CHAPTER-VIII

GENERATION OF NANOSCALE BESSEL BEAM USING LENS AND CUBIC PHASE PLATE
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8.1 Extending Depth of Focus using cubic phase plate

A wavefront coding microscope is a relatively simple modification of a modern microscope. A system overview is shown in Figure (8.1). The key optical element in a wavefront coding system is the wave plate. This is a transparent molded plastic disc with a precise aspheric height variation. Placing the waveplate in the back focal plane of a lens introduces a phase aberration designed to create invariance in the optical system against some chosen imaging parameter.

![Diagram of wavefront coding microscope system](image)

Figure (8.1): An overview of a wavefront coding microscope system. The image-forming light from the object passes through the objective lens and phase plate and produces an intermediate encoded image on the CCD camera. This blurred image is then digitally filtered (decoded) to produce the extended depth of field result.
A cubic phase function on the waveplate is useful for microscopy, as it makes the low aperture optical transfer function (OTF) insensitive to defocus. While the optical image produced is quite blurry, it is uniformly blurred over a large range along the optical axis through the specimen Figure (2.3). From this blurred intermediate image, we can digitally reconstruct a sharp an extended depth of field (EDF) image, using a measured PSF of the system and a single step deconvolution. The waveplate and digital filter are chosen to match a particular objective lens and imaging mode, with the digital filter further calibrated by the measured PSF. Once these steps are carried out, wavefront coding works well for any typical specimen. The EDF behaviour relies on modifying the light collection optics only, which is why it can be used in other imaging systems such as photographic cameras, without needing precise control over the illumination light. In epi-fluorescence both the illumination light and the fluorescent light pass through the waveplate. The CPM provides a beneficial effect on the illumination side, by spreading out the axial range of stimulation in the specimen, which will improve the SNR for planes away from best focus.
8.2 Derivation of the Cubic Phase Function

The rectangular nature of the cubic phase mask function

\[ \varphi(m,n) = A(m^3 + n^3) \]  

(8.1)

There are various methods that may be used to derive a pupil phase function which has the desired characteristics for EDF imaging. The general form of a phase function in Cartesian co-ordinates is

\[ T(m,n) = \exp[ik\varphi(m,n)] \]

(8.2)

Where \( m, n \) are the lateral pupil co-ordinates and \( k = 2\pi/\lambda \) is the wave-number. The cubic phase function was found by Dowski and Cathey [100] using paraxial optics theory by assuming the desired phase function is a simple 1D function of the form

\[ \varphi(m) = Am^\gamma \]

(8.3)

By searching for the values of \( A \) and \( \gamma \) which give an OTF which does not change through focus, they found, using the stationary phase approximation and the ambiguity function, that the best solution was for \( A = 20/k \) and \( \gamma = 3 \). Multiplying out to 2D, this gives the cubic phase function in (8.1).

8.2.1 Paraxial Model

Using the Fraunhofer approximation, as suitable for low NA, we can write down a 1D pupil transmission function encompassing the effects of cubic phase (8.1) and defocus,

\[ T(m) = \exp[ik\varphi(m)]\exp(im^3\varphi) \]

(8.4)
where $\psi$ is a defocus parameter. We then find the 1D PSF is

$$E(x) = \int_{-1}^{1} T(m) \exp(ixm) \, dm$$  \hspace{1cm} (8.5)$$

where $x$ is the lateral co-ordinate in the PSF. The 1D OTF is

$$c(m) = \int_{-1}^{1} T\left(m' + \frac{m}{2}\right) T^*\left(m' - \frac{m}{2}\right) \, dm'$$  \hspace{1cm} (8.6)$$

The 2D PSF is simply $E(x)E(y)$. Naturally this 1D CPM gives behaviour in which, in low aperture systems at least, the lateral $x$ and $y$ imaging axes are independent of each other. This gives significant speed boosts in digital post-processing. This paraxial model for the cubic phase mask has been thoroughly verified experimentally for low NA systems [99 &100].

8.3 Extending the depth of focus of the Nanoscale Beam using Symmetrically Cubic Phase Plate

Optical elements that have long focal depth as well as high lateral resolution are needed for a variety of applications, including precision alignment and profile measurements. Overcoming the limits imposed by diffraction in imaging systems has long been a topic of much interest because of its potential use in above mentioned applications. Considerable amount of effort is already dedicated to the design of well-corrected objective lenses with high numerical aperture to obtain an optical stylus with cross-sectional dimensions in nanoscale.
However, such lenses present a strong sensitivity to aberrations and misalignment. Near-field techniques that enhance localized surface plasmons are also potential candidates to obtain intense optical spots beyond the diffraction limit. The Optical super resolution Techniques [101 & 102] offer alternative means to reduce the focused spot size with respect to the diffraction limit embodied in the Airy disk pattern by changing the properties of the focused beam distribution.

