CHAPTER I

DEVELOPMENT AND SCOPE OF FLUID DYNAMICS
CHAPTER 1

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1.1 SHORT HISTORICAL REVIEW

Fluid dynamics is a branch of mechanics, which deals with the study of fluid in motion and the subsequent effect of the fluid on the boundaries, which may be either solid surface or other fluids. The essence of the subject of fluid flow is that of judicious compromise between theory and experiment. Since fluid dynamics is the branch of mechanics, its fundamental principles are based on Newton’s laws of motion, the indestructibility of matter and conservation of energy.

The earliest significant contribution to the field were undoubtedly made by Archimedes, who lived in Syracuse between the year 285-212 B.C., especially noteworthy was his analysis of the buoyancy of submerged bodies, which was applied successfully to the determination of the gold content of the crown of king Hiero I.

The next significant advancement came at the end of the 19th century when the science of fluid mechanics began to develop in two directions, which had practically no point in common. On the one side there was the science of theoretical hydrodynamics, which was evolved from Euler’s equation of motion for frictionless, non-viscous fluid and which achieved a high degree of completeness. Since, however, the result of the so-called classical science of hydrodynamics stood in glaring contradiction to experimental results- in particular as regard the very important problem of pressure loss in pipes and channels, as well as with regard to the drag of a body which moves through a mass of fluid, which had a little practical importance. For this reason, practical engineers
prompted by the need to solve important problems arising from the rapid progress in technology, developed their own highly empirical science of hydraulics.

At the beginning of 19th century L. Prandtl [1904] distinguished himself by showing how to unify these two divergent branches of fluid dynamics. He achieved a high degree of correlation between theory and experiment and paved the way to the remarkable successful development of fluid mechanics. It was already known that the great discrepancy between results in classical hydrodynamics and reality was, in many cases, due to neglecting the viscosity effects in the theory. Now the complete equations of motion of viscous flows (the Navier-Stockes equation) had been known for some time however due to the great difficulty of these equations, no approach had been found to the mathematical treatment of viscous flows. For technically important fluids such as water and air, the viscosity is very small, and thus the resulting viscous forces are small compared to the remaining forces (gravitational force, pressure force). For this reason it took a long time to see why the viscous forces ignored in the classical theory while these forces should have an important effect on the motion of the flow.

In 1904, L Prandtl presented his lecture “Über Flüssigkeitbewegung bei sehr kleiner Reibung,” (On fluid motion with very small friction) at the Heidelberg mathematical congress how it was possible to analyse viscous flow precisely in the case which had great practical importance. He introduced the “Concept of Boundary Layer” with the aid of this hypothesis. Prandtl succeeded in physical explanation of the theoretical analysis of viscous flows. This boundary layer theory proved extremely fruitful because it provided an effective tool for the development of fluid dynamics. The boundary layer is applicable in the skin friction drag, which acts on a body as it moves
through a fluid. For example the drag experienced by a flat plate at zero incidence, the
drag of ship or an aeroplane wing, aircraft nacelle or turbine blade.

Boundary layer theory gives an answer to the very important question of what
shape must a body be given in order to avoid the separation. Due to rapid increase in the
shape of flight of modern aircraft, boundary layer flows develop a “Thermal Boundary
Layer” and its existence plays an important role in the process of heat transfer between
the fluid and the solid body. The process of heat transfer and fluid flow plays a vital role
in engineering equipments. The same process also governs in the natural environment,
power production, the heating and air conditioning of building. Major segment of the
chemical and metallurgical industries use the component such as furnace, heat
exchangers, condenser and reactor where thermo fluid process works. The pollution of
natural environment is largely caused by heat and mass transfer and so are streams,
floods and fires. The process of heat transfer and fluid flow also be in use in the design of
electrical machinery and electronic circuits.

In world war 2nd, convective heat and mass transfer were largely empirical
sciences, and engineering design was accomplished almost exclusively by the use of
experimental data, generalized to some degree by dimensional analysis. During the past
two decades great strides have been made in developing analytic methods of convection
analysis, to the point where today experiment is assuming more its classical role of
testing the validity of theoretical models.

