CHAPTER 6

REAL TIME IMPLEMENTATION OF THE LPV MODEL AND CONTROL OF THE CONICAL TANK SYSTEM

6.1 INTRODUCTION

The simulation of the linear parameter varying modeling of the Boiler furnace, Boiler drum and Integrated Boiler model and control of Boiler drum and Integrated Boiler model was in the previous chapters. In order to validate the LPV model in real time, the benchmark process, the conical tank system available in the Department of Instrumentation Engineering, MIT campus, Anna University, Chennai, is used. The experimental setup of the conical tank system is used to collect real-time data sets by conducting an identification test with various operating regions over an operating trajectory. With the help of the data sets obtained, linear and LPV models are developed. Then the multi-model PI and Adaptive PI controller for real time and the LPV model of the conical tank system are designed and analyzed.

6.2 CONICAL TANK SYSTEM

Conical tanks find wide applications in process industries, namely, hydro metallurgical industries, food process industries, concrete mixing industries and waste water treatment industries. Their shape contributes to better drainage of solid mixtures, slurries and viscous liquids. So, the control
of the conical tank presents a challenging task due to its non-linearity and constantly changing cross section. It is considered to be a benchmark system, which mimics the non-linearity and time varying nature of many industrial applications. Hence, the conical tank process is considered for implementing the concept of the LPV model.

The parameters which contribute to the time varying nature of the system are called as scheduling variables. They may either be time varying setpoints (like in the case of robotic arms) or physical parameters of the system (like radius in the conical tank). By considering the variation of the scheduling variable, the LPV model will be formed and updated correspondingly.

The experimental setup of the Conical Tank System is shown in Figure 6.1. It consists of a conical tank, water reservoir, pump, rotameter, pressure transmitter, electro pneumatic converter (I/P converter), pneumatic control valve, interfacing module and a personal Computer (PC). The level of the liquid in the tank is measured by the EMERSON make (Model: 1151DP SMART) differential pressure transmitter, whose output is 4-20 mA current signal. The control valve is fitted with the EMERSON make smart valve positioner, which will take 0.020684272 – 0.103421359 MPa as an input signal. The level transmitter and the control valve are interfaced to a PC, with the help of the National Instruments Educational Laboratory Virtual Instrumentation Suite (NI-ELVIS) N114 Multifunction DAQ board. It has eight analog input channels and two analog output channels.
Figure 6.1 Experimental setup of the conical tank system

The current signal from the differential pressure transmitter is converted into a voltage signal by a current to voltage (I-V) converter, so that it could be directly fed into the interfacing unit. Similarly, the voltage signal from the interfacing unit is converted into a current signal by a voltage to current (V-I) converter, and then to a pressure signal by a current to pressure (I-P) converter, so that it could be fed to the control valve to take corresponding control action.

The level transmitter is connected with the input channel AI-0 of the NI-ELVIS N114 Multifunction DAQ board through the I-V converter. The control signal in the form of a 1-5 V voltage signal is generated from the output channel AO-0 port, and connected to the control valve (CV) through the V-I converter and I-P converter.
The schematic diagram of the Conical Tank system is shown in Figure 6.2, which is a benchmark problem for a number of research topics. It consists of an inverted conical tank with an inlet flow ($F_{in}$) at the top, and an outlet flow ($F_{out}$) at the bottom, a pump that delivers the liquid flow, and a control valve with coefficient ($C_v$) to manipulate $F_{in}$. The operating parameters of the Conical Tank system are shown in Table 6.1.

**Table 6.1 Operating parameters of the conical tank system**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>Top radius of the conical tank</td>
<td>0.4 m</td>
</tr>
<tr>
<td>H</td>
<td>Maximum height of the tank</td>
<td>0.5 m</td>
</tr>
<tr>
<td>$F_{in\ max}$</td>
<td>Maximum in flow to the tank</td>
<td>$2.7675 \times 10^{-4}$ m$^3$/sec</td>
</tr>
</tbody>
</table>
The Conical Tank System (CTS) is a single input single output (SISO) process, in which the tank liquid level \( h \) is considered as the measured variable, and the inlet flow \( F_{in} \) is considered as the manipulated variable. The radius (\( r \)) of the tank is a varying parameter; so, it is expressed as the ratio of the maximum radius (\( R \)) to the maximum height (\( H \)) of the Conical Tank.