Recently Axicons, the optical elements that produce focal segments within a specified range, have attracted considerable interest because of their unusual properties and versatility in practical applications [103-108]. In diffraction theory, the focal segment of Axicon is characterized by non-diffraction Bessel beams [109]. In application such as multi layer optical recording, nanoscale resolution and delocalisation of uniform axial intensity along the element axial within the specified range is very important. It has been proved that a cubic phase plate has some special characteristics for correcting chromatic aberration and extending the depth of the field [110 & 111]. In this work, an axicon, which is a combined symmetrically Cubic lens phase plate with a perfect lens, is introduced to achieve nanoscale resolution and to delocalize the uniform axial intensity. Based on the stationary phase method and numerical calculation, the intensity distribution along the diffracted field behind the axicon is evaluated. It is shown that near- Bessel beams can be obtained within the desired range and the axial intensity distribution is more uniform. The spot size of the shaped beam, the intensity ratio of the shaped beam side lobe to the central-lobe peak, and the depth of focus are
discussed. It is well known that a laser operating in a TEM$_{00}$ mode delivers a Gaussian beam. However, in studying the imaging Characteristics of the axicon, the illuminating light is usually assumed to be a plane wave. However a Gaussian beam affects the imaging characteristics of the axicon is less investigated. In this work effect of Gaussian beam radius in the intensity profiles of the Axicons is also investigated.

8.4 Uniform Intensity Axicon

![Diagram](image)

Figure (8.2): Illustration of the geometry of the lens coded with symmetrical cubic phase plate.

One can generate the sharper beam effectively by converting an incident Gaussian beam using a specially designed phase plate before the lens focusing. A schematic of the beam shaping is depicted in Figure (8.2). A Gaussian beam from the laser is transmitted through a special cubical phase plate (F) at plane I and focusing lens L and is diffracted into the required shaped beam intensity distribution at focal plane II of the lens. It is assumed that the waist of the Gaussian beam is located just at
the phase plate, denoted as the $z = 0$ plane. Thus the optical field in front of the plate is given by

$$U_0(\rho) = A_0 \exp(-\rho^2 / \omega^2)$$  \hspace{1cm} (8.7)

Where $A_0$ is the constant amplitude factor, $\rho$ is radial coordinate, and $\omega$ is the beam radius, defined as the distance from the beam center to the point where the normal irradiance equals $\exp(-2) = 0.135$. If $\alpha$ be the amount of optical path difference produced by the phase plate then the transmittance of the symmetrically cubic phase plate is given by

$$T(\rho) = \begin{cases} \exp(ik\alpha \rho^3)R_1 < \rho < R_2 \\ = 0 \text{ otherwise.} \end{cases}$$  \hspace{1cm} (8.8)

Where, $R_1$ and $R_2$ are the inner and outer radii of the phase plate. According to the Fresnel diffraction theory, the field intensity of the beam at a plane behind the lens can be determined as

$$I(r, z) = \frac{k^2 A_0^2}{z^2} \left| \int_{R_1}^{R_2} J_0 \left( \frac{kr \rho}{z} \right) \exp\left(-\rho^2 / \omega^2\right) \exp\{ik\phi(p, z)\} \rho d\rho \right|^2$$  \hspace{1cm} (8.9)

Where

$$\phi(\rho, z) = \alpha \rho^3 - \left( \frac{1}{2f} - \frac{1}{2z} \right) \rho^2$$  \hspace{1cm} (8.10)

And $r$ is the transverse coordinate at the $z$ plane. Applying the stationary phase method the approximation solution of equation (8.9) is given by:

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\[ l(r,z) \approx \frac{k^2 A_0}{z^2} \left[ \frac{2\pi}{k\phi^{(2)}(\rho,z)} \right]^{\frac{1}{2}} \rho_c j_0 \left( \frac{kr \rho_c}{z} \right) \exp \left( -\rho^2 / \omega^2 \right) \exp \left[ i k \phi(\rho_c,z) \right] \exp \left[ i \left( \frac{\pi}{4} \right) \right] \]

\[ \rho_c \in (R_1, R_2). \]  

(8.11)

Where \( \phi^{(2)}(\rho,z) \) denotes the second partial derivative of \( \phi(\rho,z) \), and the stationary phase point \( \rho_c \) is determined by

\[ \frac{\partial \phi(\rho,z)}{\partial \rho} = 0 \]  