Among the tasks facing the engineering problems is the calculation of energy
transfer rate and mass transfer rate at the interface between phases in a fluid system. Most
often we are concerned with transfer at a solid-fluid interface where the fluid may be
visualized as moving relative to a stationary solid surface, but there are also important application where the interface is between a liquid and gas. If the fluids are everywhere at rest, the problem becomes one of either simple heat conduction where there are temperature gradients normal to interface or simple mass diffusion where there are mass concentration gradients normal to the surface. However, if there is fluid motion, energy and mass are transported both by potential gradients (as in simple conduction) and by movement of the fluid itself. This complex of transport process is usually referred to as convection. Thermal convection occurs in so many forms in nature and over such a wide range of scales that it could be claimed with some justification that convection represents the most common fluid flow in the Universe. Convection accomplishes the heat transport in stars wherever the radiative heat transfer is not sufficient enough. It warms the Earth's atmosphere by the upward transfer of heat absorbed at the ground. Convective motions are responsible for much of the mixing of water masses occurring in the oceans, and it is widely believed that thermal convection is the basic cause of most tectonic processes in the Earth's crust, including the phenomenon of continental drift. It is also likely that the geomagnetic field is produced by the dynamo action of convection flow in the liquid core of the Earth. Part of the fascination of the subject of convection stems from the fact that the motions in an evaporating puddle of water are described by essentially the same equations as the huge turbulent eddies visible on the surface of the Sun.

But convection is not confined to the natural environment. Wherever heat transfer must be considered in industrial applications thermal convection enters in various forms. In nuclear reactors, in crystallisation processes and in solar heating devices convection
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plays a crucial role and the rapidly expanding literature on applications of convection indicates a continuing demand for an improved understanding of its properties.

On a more fundamental level thermal convection has received attention as a particularly simple system in which the transition to turbulence can be studied. Experiments on turbulent convection have yielded remarkable and unexpected results which have led to new insights into the nature of turbulent fluid flow. These will be pointed out in various sections of this review. The relative simplicity of convection patterns even in the case of turbulent motion is also one of the reasons for the esthetic attraction of convection. The delightful experience of the visualisation of the spontaneously occurring cellular patterns of convection and their changes in time has always provided a strong and not often recognised motivation for the scientific research. The field of thermal convection and related flow phenomena has expanded rapidly in the past two decades and even a sizeable monograph could hardly do justice to the large body of scientific results. In this review our attention will be focused on those non-linear properties of convection in a horizontal layer heated from below which seem to have a general importance. No attempt is being made to give a complete review of particular topics. Because the literature on convection probably includes more than a thousand titles even if the numerous papers on applications are not counted, the references listed at the end represent only a tiny and sometimes arbitrarily selected fraction of the published work.
1.2 PHYSICAL CHARACTERISTIC OF THE FLUID STATE

The matter is usually divided into two classes, the solid and the fluid. These can be expressed in technical language by the statement that solids are supported by both normal and tangential internal stresses while a fluid can sustain only normal stresses which are equal at rest. The normal stress is termed as fluid pressure and the stress at any point occupying in a region $A \lim_{0} \left( \frac{F}{A} \right)$. It is true only when the medium is continuous. Therefore, it is essential to describe continuum hypothesis, which is given in the following title.

CONTINUUM HYPOTHESIS

It is well known fact that fluids are aggregation of molecules, widely spaced for a gas and closely spaced for a liquid. The distance between molecules is very large in comparison with the molecular diameter. The molecules are not fixed in a lattice but move freely relative to each other. These molecules are in constant random motion and collision. For example, in air at standard conditions there are $2 \times 10^{19}$ molecules per cubic centimeter with mean free path (distance between molecular collisions) of $6.35 \times 10^{-6}$ cm. When we consider the fluid to consist of discrete molecules moving randomly, the mathematical tools become inapplicable and thus increase the complexity of the problem. To avoid this difficulty we consider the fluid to be continuously distributed in a given space. Thus a fluid element can be subdivided indefinitely. This hypothesis of continuity is called “Continuum Hypothesis”. Under the hypothesis we consider that the volume of fluid particle is infinitely small compared to the whole volume occupied by the fluid and the fluid particle to be a material point and the density of the fluid to be continuous function of place and time. Thus, we define pressure at a point in continuum as,
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\[ P = \lim_{A \to A^*} \left( \frac{F}{A} \right) \]

where \( F \) is the force normal to the surface \( A \) and \( A^* \) is the smallest area surrounding the point in consistence with continuum approach.

Also, the density in a continuum is given by

\[ \rho = \lim_{V \to V^*} \left( \frac{M}{V} \right) \]

where \( M \) is the mass contained in volume \( V \) and \( V^* \) is the smallest volume surrounding the point in consistence with continuum approach.

