CHAPTER III
SINGLE-LEVEL EXTENDED SKIP-LOT SAMPLING PLANS
SECTION – 3.1
MODIFIED SKIP-LOT PLAN MSkSP-1

- In section 3.1, designing of modified skip-lot sampling plan MSkSP-1 with chain sampling ChSP-1 plan as reference plan for given $p_1$, $p_2$, $\alpha$ and $\beta$ is made. Tables are constructed under the conditions for application of Poisson model for OC curve using which one can select MSkSP-1 plan under various sets of incoming and outgoing qualities. Conversion of a given set of parameters to the other familiar sets of parameters is provided.

MODIFIED SKIP-LOT PLAN MSkSP-1

A modified skip-lot plan designated as MSkSP-1 was introduced by Parker and Kessler (1981) to give protection against lots produced completely “out of control.”

OPERATING PROCEDURE

The operating procedure of MSkSP-1 plan is

1. Beginning in the normal inspection mode, a reference plan is applied to each lot.
2. When i consecutive lots are accepted according to the reference plan, a switch is made to the skipping mode.
3. In the skipping mode, a fraction $f$ of the incoming lots are inspected using the reference plan. For the remaining (1-$f$) of the incoming lots a sample of size one is taken. If the unit is “nondefective,” the lot is accepted and the skipping mode continues unabated; if the unit is “defective,” the reference plan is then applied to the lot without regard for the value of the unit sampled and the lot is rejected or accepted accordingly.
4. If any lot is rejected while in the skipping mode, a change to normal inspection mode is made and the procedure begins again at 1 above.
OPERATING CHARACTERISTIC FUNCTION

The probability of acceptance under MSkSP-1 is derived as

\[ P'_a = P + (1-P) (1-f) (1-d) \eta_i+1 \]  

(3.1)

where  \( P \) = probability of acceptance of the reference plan

\( d \) = probability that the unit sampled during a “skip” is defective

and  \( \eta_i+1 = \alpha / \left[ 1 + \frac{d (1 - f) (1 - P^i)}{f + (1 - f) P^i} \right] \)  

(3.2)

where  \( \alpha = \frac{P^i}{f + (1 - f) P^i} \)  

(3.3)

SELECTION OF PLANS

Selection procedures of MSkSP-1 Plan with ChSP-1 as reference plan will now be indicated under the conditions for application of Poisson model for OC curve. Table 3.1 has been constructed for various parametric values indexed by \( d, i_c, f, i \), which can be used for selection of such plans.

Designing plans for Given AQL and LQL

Column 5, 6 and 10 of Table 3.1 is used to design plans for a given AQL and LQL.

For example, for given AQL = 0.008 and LQL = 0.10, one can compute \( \text{LQL} / \text{AQL} = 0.10/0.008 = 12.5 \).

The value closest to 12.5 in the \( p_2 / p_1 \) column of Table 3.1 is 12.514.

This corresponds to \( d, i_c, f, i \) and \( np_1 \) values of 0.2, 2, 2/3, 12 and 0.185 respectively.

Now one can calculate \( n = np_1 / p_1 = 0.185 / 0.008 = 23.125 \).

Rounding \( n \) to the nearest integer one obtains a sample size of 23.

Thus, the desired plan can be specified by \( n = 23, d = 0.2, i_c = 2, f = 2/3 \) and \( i = 12 \).

Designing Plans for Given AQL and IQL

The entries in Column 5, 7 and 11 of Table 3.1 can be used to design plans for given AQL and IQL.
For example, for given $AQL = 0.008$ and $IQL = 0.025$, one can compute $LQL / AQL = 0.025 / 0.008 = 3.125$.

The value closest to 3.125 in the $p_2 / p_1$ column of Table 3.1 is 3.143.

The $d$, $i_c$, $f$, $i$ and $n_{p_1}$ values corresponding to this value are 0.4, 3, $1/5$, 6 and 0.258 respectively.

Now the sample size $n = n_{p_1} / p_1 = 0.258 / 0.008 = 32.25 = 32$.

Thus, the desired plan can be specified by $n = 32$, $d = 0.4$, $i_c = 3$, $f = 1/5$ and $i = 6$.

**Designing plans for Given AQL and AOQL**

Column 5, 9 and 12 of Table 3.1 is used to design plans for a given AQL and AOQL.

For example, for given $AQL = 0.014$ and $AOQL = 0.022$, one can compute $AOQL / AQL = 0.022 / 0.014 = 1.571$.

The value closest to 1.571 in the $AOQL / AQL$ column of Table 3.1 is 1.574.

