynamics. This assumption, however, may not be valid in some cases, where the scheduling variable varies slowly with respect to time. In this case, it is impossible to perform a global experiment. So, it is appropriate to use the local LPV identification approach, based on the interpolation of a set of local LTI models. The LTI models are estimated using a set of local measurements. The
local measurements are obtained by exciting the system at different fixed operating conditions, that is, for constant values of the scheduling variables.

The local approach has the important practical advantage that many engineers are well-experienced in LTI identification experiments. The local LTI models can be estimated using a wide variety of well established and widely spread LTI identification algorithms. Afterwards, an appropriate methodology is applied to construct an LPV model that interpolates these consistent local models. The local approach is used to model the industrial processes proposed in this thesis.

2.4 LPV MODEL IDENTIFICATION

It is well known, that one convenient way to represent an operating-trajectory model is to use the so called LPV model. In this work, the LPV modeling is described briefly for a multi-input single-output (MISO) process.

Let $y(t)$ represent the process output at discrete time $t$, $u(t)$ the input vector at time $t$ and $w(t)$, the working-point variable (scheduling variable) that determines the working point of the process operation. The scheduling variable is a measured variable from the process or can be calculated from measurable process variables. The Scheduling variable can be an input, output, or independent variable of the process. Examples of working point variables are: the load of a power plant (independent variable), air feed rate of an air separation process (input variable) and product viscosity of a lubricant oil unit (output variable), where

$$w(t) \in [w_l, w_h]$$
$w_l$ and $w_h$ are the low and high limits of $w(t)$, which projects the operating trajectory of the process.

### 2.5 IDENTIFICATION OF LINEAR MODELS

For linear model identification, the process is perturbed, using test signals at two different conditions of operation. They are the normal operating condition and the transition condition. The working point test is conducted during the normal operating condition. At each working point, perform a normal identification test for linear model identification using small test signals. The test can be performed in an open or closed-loop of the process. Proper test signals (excitation) should be used during the tests. The transition test is conducted during the transition condition. In the transition test, add small test signals to the inputs during the transition periods to develop linear models.

Industrial experiences have shown that working point tests can be applied without problems in linear MPC control. Transition tests do not add additional production cost than normal transition control, because the only difference is the addition of small test signals. Therefore, this test approach is low cost.

In this research work, only the working point test is conducted to develop the linear models. The identification tests are performed by adding small test signals during normal operation of the process. The test signal amplitudes are determined in such a way that they will not cause any problem in safe operation. The entire operating trajectory is divided into a number of linear regions. In each region, the linear model is identified using the input and output data. Several linear identification methods can be used, such as the
prediction error method, subspace method and asymptotic method. Assume that the MISO process has p working points and the linear models are denoted as follows:

\[
y(t) = G_1^1(q)u_1(t) + \ldots + G_m^1(q)u_m(t) \quad w(t) = w_1
\]

\[
y(t) = G_1^2(q)u_1(t) + \ldots + G_m^2(q)u_m(t) \quad w(t) = w_2
\]

\[
\vdots
\]

\[
y(t) = G_1^p(q)u_1(t) + \ldots + G_m^p(q)u_m(t) \quad w(t) = w_p
\]  \hspace{1cm} (2.1)

where \( G \) is the linear transfer function, \( m \) is the number of inputs and \( p \) represents number of operating conditions along operating trajectories.

### 2.6 LPV MODEL BY INTERPOLATION

The LPV model is obtained by interpolating the linear models, using the total data (Zuhua Xu et al 2009) denoted as follows

\[
y(t) = \alpha_1(w)y^1(t) + \alpha_2(w)y^2(t) + \ldots + \alpha_p(w)y^p(t)
\]  \hspace{1cm} (2.2)

where \( \alpha_1(w), \alpha_2(w), \ldots, \alpha_p(w) \) are weights which are the functions of the working point variable or scheduling variable \( w(t) \).

A good way to determine the weights is to estimate them from the total testing data, which should include the transition periods. The weight functions \( \alpha_1(w), \alpha_2(w), \ldots, \alpha_p(w) \) can be parametrized as cubic splines, polynomials, or piece-wise linear function.

The weight estimation method is illustrated below using cubic splines.
Denote a set of knots \( \{k_1, k_2, \ldots, k_s\} \) for the working-point variable \( w(t) \) which are real numbers and satisfy
\[
k_1 = k_{\text{min}} \leq k_2 \leq k_3 \leq \cdots \leq k_s = k_{\text{max}} \tag{2.3}
\]

A cubic spline function for \( \alpha_i(w) \) is given as
\[
\alpha_i(w) = \beta_i^1 + \beta_i^2 w + \sum_{j=2}^{s} \beta_i^j \left| w - k_j \right|^3
\tag{2.4}
\]
where \( \left[ \beta_i^1, \beta_i^2, \ldots, \beta_i^s \right] \) are the parameters to be estimated. Here, \( s \) can be called as the order of the cubic splines. The other weight functions are also defined in the same manner as in Equation (2.4). Assume that, for the moment, all weighting functions use the same knots as in Equation (2.3). The knots should span in the process operation range. A convenient way is to let the knots distribute uniformly in the range
\[
k_1 = k_{\text{min}} = w_1 \quad \text{and} \quad k_s = k_{\text{max}} = w_h \tag{2.5}
\]

The order of the cubic splines \( s \) depends on the number of working points and the amount of data.