**HOMOGENEITY**

The fluid properties are assumed to be the same in all parts of the system. A suspension, for example in which the particles are not uniformly distributed would violate this assumption.

**ISOTROPY**

A fluid is said to be isotropic with respect to some property (pressure density etc.) if that property is same in all directions at a point. A fluid is said to be anisotropic with respect to a property if that property is not the same in all directions.

**1.3 FLUID STATE**

The fluid state is commonly divided into liquid, gaseous and plasma. The study of former two states comes under fluid dynamics and the study of latter one comes under plasma dynamics. Again, the two corresponding branches of fluid dynamics are called hydrodynamics and aerodynamics, the former relating to water as well as other liquids and the latter two air and other gases.
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Another customary division of the subject depends on the practical importance of fluid friction. The “prefect fluids” are treated as if all the tangential stresses caused by friction were ignored. The “real fluids” refer to the cases in which friction is properly taken into account.

1.3.1 IDEAL FLUID OR INVISCID FLUID

“An Ideal fluid is one, which has no property other than density. No resistance is encountered when such a fluid flows” or Ideal fluids or Inviscid fluids are those fluids in which two contacting layers experience no tangential force (shearing stress) but act on each other with normal force (pressure) when the fluid is in motion. This is equivalent to stating that inviscid fluid offers no internal resistance to change in shape. The pressure at every point of an ideal fluid is equal in all directions, whether the fluid is at rest or in motion. Inviscid fluids are also known as prefect fluids or frictionless fluids. In true sense, no such fluid exists in nature. The assumption of ideal fluids helps in simplifying the mathematical analysis. However fluids which have low viscosities such as water and air can be treated as ideal fluids under certain conditions.

1.3.2 VISCOUS FLUID OR REAL FLUID

“Viscous fluid or real fluid are those, which have viscosity, surface tension and compressibility in addition to the density” or viscous fluid or real fluid are those when they are in motion the two contacting layers of those fluids experience tangential as well as normal stresses. This being also the case near solid wall wetted by a fluid. The property of exerting tangential or shearing stress and normal stress in a real fluid when the fluid is in motion is known as viscosity of the fluid. In viscous fluid internal friction plays an important role during the motion of the fluid. One of the important
characteristics of viscous fluid is that it offers internal resistance to motion of the fluid. Viscosity, being the characteristic of the real fluids, exhibits a certain resistance to alter the form also. Viscous or real fluids are classified into following two categories.

(i) Newtonian Fluid

(ii) Non Newtonian Fluid

(i) **NEWTONIAN FLUID**

To understand the concept of Newtonian fluid, let us consider a thin layer of fluid between two parallel plates at distance \( dy \) as shown in the figure.

![Newtonian Fluid Diagram](image)

Fig.1.3.1

Here one plate is fixed and a shearing force \( F \) is applied to the other. When conditions are steady the force \( F \) will be balanced by an internal force in the fluid due to its viscosity.

Newton, while discussing the properties of fluid, remarked that in a simple rectilinear motion of a fluid two neighbouring fluid layers, one moving over the other with some relative velocity, will experience a tangential force proportional to the relative
velocity between the two layers and inversely proportional to the distance between the layer, that is if the two neighbouring fluid layers are moving with velocities \( u \) and \( u + \Delta u \) are at a distance \( \delta y \), then, the shearing stress.

\[
\tau \propto \frac{\delta u}{\delta y} \quad \text{or} \quad \tau = \mu \frac{du}{dy}
\]  

(1.3.1)

This is called Newtonian hypothesis and a fluid satisfying this hypothesis is called a Newtonian fluid. It is clear from the Newton’s law that

(i) If \( \tau = 0 \) then \( \mu = 0 \), equation (1.3.1) will represent an ideal fluid.

(ii) If \( \frac{du}{dy} = 0 \) then \( \mu = \infty \), equation (1.3.1) will represent the elastic bodies.

(iii) A fluid for which the constant of proportionality \( \mu \) does not change with rate of deformation (shear strain \( \frac{du}{dy} \)) is said to be Newtonian fluid and graph \( \tau \) versus \( \frac{du}{dy} \) is a straight line.

Where \( \mu \), is known as Newtonian viscosity. It will be seen that \( \mu \) is the tangential force per unit area exerted on layers of fluid a unit distance apart and having a unit velocity difference between them.

The diagram relating to shear stress and rate of shear for Newtonian fluids represents flow curve of the type straight line.