This value corresponds to $d$, $i_c$, $f$, $i$ and $n_{AOQL}$ values of 0.2, 3, $1/5$, 6 and 0.258 respectively.

Now one can calculate $n = n_{AOQL} / AOQL = 0.258 / 0.014 = 18.428$.

Rounding $n$ to the nearest integer one obtains a sample size of 18.

Thus, the desired plan can be specified by $n = 18$, $d = 0.2$, $i_c = 3$, $f = 1/5$ and $i = 6$.

**CONVERSION OF PARAMETERS**

Now converting a given set of parameters to the other set of equivalent (resulting plans having nearly identical OC curves) parameters will be discussed through an example.

For example $p_1 = 0.02$, $p_L = 0.03$, $p_L / p_1 = 0.03 / 0.02 = 1.5$, which corresponds to the table value of 1.500 from Table 3.1. Now, $n = n_{p_1} / p_1 = 0.298 / 0.02 = 14.9 = 15$. The parameters of MSkSP-1 plan satisfying the requirements are found from Table 3.1 as $n = 15$, $d = 0.8$, $i_c = 2$, $f = 1/5$, $i = 6$. 
The other measure can be calculated from Table 3.1 as
\[
\begin{align*}
p_2 &= np_2 / n = 2.316 / 15 = 0.154 \\
p_0 &= np_0 / n = 0.883 / 15 = 0.059 \\
p_m &= np_m / n = 0.762 / 15 = 0.051
\end{align*}
\]

The other similar sets of parameters can be specified as
\[
\begin{align*}
(AQL, LQL) &= (0.02, 0.154) \\
(AQL, IQL) &= (0.02, 0.059) \\
(AQL, p_m) &= (0.02, 0.051)
\end{align*}
\]

CONSTRUCTION OF TABLES 3.1

The probability of acceptance under MSkSP-1 is derived as
\[
P'_a = P + (1-P) (1-f) (1-d) \eta_{i+1}
\]
where
\[
P = \text{probability of acceptance of the reference plan}
\]
\[
d = \text{probability that the unit sampled during a “skip” is defective}
\]
and
\[
\eta_{i+1} = \alpha / \left[ 1 + \left[ d (1 - f) (1 - P_i) \right] / \left[ f + (1 - f) P_i \right] \right]
\]
where
\[
\alpha = P_i / \left[ f + (1 - f) P^i \right].
\]

Here,
\[
P = e^{-np} + (np) e^{-np(ic + 1)},
\]
which is the OC function for ChSP-1 reference plan having parameters n and i, as derived by Dodge (1955a), under the conditions for application of Poisson model for OC curve.

For given values of d, i, f, i and P_a (p), equation (3.1) can be solved for np by the method of iterations. The entries of the columns np_1, np_2 and np_0 of Table 3.1 provides such np values with P_a (p) = 0.95, 0.10 and 0.50 respectively for given d, i, f, i and P_a (p).

Assuming nAOQ = np P_a (p), for given values of d, i, f and i, the values of np_m that maximizes nAOQ, can be obtained from equation (3.1) and are tabulated. Values of nAOQL are obtained using np_L = np_m P_a (p_m).

The remaining columns of the table of 3.1 are the ratios of corresponding parametric values.
In section 3.2, selection of CSP-M SkSP with chain sampling ChSP-1 plan as reference plan is outlined. Tables are constructed under the conditions for application of Poisson model for OC curve using which one can select SkSP-2 under various sets of points of OC curve and quality levels. Conversion of a given set of parameters to the other equivalent sets of parameters is discussed through an example.

Carr (1982) has proposed CSP-M one-level skip-lot plan designated as CSP-M SkSP, which is similar to the multi-level continuous sampling plan CSP-M.

OPERATING PROCEDURE

The operating procedure of this plan is given below:

1. Start with normal inspection, using the reference plan.
2. When i consecutive lots are accepted on normal inspection, switch to skipping inspection (inspect only a fraction, f, of the lots). When a lot is rejected, proceed as in step 3.
3. Inspect next four lots only. If all the four lots are accepted, proceed as in step 4. If one or more lots are rejected, go to step 1.
4. Inspect only a fraction, f, of the lots at random. If a lot is rejected, go to step 1. If (i - 4) lots are accepted, go to step 2.
OPERATING CHARACTERISTIC FUNCTION

The OC function of the CSP-M SkSP –plan is given by

\[ P_a = (1 - F) + F P \]  \hspace{1cm} (3.4)

where \( P \) = probability of acceptance of the reference plan
and \( F \) = the probability that a lot is sampled

\[ = \frac{[ f \left( A^2 + q^2i B + ( C + 4 p ) q^i \right) ]}{[ f A^2 + q^2i B + q^i ( C + 4 f p ) ]} \]

where \( p = 1 - P \)
\( q = 1 - p \)
\( A = 1 - q^i \)
\( B = 2 - q^i \)
\( C = 1 - 3q^i + q^{2i} + q^4 \)

SELECTION OF PLANS

Selection procedures of CSP-M SkSP Plan with ChSP-1 as reference plan will now be indicated under the conditions for application of Poisson model for OC curve. Tables 3.2 and 3.3 have been constructed for various parametric values indexed by \( i_c, f, i \), which can be used for selection of such plans.