According to the mass balance equation:

The mathematical model of the CTS is given by

\[
\frac{d(M(h))}{dt} = \rho_1 F_{in} - \rho_2 F_{out} \tag{6.1}
\]

where \( M(h) = \rho V(h) \)

\( V(h) \) is the volume of the liquid in the tank, \( \rho \) is the density of the liquid in the tank, \( \rho_1 \) is the density of the liquid in the inlet stream and \( \rho_2 \) is the density of the liquid in the outlet stream. Assuming the room temperature as constant, the density of the liquid is the same throughout. Therefore,

\[
\rho_1 = \rho_2 = \rho \tag{6.2}
\]

Substituting Equation (6.2) in Equation (6.1), it becomes

\[
\frac{dV(h)}{dt} = F_{in} - F_{out} \tag{6.3}
\]

\[
V(h) = \frac{\pi r^2 h}{3} \tag{6.4}
\]

\[
\tan \theta = \frac{R}{H} \tag{6.5}
\]
At any level (h)

\[ \tan(\theta) = \frac{r}{h} \quad (6.6) \]

Equating Equation (6.5) and Equation (6.6)

\[ \frac{r}{h} = \frac{R}{H} \]

\[ r = \frac{Rh}{H} \quad (6.7) \]

Substituting Equation (6.7) in (6.4)

\[ V(h) = \frac{\pi R^2 h^3}{3H^2} \quad (6.8) \]

Differentiating the volume of the tank with respect to time

\[ \frac{dV(h)}{dt} = \frac{\pi R^2 h^2 dh}{H^2 dt} \quad (6.9) \]

The cross sectional area of the tank at any level h

\[ A(h) = \pi r^2 \quad (6.10) \]

Substituting Equation (6.7) in Equation (6.10)

\[ A(h) = \frac{\pi R^2 h^2}{H^2} \quad (6.11) \]

Substituting Equation (6.11) in Equation (6.9)

\[ \frac{dV(h)}{dt} = A(h) \frac{dh}{dt} \quad (6.12) \]

Substituting Equation (6.12) in Equation (6.3)
\[ A(h) \frac{dh}{dt} = F_{in} - F_{out} \]

\[ \frac{dh}{dt} = \frac{F_{in} - F_{out}}{A(h)} \quad (6.13) \]

where, \( F_{out} = C_v \sqrt{2gh} \) \quad (6.14)

Substitute Equation (6.11) and Equation (6.14) in (6.13) to get

\[ \frac{dh}{dt} = \frac{F_{in} - C_v \sqrt{2gh}}{\pi \left( \frac{R}{H} \right)^2 h^2} \quad (6.15) \]

where \( h \) is the liquid level in the conical tank in meters, \( R \) is the top radius of the tank in meters, \( H \) is the maximum height of the tank in meters, \( C_v \) is the valve coefficient, \( F_{in} \) is the liquid inlet flow rate in \( \text{m}^3/\text{sec} \), \( \theta \) is the half cone angle of the conical tank, and \( g \) is the acceleration due to gravity in \( \text{m/sec} \).

### 6.3 Linear Transfer Function Model Identification

The model identification of the conical tank system is done by conducting open loop tests in the experimental setup. The input, the inlet flow \( F_{in} \), is varied in steps, and the corresponding changes in the liquid level of the tank, are observed. The obtained liquid level of the tank is termed as the real time data. For the given first step input, the system attains the steady state at 0.1152m. The same procedure is repeated at different operating regions in the conical tank system. It is necessary to maintain the liquid level in the tank within the maximum selected height of 0.45m since the maximum height of the tank is 0.5m. Figure 6.3 shows the liquid level response of the system at various step changes in the input flow. From the response, the non-linear process can be split into four linear regions, using the input-output data.
The inlet flow rate to the tank is taken as the input, whereas the water level in the tank is taken as the output. For individual regions, linear transfer function models are developed, using the system identification toolbox for every input-output combination. The linear transfer function models are denoted as \( \frac{k_p}{\tau_p s + 1} \), where \( k_p \) is the process gain and \( \tau_p \) is the time constant. The linear transfer function model obtained for the four linear regions are reported as the process model parameters at different operating points for the conical tank process in Table 6.2.