(8.12)

On substituting Equation (8.10) in to Equation (8.12), we have

\[ \rho_c = \frac{z - f}{3\alpha f z} \]  

(8.13)

In order to achieve the desired range of focal segment, the focal length \( f(\rho) \) in the presence of symmetrically cubic phase plate is given by

\[ \frac{1}{f(\rho)} = \frac{1}{f} - 3\alpha \rho \]  

(8.14)

Therefore the focal segment parameters are given by

\[ d_{1,2} = \frac{1}{1 - 3\alpha f R_{1,2}} \]  

(8.15)

Hence the particulars values \( \alpha \) and \( f \) are determined by \( d_{1,2} \) and \( R_{1,2} \) i.e.,

\[ \alpha = \frac{d_2 - d_1}{3d_1 d_2 (R_2 - R_1)} \]  

(8.16)

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$f = \frac{d_i}{1 + \frac{d_2 - d_i}{d_2} \frac{R_i}{R_2 - R_i}}$  \hspace{1cm} (8.17)

The main difference between the stationary-phase solution and the more exact solution from the Fresnel diffraction integral is that, within the focal segment, the exact solution exhibits rapid oscillation around the average smooth focal segment, and the stationary-phase solution is just the average itself.

**8.5 Results and Discussion**

The characteristics of the axicon used are analyzed by means of numerical calculation. To achieve a more exact solution, the numerical calculation on the diffraction integral shown in the equation (8.9), describing the diffractive field is performed. The parameters of the focal segment are chosen as $d_1 = 5$ mm, $d_2 = 6$ mm, $R_1 = 0.3$ mm, and $R_2 = 0.5$ mm, which result in $\alpha = 0.11 \times 10^{-4}$ mm$^{-2}$, and $f = 4.00$ mm. The axial intensity distribution of the focal segment generated by the axicon corresponding to these values of $\alpha$ and $f$ with different values of $R_1, R_2$ and beam radius ($\omega$) is shown in Figure (8.3, 8.4& 8.6). Figure 8.3 shows that the axial intensity distributions decays rapidly when the system is illuminated with the Gaussian beam of beam radius $\omega = 0.4$ mm. However, by Gaussian beam illumination of beam radius $\omega = 0.6$ mm, uniform-axial-intensity distribution is obtained at different focal segments and is shown in figure 8.4. It is noted that further increasing the beam radius results in oscillatory intensity and is shown in
From the Figure 8.3 & 8.5 it is observed that by illuminating the system with a Gaussian beam of beam radius \( \omega = 4 \text{ mm and 8 mm} \) results in decaying and oscillating on axial intensity for the parameters mentioned in the Figure. However for the same parameters and by illuminating the system with the Gaussian beam of beam radius \( \omega = 0.6 \text{ mm} \) results in constant on axial segments localized in the range of 5 mm-6mm, 6 mm-7 mm & 7 mm-8 mm along the focal axis corresponding to different values of \( R_1 \) and \( R_2 \) as shown in Figure(8.4). Thus by changing the inner and outer radii of the phase plate and by illuminating with a beam of suitable radius, it is possible to localize the focal segments of constant on axial intensity to desired ranges. Based on equation (8.9), we can also calculate the transverse intensity distribution along the focal segments. The transverse intensity distribution corresponding to \( \alpha = 0.111 \times 10^{-4} \text{ mm}^{-2}, f = 4.00 \text{ mm} \) and \( \omega = 0.6 \text{ mm} \) at different axial distance \( z \) is presented in Figures (8.6-8.8). The transverse profile shows that the FWHM value of the calculated spot size is around 800nm for \( R_1=0.25 \text{ mm} \) & \( R_2 = 0.4 \text{ mm} \) and is shown in Fig. (8.6). It is also observed from the Figures (8.6-8.8) that the calculated spot size remains constant in the range of 5 mm-6 mm along the axial segment and the ratio of the first side lobe peak to the central-lobe peak of the beam increases from 16.2% to 16.8% when the axial distance increases from 5mm to 6mm. It is found that by changing \( R_1=0.3 \text{ mm} \) & \( R_2 = 0.5 \text{ mm} \), and by keeping the other parameters \( (\alpha & f) \) constant, we can obtain a spot having FWHM around 900 nm and which is found to remains constant in the range of 6mm – 7mm along the axial segment. The ratio of the first side lobe
peak to the central-lobe peak of the beam is observed to be increases from 17.1% to 17.62% when the axial distance increases from 6mm to 7mm. When changing $R_1 = .2 \text{ mm} \& R_2 = .5 \text{ mm}$ the FWHM value of the spot size further increases to 920nm and found to remain constant in the range 7mm-8mm along the axial segment. The ratio of the first side lobe peak to the central-lobe peak of the beam is observed to be increases from 17.81% to 18.762% when the axial distance increases from 7mm to 8mm. In multilayer optical recording applications, this feasibility of localizing the constant on axial intensity of the nanoscale Bessel beam at a specific depth within its range avoids the mechanical movement of lens (i.e., by using a variable ring aperture mask, which could be implemented using a liquid crystal shutter) in the conventional system and will increase memory access speed by a factor of $\sim 50$ over that of the current systems, i.e., several hundred Mbytes/second in high density/speed optical recording.