Designing plans for Given AQL and LQL

Column 4, 5 and 9 of Table 3.2 is used to design plans for a given AQL and LQL.

For example, for given AQL = 0.008 and LQL = 0.09, one can compute \( \text{LQL} / \text{AQL} = 0.09/0.008 = 11.25 \).

The value closest to 11.25 in the \( p_2 / p_1 \) column of Table 3.3 is 11.361.

This corresponds to \( i_c, f, i \) and \( np_1 \) values of 3, 1/3, 12 and 0.202 respectively.

Now one can calculate \( n = np_1 / p_1 = 0.202 / 0.008 = 25.25 \).
Rounding \( n \) to the nearest integer one obtains a sample size of 25.

Thus, the desired plan can be specified by \( n = 25, i_c = 3, f = 1/3 \) and \( i = 12 \).

**Designing Plans for Given AQL and IQL**

The entries in Column 4, 6 and 10 of Table 3.2 can be used to design plans for given AQL and IQL.

For example, for given AQL = 0.004 and IQL = 0.017, one can compute \( \frac{IQL}{AQL} = \frac{0.017}{0.004} = 4.25 \).

The value closest to 4.25 in the \( p_0/p_1 \) column of Table 3.3 is 4.256.

The \( i_c, f, i \) and \( np_1 \) values corresponding to this value are 5, 1/3, 12 and 0.168 respectively.

Now the sample size \( n = np_1/p_1 = 0.168/0.004 = 42 \).

Thus, the desired plan can be specified by \( n = 42, i_c = 5, f = 1/3 \) and \( i = 12 \).

**Designing plans for Given AQL and AOQL**

Column 4, 8 and 11 of Table 3.2 is used to design plans for a given AQL and AOQL.

For example, for given AQL = 0.012 and AOQL = 0.026, one can compute \( \frac{AOQL}{AQL} = \frac{0.026}{0.012} = 2.167 \).

The value closest to 2.167 in the AOQL / AQL column of Table 3.3 is 2.169.

This value corresponds to \( i_c, f, i \) and \( np_1 \) values of 2, 2/3, 6 and 0.195 respectively.

Now one can calculate \( n = np_1/p_1 = 0.195/0.012 = 16.25 \).

Rounding \( n \) to the nearest integer one obtains a sample size of 16.

Thus, the desired plan can be specified by \( n = 16, i_c = 2, f = 2/3 \) and \( i = 6 \).
**Designing plans for Given (p₁, h₁)**

Column 4 and 7 of Table 3.3 is used to design plans for a given p₁ and h₁.

For example, for given p₁ = 0.02 and h₁ = 0.14, from Table 3.4 under column headed h₁, locate the value equal to or just greater than specified h₁, which is 0.140.

Corresponding to this h₁, the iₖ, f, i, and np₁ values associated are 2, 1/4, 10 and 0.262 respectively.

From this one can obtain the sample size n=np₁ / p₁ = 0.262 / 0.02 = 13.1 = 13.

Thus the selected parameters for CSP-M SkSP plan are n = 13, iₖ = 2, f = 1 / 4 and i = 10.

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**Designing plans for Given (p₂, h₂)**

Column 5 and 8 of Table 3.3 is used to design plans for a given p₂ and h₂.

For example, for given p₂ = 0.3 and h₂ = 2.41, from Table 3.4 under column headed h₂, locate the value equal to or just greater than specified h₂, which is 2.418.

Corresponding to this h₂, the iₖ, f, i, and np₂ values associated are 2, 1/5, 6 and 2.316 respectively.

From this one can obtain the sample size n = np₂ / p₂ = 2.316 / 0.3 = 7.72 = 8.

Thus the selected parameters for CSP-M SkSP plan are n = 8, iₖ = 2, f = 1 / 5 and i = 6.

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**Designing plans for Given (p₀, h₀)**

Columns 6 and 9 of Table 3.3 can be used to design plan for a given p₀ and h₀.

For example, for given p₀ = 0.08 and h₀ = 0.9, from Table 3.4 under column headed h₀, locate the value equal to or just greater than specified h₀, which is 0.908.