(ii) **NON-NEWTONIAN FLUIDS**

Non-Newtonian fluids are those fluids which does not obey Newtonian law. It can also be stated as “the non-Newtonian fluids are those for which the flow curve is not
linear”, i.e. the ‘viscosity’ of a non-Newtonian fluid is not constant at a given temperature and pressure but depends on other factors such as the rate of shear in the fluid, the apparatus in which the fluid is contained or even on the previous history of the fluid.

1.4 CLASSIFICATION OF NON-NEWTONIAN FLUIDS

The non-Newtonian fluids for which the flow curves are not linear may be classified into three broad types.

(i) Time independent non-Newtonian Fluids are those fluids for which the rate of shear at any point is some function of the shearing stress at that point and depends on nothing else.

(ii) More complex system exists in which the relation between shear stress and shear rate depends on the time the fluid has been sheared or on its previous history during its motion. These complex systems of fluids are known as time dependent non-Newtonian fluids.

(iii) There exist some systems of fluids, which have characteristics of both solids and fluids and exhibit partial elastic recovery after deformation. These systems of fluids are known as visco-elastic fluids.

1.4.1 TIME INDEPENDENT NON-NEWTONIAN FLUIDS

Fluids of the first type whose properties are independent of time may be described by a rheological equation of the form

\[
\frac{du}{dy} = f(\tau).
\]

The above equation implies that the rate of shear at any point in the fluid is a simple function of the shear stress at that point. Such fluids are practically known as non-Newtonian viscous fluids.
These fluids may conveniently be subdivided into three distinct types depending on the nature of the function.

(i) Bingham Plastics

(ii) Pseudoplastic Fluids

(iii) Dilatant Fluids

The typical flow curves for these three fluids are shown in Fig.1.1 and compared with the linear relation typical of Newtonian fluids.

**BINGHAM PLASTICS**

A Bingham Plastic is characterised by a flow curve which is a straight line having an intercept on the shear-stress axis. The yield stress \( \tau_y \) is the stress, which must be exceeded before flow starts. The rheological equation for Bingham plastic may be written as

\[
\tau = \eta \dot{\gamma} + \tau_y
\]
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\[ \tau - \tau_y = \mu_p \frac{du}{dy}; \quad \tau > \tau_y. \]

Where \( \mu_p \), the plastic viscosity or coefficient of rigidity represented by the slope of flow curve.

**PSEUDOPLASTIC FLUIDS**

Pseudoplastic fluids show no yield value and the typical flow curve for these materials indicates that the ratio of shear stress to the rate of shear, which may be termed the apparent viscosity, \( \mu_a \) falls progressively with shear rate and the flow curve becomes linear only at very high rates of shear. This limiting slope is known as the viscosity at infinite shear and is designated \( \mu_\infty \).

An empirical functional relation known as the power law is widely used to characterize fluids of this type

\[ \tau = K \left( \frac{du}{dy} \right)^n \]

Here \( K \) and \( n \) are constants (\( n<1 \)) for the particular fluid. \( K \) is the measure of consistency of the fluid. Higher \( K \) means the fluid is more viscous. \( n \) is a measure of the degree of non–Newtonian behaviour.

**DILATANT FLUIDS**

Dilatant fluids show no yield stress they are similar to pseudoplastic in that they show no yield stress. However, but they differ from pseudoplastic in that the apparent viscosity for these materials increases with increasing rates of shear. The power law equation is again often applicable but in this case the index \( n \) is greater than unity.
This type of behaviour was first found in suspension of solids at high solid content by Osborne Reynolds. He suggested that when this concentrated suspension is at rest the voidage is at a minimum, and the liquid is only sufficient to fill these voids.

### 1.4.2 TIME DEPENDENT NON-NEWTONIAN FLUIDS

Many real fluids cannot be described by a simple rheological equation, which applies to fluids for which the relation between shear stress and shear rate is independent of time. The apparent viscosity of more complex fluids depends not only on the rate of shear but also on the time the shear has been applied. These fluids are termed as time dependent non-Newtonian fluids. These fluids may be subdivided into two classes.

(a) Thixotropic Fluids

(b) Rheopectic Fluids

This classification is being done on whether the shear stress decreases or increases with time when the fluid is sheared at a constant rate. For thixotropic fluids, the shear stress decrease with time as the fluid is sheared while for a rheopectic fluid, the shear stress increase with time as the fluid is sheared. An example of a thixotropic fluid is printer’s ink.