**Table 6.2  Process model parameters at different operating points for the conical tank system**

<table>
<thead>
<tr>
<th>Operating region</th>
<th>Level of tank (m)</th>
<th>Process gain ( k_p )</th>
<th>Time constant ( \tau_p ) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>0.1152</td>
<td>0.88142</td>
<td>27.2208</td>
</tr>
<tr>
<td>Region 2</td>
<td>0.2048</td>
<td>1.1071</td>
<td>33.5472</td>
</tr>
<tr>
<td>Region 3</td>
<td>0.3329</td>
<td>1.2001</td>
<td>62.0102</td>
</tr>
<tr>
<td>Region 4</td>
<td>0.4269</td>
<td>1.381</td>
<td>79.5361</td>
</tr>
</tbody>
</table>
The developed linear transfer function models are validated with the help of the real time open loop data of the Conical Tank system.

Figure 6.4(a-d) shows the comparison of the level of the conical tank in each linear region of the linear transfer function model and real time open loop data of the Conical Tank system. In all the figures, the red line represents the model output, and the blue line represents the real time data. It is inferred that, the model output tracks the real time data accurately in all the four regions.

Figure 6.4(a-d) Validation of the transfer function model at four regions with real time data
6.4 LINEAR PARAMETER VARYING MODEL FORMULATION AND VALIDATION

The LPV model denoted in Equation 6.16 is formed, based on the weights that are computed, based on the variation of the liquid level in the conical tank system.

**In general the LPV model is given by,**

\[
y(t) = w_1(h)y^1(t) + w_2(h)y^2(t) + w_3(h)y^3(t) + w_4(h)y^4(t)
\]  
(6.16)

where

\[
y^1(t), \ y^2(t), \ y^3(t) \text{ and } y^4(t) \text{ are the respective linear transfer function model outputs for the four operating regions.}
\]

\[
w_1(h), \ w_2(h), \ w_3(h) \text{ and } w_4(h) \text{ are the weights, which are the functions of the scheduling variable, } h, \text{ the liquid level in the conical tank system.}
\]

These weights can be determined, using the Triangular Weighting function, which is pre-assigned and needs no estimation, as represented in (Zhu & Ji 2009).

\[
y(t) = \begin{cases} 
  y^1(t) & h(t) \leq w_1 \\
  \frac{w_2 - h(t)}{w_2 - w_1} y^1(t) + \frac{h(t) - w_1}{w_2 - w_1} y^2(t) & w_1 < h(t) \leq w_2 \\
  \frac{w_3 - h(t)}{w_3 - w_2} y^2(t) + \frac{h(t) - w_2}{w_3 - w_2} y^3(t) & w_2 < h(t) < w_3 \\
  \frac{w_4 - h(t)}{w_4 - w_3} y^3(t) + \frac{h(t) - w_3}{w_4 - w_3} y^4(t) & w_3 < h(t) < w_4 \\
  y^4(t) & w_4 \leq h(t)
\end{cases}
\]  
(6.17)

where \( h(t) \) is the scheduling variable, and \( w_1, w_2, w_3 \) and \( w_4 \) are the nominal values of the scheduling parameter in each operating region. The variation of the weights with respect to time is shown in Figure 6.5. In Figure 6.5,
W1, W2, W3 and W4 represents the weights as a function of the scheduling variable (liquid level of the tank) associated with the transfer function output of the first, second, third and fourth regions respectively.

![Variation of weights in the formation of the LPV model for the conical tank system](image)

**Figure 6.5 Variation of weights in the formation of the LPV model for the conical tank system**

Once the weights are computed, they are multiplied by their respective linear transfer functions. Then, the products are summed together to form an interpolated model, which is called as the Linear Parametric Varying (LPV) model.

Thus, the interpolation of the linear transfer function models with the weights, gives the LPV model. The formulated LPV model is validated with the real time data of the Conical Tank system. The same input is given for both the plant and the LPV model, and the output responses are captured. The validation of the LPV model with the open loop data of the Conical Tank system is shown in Figure 6.6. It is clear that the LPV model is able to track the response of the real time Conical Tank system in all the regions. Therefore, the LPV model is good enough to capture the non-linearity of the conical tank process over the entire operating trajectory.
Figure 6.6 Validation of the LPV model with real time data of the conical tank system

6.5 IMPLEMENTATION AND ANALYSIS OF VARIOUS CONTROL SCHEMES

6.5.1 Multi-Model PI Control Scheme

Conventional PI controllers are widely used in industries, as they are simple and robust, provided the system is linear. There are a number of techniques for tuning the parameters of the PID controllers. The direct synthesis method is applied for minimum phase systems, which do not have time delays and right half plane zeroes. The controller settings ($K_c$, proportional gain; $\tau_i$, integral time) can be calculated using these tuning rules. The PI settings for the first order process ($k_p$, process gain and $\tau_p$, process time constant), based on Direct Synthesis, are given as

$$K_c = \frac{\tau_p}{k_p\tau_i}, \quad \tau_i = \tau_p$$  \hspace{1cm} (6.18)
where $\lambda$ is the user defined closed-loop time constant, and can be chosen as equal to the process time constant. The process time constant is in seconds. The $\lambda$ value is very small, which gives a faster closed loop response.