Corresponding to this h₀, the iₖ, f, i, and np₀ values associated are 3, 1/3, 8 and 0.775 respectively.

From this one can obtain the sample size n = np₀ / p₀ = 0.775 / 0.08 = 9.6875 = 10.
Thus the selected parameters for CSP-M SkSP plan are \( n = 10, i_c = 3, f = \frac{1}{3} \) and \( i = 8 \).

**Designing plans through the Ratio of Relative slope \( h_2 / h_1 \)**

Column 4 and 10 of Table 3.3 is used to design plans the ratio of relative slopes \( h_2/h_1 \).

For example, for given \( p_1 = 0.02 \) and the ratio of \( h_2/h_1 = 21.71 \).

By using Table 3.4, under column headed \( h_2/h_1 \) one can locate the value equal to or just greater than desired ratio, which are 21.722.

Corresponding to this located value, one can find \( i_c, f, i, \) and \( np_1 \) values as 3, 1/3, 4 and 0.227 respectively.

From this one can obtain the sample size \( n = np_1 / p_1 = 0.227 / 0.02 = 11.35 = 11 \).

Thus the selected parameters for the plan are \( n = 11, i_c = 3, f = 1/3 \) and \( i = 4 \).

**CONVERSION OF PARAMETERS**

Now converting a given set of parameters to the other set of equivalent (resulting plans having nearly identical OC curves) parameters will be discussed through an example.

For example \( p_1 = 0.03, p_0 = 0.09, p_0 / p_1 = 0.09 / 0.03 = 3 \), which corresponds to the table value of 3.036 from Table 3.2. Now, \( n = np_1 / p_1 = 0.278 / 0.03 = 9.267 = 9 \). The parameters of CSP-M SkSP plan satisfying the requirements are found from Table 3.2 as \( n = 9, i_c = 2, f = 1/5, i = 10 \).

The other measure can be calculated from Table 3.2 and Table 3.3 as

\[
\begin{align*}
p_2 &= np_2 / n = 2.315 / 9 = 0.257 \\
p_0 &= np_0 / n = 0.930 / 9 = 0.103 \\
p_L &= np_L / n = 0.421 / 9 = 0.047 \\
h_0 &= 0.965 \\
h_1 &= 0.150 \\
h_2 &= 2.419
\end{align*}
\]
The other similar sets of parameters can be specified as

\[(AQL, IQL) = (0.03, 0.09)\]
\[(AQL, AOQL) = (0.03, 0.047)\]
\[(p_0, h_0) = (0.09, 0.965)\]
\[(p_1, h_1) = (0.03, 0.15)\]
\[(p_2, h_2) = (0.257, 2.419)\]

**CONSTRUCTION OF TABLES 3.2 AND 3.3**

The OC function of the CSP-M SkSP –plan is given by

\[P_a = (1 - F) + F P\]

where \(P\) = probability of acceptance of the reference plan

and \(F\) = the probability that a lot is sampled

\[= \left[ \frac{f [A^2 + q^{2i} B + (C + 4 p) q^i]}{f A^2 + q^{2i} B + q^i (C + 4 f p)} \right]\]

where

\[p = 1 - P\]
\[q = 1 - p\]
\[A = 1 - q^i\]
\[B = 2 - q^i\]
\[C = 1 - 3q^i + q^{2i} + q^4\]

Here, \(P = e^{np} + (np) e^{np(ic + 1)}\),

which is the OC function for ChSP-1 reference plan having parameters \(n\) and \(i_c\), as derived by Dodge (1955a), under the conditions for application of Poisson model for OC curve.

For given values of \(i_c, f, i\) and \(P_a (p)\), equation (3.4) can be solved for \(np\) by the method of iterations. The entries of the columns \(np_1, np_2\) and \(np_0\) of Table 2.1 provides such \(np\) values with \(P_a (p) = 0.95, 0.10\) and 0.50 respectively for given \(i_c, f, i\) and \(P_a (p)\).

Assuming \(n_{AOQ} = np \ P_a (p)\), for given values of \(i_c, f\) and \(i\), the values of \(np_m\) that maximizes \(n_{AOQ}\), can be obtained from equation (3.4) and are tabulated. Values of \(n_{AOQL}\) are obtained using \(np_{L_1} = np_m \ P_a (p_m)\).

The entries of column \(h_1, h_2\) and \(h_0\) are calculated through the expression

\[h = -(p / P_a (p)) \ (d P_a (p) / d p), \text{ for } p = p_1, p_2 \text{ and } p_0.\]

The remaining columns of the tables of 3.2 and 3.3 are the ratios of corresponding parametric values.