The side lobe may cause pit writing in recording. In the optical recording, however, a recording medium has the threshold characteristics for recording power. The recording power for a practical medium is usually set $\sim 2 - 3$ times as high as the threshold power to assure stable recording. Therefore, to avoid recording error, the allowable side lobe intensity must be $\leq 30\%$ of the main lobe intensity. On the other hand, the readout signal may degrade due to side lobe influence. Since the maximum side lobe intensity is only 18.760% of the main lobe intensity corresponding to $\omega = 0.6 \text{ mm}$, the recording and reading errors due to side lobe can be effectively avoided in this system.
In conclusion, an Axicon, which is a combined symmetrically Cubic lens phase plate with a perfect lens, is introduced to achieve nanoscale resolution and to delocalize the uniform axial intensity. Both the stationary phase method and numerical calculation are applied to solve the diffraction integral describing the characteristics of the focal segment. The results show that this kind of axicon can generate a focal segment with a uniform axial intensity, as expected. By fixing the $\alpha$ and $f'$ values and by changing the inner and outer radii of the phase plate, it is possible to delocalize the constant on axial intensity segments of the generated Bessel beam in to a desired ranges. It is also observed that the spot size is in nanoscale and remains constant to certain range in the localized constant on axial segments. The large Depth of Focus (DOF) of axicon lens avoids the mechanical damage of the lens surface due to its friction with the spinning disk and helps to integrate other components easily with the system and more over this system totally eliminates the requirement for precise z adjustment of the read/write focused beams in conventional systems, which is a major factor limiting the speed as well as the resolution ($\sim1\ \mu m$) of these systems. The possibility of localization of the constant on axial segments helps for multi layer optical recording to increase the storage capacity to several gigabytes. Thus proposed axicon based system can achieve High speed/ high density optical storage.
Figure (8.3): Axial intensity distribution at the diffraction field when Gaussian beam of beam radius $a=0.4\,\text{mm}$ is focused by a lens coded with a symmetrically cubic phase plate. The parameters are $\alpha = 0.111 \times 10^{-4}\,\text{mm}^{-2}$ and $f=4.0\,\text{mm}$. 
Figure (8.4): Axial intensity distribution at the diffraction field when Gaussian beam of beam radius $\omega=0.6 \text{ mm}$ is focused by a lens coded with a symmetrically cubic phase plate. The parameters are $\alpha = 0.111 \times 10^{-3} \text{ mm}^{-2}$ and $f=4.0 \text{ mm}$. 
Figure (8.5): Axial intensity distribution at the diffraction field when Gaussian beam of beam radius $\omega = 0.8\text{mm}$ is focused by a lens coded with a symmetrically cubic phase plate. The parameters are $\alpha = 0.111 \times 10^{-2} \text{mm}^{-2}$ and $f = 4.00 \text{ mm}$. 
Figure (8.6): Transverse intensity distribution at the diffraction field when Gaussian beam of beam radius $\omega = 0.6 \text{ mm}$ is focused by a lens coded with a symmetrically cubic phase plate. The parameters are $a = 0.111 \times 10^{-4} \text{ mm}^2$, and $f = 4.00 \text{ mm}$. 
Figure (8.7): Transverse intensity distribution at the diffraction field when Gaussian beam of beam radius $\omega=0.6$ mm is focused by a lens coded with a symmetrically cubic phase plate. The parameters are $\alpha = 0.111\times10^{-4}$ mm$^2$, and $f = 4.00$ mm.
Figure (8.8): Transverse intensity distribution at the diffraction field when Gaussian beam of beam radius \( \omega = 0.6 \text{ mm} \) is focused by a lens coded with a symmetrically cubic phase plate. The parameters are \( \alpha = 0.111 \times 10^{-4} \text{ mm}^{-2} \), and \( f = 4.00 \text{ mm} \).