In this method, the only parameter to tune is the proportional constant, since the integral constant is equal to the time constant of the process. Tuning a single controller parameter is easier than tuning two or three. Here, the non-linear process is split into four linear regions, and a single PI controller is designed, using the direct synthesis method for the first region. The first region controller setting is used in all the four operating regions. It is implemented using LabVIEW and the block diagram representation using LabVIEW for the conical tank system with single PI controller, as shown in Figure 6.7.

![Figure 6.7 Block diagram representation using LabVIEW for the conical tank system (single PI controller)](image)

The real time responses were obtained for multi step inputs, as shown in Figure 6.8. R1, R2, R3 and R4 in Figure 6.8 represent the four regions of the conical tank system. It is inferred that the servo performance of the controller is not satisfactory in all regions, except the first, for which it is
designed. Due to gain mismatch, oscillations occur, and set point tracking is not satisfactory in R2, R3 and R4 regions.

![Figure 6.8](image)

**Figure 6.8  Response of the conical tank system with the single PI controller acting for the entire region**

Hence, to improve the performance of the controller over the entire process, the multi-model PI control scheme is used. The PI controller is designed for all the four regions independently, using the direct synthesis method and implemented in real time. The PI controller parameters for each operating region are calculated, using the direct synthesis method mentioned in Equation 6.18. The controller parameters for their respective regions are shown in Table 6.3. As the process variable goes through various operating regions, the respective controller parameters are switched. The servo response is checked in all the regions, while the regulatory response is also checked in Region 2. The multi-model PI control scheme is implemented using LabVIEW, and is shown in Figure 6.9.
Table 6.3  Controller settings for various operating regions of the conical tank system

<table>
<thead>
<tr>
<th>Operating Regions</th>
<th>proportional gain ((K_c))</th>
<th>Integral Time (\tau_i) (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>1.1345</td>
<td>0.45368</td>
</tr>
<tr>
<td>Region 2</td>
<td>0.903</td>
<td>0.55912</td>
</tr>
<tr>
<td>Region 3</td>
<td>0.8332</td>
<td>1.0335</td>
</tr>
<tr>
<td>Region 4</td>
<td>0.7241</td>
<td>1.3257</td>
</tr>
</tbody>
</table>

Figure 6.9  Block diagram representation using LabVIEW for the conical tank system (multi-model PI control scheme)

Figure 6.10 shows the response of the conical tank system with the multi-model PI control scheme. In each region, the set point is tracked by the level of the conical tank system, and the disturbance caused is also rejected.
The multi-model PI control scheme adopted for the real time system is also implemented for the developed LPV model of the conical tank system. At each sampling instant, the weighting function will assign weights to each PI controller output, and the weighted sum of this output will be applied as input to the LPV model of conical tank system, as shown in Figure 6.11. The weighting function that determines the weights $W_1$, $W_2$, $W_3$ and $W_4$ is based on the scheduling variable $h$. The weights are in the range of 0 to 1. The weight calculation algorithm followed in this work is shown in Figure 6.12.
Figure 6.11  Block diagram of the multi-model PI control scheme for the LPV model of the conical tank system

Figure 6.12  Calculation of weights based on the scheduling variable (level of the tank)

The response of the LPV model of conical tank system with the multi-model PI control scheme is shown in Figure 6.13. The level of the conical tank system tracks the variation in set point in all the four regions.
6.5.2 Adaptive PI Control Scheme