Now, the weighting functions \( \alpha_1(w), \alpha_2(w), \ldots, \alpha_p(w) \) will be estimated using the total testing data. Denote the total data set \( Z^N \) as follows
\[
Z^N = \{u_1(t), u_2(t), \ldots, u_m(t), y(t), w(t) \mid t = 1, 2, \ldots, N\} \tag{2.6}
\]
where \( u_1(t), u_2(t), \ldots, u_m(t) \) represents the input data, \( y(t) \) represents the output data, and the scheduling variable as \( w(t) \).
Simulate the \( p \) working point models using the total test data \( Z^{N} \) as:

\[
y^1(t) = G^1(q)u_1(t) + \ldots + G^m(q)u_m(t) \\
y^2(t) = G^1(q)u_1(t) + \ldots + G^m(q)u_m(t) \\
\vdots \\
y^p(t) = G^1(q)u_1(t) + \ldots + G^m(q)u_m(t)
\]  
(2.7)

Denote the parameter vector as,

\[
\theta = \begin{bmatrix}
\beta_{11}^1, & \beta_{12}^1, & \ldots, & \beta_{11}^2, & \beta_{12}^2, & \ldots, & \beta_{11}^p, & \beta_{12}^p, & \ldots, & \beta_{1p}^p
\end{bmatrix}^T
\]  
(2.8)

Note that the superscripts in Equations (2.7) and (2.8) are used for numbering, and they not as powers. Then the parameters of the weighting functions can be determined by minimizing the output error loss function:

\[
\theta = \min_{\theta} \sum_{t=1}^{N} \sum_{i=1}^{p} \left[ e_{OE}(t) \right]^2
\]  
(2.9)

where \( e_{OE}(t) \) is the output error of model in Equation (2.2).

\[
e_{OE}(t) = y(t) - \left[ a_1(w)y^1(t) + a_2(w)y^2(t) + \ldots + a_p(w)y^p(t) \right]
\]  
(2.10)

Now, the data of the working-point variable \( w(t) \) is used.

Denote the data vector related to the cubic spline’s weighting functions as,
\[
\phi(t) = \begin{bmatrix}
1 & w(t) & |w(t)-k_{s1}| & \cdots & |w(t)-k_{s1}|^3
\end{bmatrix}
\] (2.11)

Then the output error can be written as

\[
e_{oe}(t) = y(t) - \begin{bmatrix}
\phi(t)y^1(t) & \phi(t)y^2(t) & \cdots & \phi(t)y^p(t)
\end{bmatrix} \hat{\theta}
\] (2.12)

Because the output error \(e_{oe}(t)\) is linear in the weighting function parameters, the optimization in Equation (2.9) is a linear least squares problem, and has the following solution:

\[
\hat{\theta} = \left[\phi^T \phi\right]^{-1} \phi^T Y
\] (2.13)

where,

\[
Y = \begin{bmatrix}
y(1), y(2), \ldots, y(N)
\end{bmatrix}^T
\] (2.14)

And,

\[
\phi = \begin{bmatrix}
\phi(1)y^1(1) & \phi(1)y^2(1) & \cdots & \phi(1)y^p(1) \\
\phi(2)y^1(2) & \phi(2)y^2(2) & \cdots & \phi(2)y^p(2) \\
\vdots & \vdots & \ddots & \vdots \\
\phi(N)y^1(N) & \phi(N)y^2(N) & \cdots & \phi(N)y^p(N)
\end{bmatrix}
\] (2.15)

The data vector related to the cubic spline’s weighting functions is determined. The data should contain both the working point tests and transition point tests.
Assume that no tests in the transition periods are permitted due to economical considerations. Then the weighting function is estimated, using the triangular weighting function. $\alpha_1(w)$, $\alpha_2(w)$, ...., $\alpha_p(w)$ are weights which are functions of the working point variable $w(t)$. Instead of $p$ operating regions, three operating regions are considered. Then $\alpha_1(w)$, $\alpha_2(w)$, $\alpha_3(w)$ are the weights associated with the three operating regions. These weights are calculated as follows.

The best way is to let the weightings be equal to the distances between the current working-point and the working point of the linear models. Then the weights can be given directly without estimation (Yucai Zhu & Zuhua Xu 2008).

\[
\alpha_1(w) = \begin{cases} 
1 & w < w_1 \\
\frac{w_2 - w}{w_2 - w_1} & w_1 \leq w \leq w_2 \\
0 & w > w_2
\end{cases}
\]

\[
\alpha_2(w) = \begin{cases} 
0 & w < w_1 \\
\frac{w - w_1}{w_2 - w_1} & w_1 \leq w \leq w_2 \\
\frac{w_3 - w}{w_3 - w_2} & w_2 < w \leq w_3 \\
0 & w > w_3
\end{cases}
\]

\[
\alpha_3(w) = \begin{cases} 
0 & w < w_2 \\
\frac{w - w_2}{w_3 - w_2} & w_2 \leq w \leq w_3 \\
1 & w > w_3
\end{cases}
\]

The LPV model using the triangular weighing function for the three operating regions is denoted as follows
The LPV model given in Equation (2.16) can give reasonably good approximation of the nonlinear process along its operating-trajectory, at least much better than an averaging linear model.

**2.7 SUMMARY**

In this chapter, the Linear Parameter Varying modeling of the MISO process is discussed. In the identification procedure, the weighting function is calculated using cubic splines or the triangular weighting method. In the absence of the transition period test, the triangular weighting method is considered as the economic one, which is used in this research work to estimate the weights. Using this principle, the LPV modeling of the boiler furnace, boiler drum, integrated boiler model and conical tank system are discussed in the subsequent chapters.