CHAPTER-3

MHD FLOW AND HEAT TRANSFER OF THE HYDRODYNAMIC
SLIP OVER A LINEAR VERTICALLY STRETCHING SHEET.
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3.1 INTRODUCTION

The study of flow over a stretching sheet has generated much interest in recent years in view of its numerous industrial applications such as the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film, condensation process of metallic plate in a cooling bath and glass, and also in polymer industries. Since the pioneering work of Sakiadis [1961] who studied the moving plate flow problem, wherein various aspects of the problem have been investigated by many authors such as Cortell[2008], Xu and Liao [2005], Hayat et al. [2008] etc.

The study of two-dimensional boundary layer flow, heat and mass transfer over a porous stretching surface is very important as it finds many practical applications in different areas. To be more specific, it may be pointed out that many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing these strips, are sometimes stretched. Viscous dissipation changes the temperature distributions by playing a role like an energy source, which leads to affect heat transfer rates. The merit of the effect of viscous dissipation depends on whether the sheet is being cooled or heated. Apart from the viscous dissipation, the Joules dissipation also acts as a volumetric heat source. Heat transfer analysis over porous surface is of much practical interest due to its abundant applications. To be more specific, heat-treated materials travelling between a feed roll and wind-up roll or materials manufactured by extrusion, glass-fiber and paper production, cooling of metallic sheets or electronic chips, crystal growing are a few practical
applications of flow over a stretching sheet. In all these cases, the final product of desired characteristics depends on the rate of cooling and also the rate of stretching. In view of all these aspects, the present work deals with the effect of viscous and Joules dissipation on MHD flow, heat and mass transfer over a porous sheet, with partial slip. Researches in these fields have been conducted by many investigators. For example, analytical results were carried out by Vajravelu and Hadjinicolaou [2006] who took into account the effects of viscous dissipation and internal heat generation. An analysis of thermal boundary layer in an electrically conducting fluid over a linearly stretching sheet in the presence of a constant transverse magnetic field with suction or blowing at the sheet was carried out by Chiam [1977].

Very recently, the viscous and joules dissipation and internal heat generation was taken into account in the energy equation. Sajid et al. [2007] investigated the non-similar analytic solution for MHD flow and heat transfer in a third-grade fluid over a stretching sheet. He found that the skin friction coefficient decreases as the magnetic parameter or the third grade parameter increases. A mathematical analysis has been carried out on momentum and heat transfer characteristics in an incompressible, electrically conducting viscoelastic boundary layer fluid flow over a linear stretching sheet by Abel et al. [2008].

A numerical reinvestigation of MHD boundary layer flow over a heated stretching sheet with variable viscosity has been analyzed by Pantokratoras [2008]. Ishak et al. [2006] studied mixed convection boundary layers in the stagnation-point flow of an incompressible viscous fluid over a stretching vertical sheet. Hossain and Takhar [1996] have investigated the radiation effect on mixed convection boundary layer flow of an optically dense viscous incompressible fluid along a vertical plate with uniform surface temperature
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The study of magneto hydrodynamics of conducting fluid finds applications in a variety of astrophysical and geophysical problems. The effects of magnetic field on the natural convection heat transfer have been discussed by Romig [1964], Elbashbeshy [1998], and considered heat transfer over a stretching surface with a variable surface heat flux. The convective heat transfer in an electrically conducting fluid at a stretching surface has been studied by Vajravelu and Hadjinicolau [1997]. Other studies dealing with hydro magnetic flows can be found in Grandet et al [1992] Takhar and Ram [1994], and Duwairi and Damseh [2004]. Crane [1970] obtained an exact solution of the two-dimensional boundary layer equations. After his pioneering work, the flow field over a stretching surface has drawn considerable attention and a good amount of literature has been generated on this problem.
Hence the present study investigates the effect of viscous and Joules dissipation on MHD flow over a vertically stretching sheet with viscous and joules dissipation with hydrodynamic/thermal slip.

3.2 MATHEMATICAL FORMULATION:

Two-dimensional, steady, MHD laminar boundary layer flow with heat transfer of a viscous, incompressible and electrically conducting fluid over a vertical stretching sheet with hydrodynamic/thermal slip, embedded in the presence of transverse magnetic field including viscous and Joules dissipation is considered for investigation. A uniform transverse magnetic field of strength $\frac{\sigma B^2}{\rho}$ is applied parallel to y-axis. Consider a stretching sheet that emerges out of a slit at $x = 0, y = 0$ and subsequently being stretched, as in a polymer extrusion process. Let us assume that the speed at a point on the plate is proportional to the power of its distance from the slit and the boundary layer approximations are applicable. In writing the following equations, it is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible.

Consider a steady, two-dimensional free convection flow adjacent to a stretching vertical sheet immersed in an incompressible electrically conducting viscous fluid of temperature $T$. The stretching velocity and the surface temperature are constants. Under these conditions, the governing boundary layer equations of momentum, energy with buoyancy, viscous and Joules dissipation, with hydrodynamic slip are
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(3.2.1)

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta (T - T_\infty) - \frac{\sigma B^2}{\rho} u
\]  
(3.2.2)

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\sigma B^2}{\rho c_p} \right) u^2,
\]  
(3.2.3)

And the boundary conditions are,

\[u(x, y) = L \frac{\partial u}{\partial y} + \alpha x, \quad v = 0, \quad T = T_\infty + \alpha_1 \left\{ \frac{x}{x_L} \right\}^2 + h \left( \frac{\partial T}{\partial y} \right) \text{ at } y=0\]

\[u \to 0, T \to T_\infty \text{ as } y \to \infty,\]  
(3.2.4)