In the conventional approach, it is necessary to tune the controller parameters for different operating conditions. Hence, an alternative approach, using the adaptive control scheme is proposed in this work. Adaptive control has always been a successful methodology to control a system with parametric variations. The tuning system of an adaptive control will sense the parametric variations, and tune the controller parameters in order to compensate for it. The parametric variation may be due to the disturbance or due to the inherent non-linearity of the system. In a conical tank the cross section area varies as a function of level, which in turn, leads to parametric variations.
The scheduling variable used in the conical tank system is the level of liquid in the tank. When the scheduling variable has been determined at each operating condition, the controller parameters are calculated at each operating condition, by using a suitable set of polynomial equations, which relate the process parameters and scheduling variable. The model parameters \((k_p \text{ and } \tau_p)\) for the regions along with their respective steady state operating point, the level of the tank \((h)\) are known. Using this, the polynomials for \(k_p\) and \(\tau_p\) in terms of \(h\) are formed separately, using the least square curve fitting method, and are given in Equations 6.19 and 6.20.

\[
k_p(h) = 0.0010h^3 - 0.0183h^2 + 0.4008h + 0.0261 \quad (6.19)
\]

\[
\tau_p(h) = 664h^3 - 6394h^2 + 2070h - 21799 \quad (6.20)
\]

The two polynomials give the model parameters for every instance; they are then converted into controller settings for every operating point of the process correspondingly, through the direct synthesis method. The PI controller settings, using the direct synthesis method, are given in Equation 6.18. The PI controller adapts its parameters with a change in the level of the liquid in the tank. A single PI controller with an adaptive mechanism takes care of the servo performance of the entire operating trajectory of the process. The adaptive PI controller scheme is implemented using LabVIEW, and is shown in Figure 6.14. The response of the conical tank system with the adaptive PI controller in real time is shown in Figure 6.15. The servo performance of the conical tank system is analysed in all the regions. It is inferred that the variation in set point in each region is tracked by the level of the conical tank system.
Figure 6.14  Block diagram representation of the conical tank system in LabVIEW (Adaptive PI control scheme)

Figure 6.15  Response of the conical tank system with the adaptive PI controller
The block diagram of adaptive PI control scheme for the LPV model of the conical tank system is shown in Figure 6.16. The LPV model of the conical tank system that generates level of the tank as output is given as input to the adaptive mechanism. The adaptive mechanism consist of polynomial equations, which relates the process parameters and scheduling variable. The process gain and time constant in terms of scheduling variable, level of the tank is obtained as the output of adaptive mechanism. The PI controller settings using the direct synthesis method given in Equation 6.18 is used to estimate the controller parameters ( $K_c$ and $\tau_i$ ).

![Figure 6.16 Block diagram of the adaptive PI control scheme for the LPV model of the conical tank system](image)

The designed adaptive PI controller is implemented on the LPV model based the conical tank system and servo performance is analysed. The response of the conical tank system with LPV model based adaptive PI controller is shown in Figure 6.17. It is inferred, that the level of the tank tracks the setpoint variation in all the operating regions.
6.5.3 Performance Analysis of the Control Schemes

The multi-model PI control scheme and adaptive PI control scheme adopted for real time and LPV model of the conical tank system are compared, using the time domain specifications. The comparative analysis for the second region is given in Table 6.4. In the LPV model based Adaptive PI control scheme, even though the settling time and rise time are comparatively larger than those of the conventional PI control scheme, the overshoot has been completely eliminated.
Table 6.4 Comparison of the time domain specifications of the implemented control schemes

<table>
<thead>
<tr>
<th>Timedomain specifications</th>
<th>Rise time (sec)</th>
<th>Settling time (sec)</th>
<th>Overshoot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-model PI controller for real time setup</td>
<td>300</td>
<td>455</td>
<td>0.3224</td>
</tr>
<tr>
<td>Adaptive PI controller for real time setup</td>
<td>338</td>
<td>475</td>
<td>0.058</td>
</tr>
<tr>
<td>LPV model based multi-model PI controller</td>
<td>200</td>
<td>332</td>
<td>0.1687</td>
</tr>
<tr>
<td>LPV model based Adaptive PI controller</td>
<td>440</td>
<td>522</td>
<td>0</td>
</tr>
</tbody>
</table>

6.6 SUMMARY

The Linear Parameter Varying model was developed for a conical tank system, which is nonlinear in nature. The linear parameter varying modeling proves to be a simple and efficient method of modeling a nonlinear process.

In the conical tank system, the conventional PI controller schemes (single PI controller, multi-model PI controller and Adaptive PI controller) and the LPV model based control schemes have been implemented at the four operating regions of the system. The comparative performance analysis, in terms of the time domain specification, is carried out for all the control schemes.