Where \(u\) and \(v\) are the velocity components along the \(x\) and \(y\) axes, respectively. Further \(\mu\), \(\rho\), \(\alpha\), \(\beta\), \(T\) and \(g\) are the dynamic viscosity, fluid density, thermal diffusivity, thermal expansion coefficient, fluid temperature in the boundary layer, and acceleration due to gravity, respectively. Introducing the following similarity transformations,

\[\eta = \left( \frac{a}{v} \right)^{1/2} y, \quad u(x, y) = ax f' (\eta), \quad v(x, y) = -\sqrt{v a} f (\eta)\]  
(3.2.5)

\[\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}.\]  
(3.2.6)

Eqns (3.2.1) to (3.2.3) takes the following form of nonlinear ordinary differential equations.

\[f'' - f^2 - Gr \theta + Mf' = 0,\]  
(3.2.7)
\[ \dot{\theta} = P_f \dot{f} \theta - 2P_r \dot{f} \theta - Ec Pr \left( f^{-2} + Mf^{-2} \right) \]  

(3.2.8)

similarly the boundary conditions (3.2.4), becomes

\[ f(0) = f_w, \quad f'(0) = 1 + \gamma f'(0), \quad \theta(0) = 1 + K \theta'(0), \quad f'(\infty) = 0, \quad \theta(\infty) = 0. \]  

(3.2.9)

where,

\[ \kappa = \kappa_1 \sqrt{\frac{a}{v}} \text{ and } Gr_s = \frac{\varepsilon \beta (T_w - T_0)}{v^2}, \quad \gamma = l \sqrt{\frac{a}{v}} \]

3.3. NUMERICAL SOLUTION

The nonlinear boundary value problem represented by Eqs. (3.2.7) to (3.2.9) is solved numerically using Fourth-order Runge Kutta shooting technique. The system of non-linear ordinary differential Eqs. (3.2.7) and (3.2.8) together with the boundary conditions Eq. (3.2.9) are similar and are solved numerically by using the fourth order of Runge Kutta integration scheme accompanied with the Shooting scheme. Making an initial guess for the values of \( f''(0), \theta'(0) \) to initiate the shooting process is very crucial in this process. The success of the procedure depends very much on how good this guess is. Numerical solutions are obtained for several values of the physical parameters i.e magnetic parameter \( M \), Prandtl number \( Pr \), hydrodynamic slip parameter \( \gamma \), thermal slip parameter \( K \), Grashof number \( Gr_s \), and Eckert number (Ec).

We have chosen a step size \( h \) of the order 0.01 to satisfy the convergence criterion of in all cases. The maximum value value of 0.01 was found to each iteration loop. The maximum value of \( h \) to each group of parameter is determined when the value of the unknown boundary conditions at \( y=0 \) is not changed to successful loop with error less than \( 10^{-6} \).

3.4. RESULTS AND DISCUSSION
In order to gain physical insight of the problem, the effects of various governing parameters on velocity, and temperature profiles have been discussed by assigning numerical values to the parameter, encountered in the problem i.e., numerical calculations were carried out for different values of, magnetic parameter $M$, Grashof number $Gr$, Prandtl number $Pr$, Eckert number $Ec$, hydrodynamic slip parameter $\beta$, and their effects on flow and heat transfer characteristics are analyzed graphically.

The influences of the magnetic parameter on the longitudinal velocity profile is depicted in fig1. It can be seen that increasing magnetic parameter is to reduce the velocity distribution in the boundary layer which results in thinning of the boundary layer thickness, and hence induces an increase in the absolute value of the velocity gradient at the surface.

The influences of the thermal slip parameter $K$, on temperature profile is depicted in fig2. It can be seen that increasing, thermal slip parameter enhances temperature, in the thermal boundary layer region resulting in thickening of thermal boundary layer thickness.

An increase in Prandtl number $Pr$ is associated with a decrease in the temperature distribution which is displayed in Fig. 3, which is consistent with the fact that thermal boundary layer thickness decreases with increase in the values of prandtl number. The rate of heat transfer increases with the increasing values of Prandtl number. The boundary layer edge is reached faster as $Pr$ increases.

Dimensionless velocity profile is presented in fig4 for some different values of the hydrodynamic slip parameter $\gamma$. It is readily seen that it, has a substantial effect on the solutions. In fact, the amount of slip increases monotonically with from the no-slip solution for and towards full slip as $\gamma$ tends to infinity. The latter limiting case implies that the
frictional resistance between the viscous fluid and the surface is eliminated, and the stretching of the sheet does no longer impose any motion of the fluid.

In fig 5, the effects of on dimensionless longitudinal velocity is shown graphically and the effects of buoyancy force (Grashof number $G_r$) is found to be more pronounced for a fluid with a small $Pr$. Thus, fluid with smaller $Pr$ is more susceptible to buoyancy force effects.

The velocity and temperature profiles presented in figs. 1-5, show that the far field boundary conditions are satisfied asymptotically, which support the validity of the numerical results presented.

**GRAPHS:**
Fig. 1. The influences of the magnetic parameter Vs velocity profile
Fig. 2. The influence of thermal slip parameter on temperature profile
Fig. 3. The effect of Prandtl number $Pr$ Vs temperature distribution

Gr=0.05, $M=0.9$, $Ec=0.1$; $\gamma=0.8$
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Fig. 4. The effects of temperature profile for various values of Eckert number $Ec$

Fig-4
Similarity Variable $\eta$

$Ec=0.2, 0.3, 0.4, 0.5.$

$m=4.0,$
$Pr=1.5,$
$Gr=6.0$
$\gamma=2.0$
Fig. 5. The effects of dimensionless velocity and the buoyancy force (Grashof number $G_r$).