### 1.4.3 VISCO-ELASTIC FLUIDS

A visco-elastic material exhibits both elastic and viscous properties. The simplest visco-elastic fluid is one which is Newtonian in viscosity and obeys Hooke’s law for the elastic part, giving the constitutive equation:

\[ \dot{\gamma} = \frac{\tau}{\mu} + \frac{\dot{\tau}}{\dot{\lambda}} \]

where \( \lambda \) is a rigidity of modulus.
Many common fluids are Non-Newtonian e.g. paints, enamels, varnish, wet clay and mud, solutions of various polymers, suspensions of particles, emulsions of oil in water etc.

1.5 SIGNIFICANCE OF THE PROBLEM

The development of numerical techniques to accurately approximate the flow of polymer melts in channels with moving boundaries is of paramount importance in polymer processing. In polymer processing, such a flow situation occurs in injection moulding, extrusion and simultaneous injection/compression moulding. In the case of injection moulding, such a flow situation occurs in the non-return valve located on the front of the machine screw. During the injection stage of the moulding process, the valve is required to close to stop the flow of polymer melt back into the screw region. Typically, a ring, ball or piston is utilized to close the flow passage into the screw to facilitate this shut-off. In extrusion, varying the geometry of the die by using choker bars or deformable lips allows the control of melt flow to obtain products according to desired specifications. In simultaneous injection/compression moulding, polymer enters a mould and is compressed by a moving boundary that is perpendicular to the flow direction. In all of the examples, the boundary moves perpendicular to the dominant flow direction.

Current research conducted in laboratory has focused on the simulation of the non-return valve during the moulding process. Figure 3 shows a typical non-return valve used in an injection moulding machine. This valve has a cylindrical ring which closes during the injection stage to close the passage into the screw. This closure stops polymer melt from flowing back into the screw. During the recovery stage, this ring opens to
allow melt to accumulate in front of the valve/screw assembly. This melt will be injected into the mould during the next injection step.

**Fig 3.** Sketch of typical ring type non-return valve for the injection moulding of thermoplastics.

A plot of the pressure traces in front of the valve on the barrel wall (downstream) and behind the valve (upstream) in the screw section and screw displacement during the injection and recovery stages. At the start of injection both pressures increase rapidly. The upstream pressure reaches a steady value, while the downstream pressure in the screw metering section decreases drastically. This decrease in pressure indicates the ring closing the flow passage into the screw. Thus, the valve is closed. The closing time of the valve can then be determined. For the recovery cycle, an oscillating pressure is observed in the upstream pressure transducer. This is caused by the movement of the flights of the rotating screw over the stationary pressure transducer. The downstream pressure measurement has a steady pressure due to the accumulation of polymer melt in the large reservoir (shot size) in front of the valve for the next injection stage. The pressure drop across the valve/screw determines the amount of resistance during the recovery stage, which affects the length of the recovery time. The ability to simulate this process of the valve opening during the recovery stage and closing during the injection stage will
determine the time for the valve to open, close and the forces acting on the ring, which will help in the development of more efficient valves.

In order to understand the phenomena that takes place during the polymer processing with moving boundaries, an experimental slit die with a moving boundary has been designed (Figure 4). The polymer flows through the slit die as a wall closes the flow passage.

![Fig 4. Sketch of slit die with moving boundary.](image)

### 1.6 FLOW INDUCED BY A STRETCHING SHEET

The flow generated due to the stretching of an elastic sheet which moves in its plane with a velocity varying with the distance from a fixed point due to the application of a stress known as the flow due to the stretching sheet. The flow of an incompressible viscous fluid over a stretching surface has important applications in polymer industry. A number of technical process containing polymers, such as extrusion of a polymer sheet from a dye of these sheets, the melt issues from a slit is subsequently stretched to achieve the desired thickness. The achievement of the desired characteristics of final product depends on the stretching rate, the rate of cooling in the process and the process of stretching. In view of these applications Sakiadis[1961] initiated the study of boundary layer flow over a continuous solid surface moving with constant speed and presented its numerical results. Due to the entrainment of the fluid, this boundary layer flow is quite
different from layer flow over a semi infinite flat plate. In 1970, Crane studied the two
dimensional laminar flow of an incompressible viscous fluid over a stretching sheet.
Crane’s problem is one of the rare problems in fluid dynamics that admits an exact closed
form solution. Not surprisingly it has been extended in various ways to include many
other important physical features such as suction or blowing, magnetic field, heat transfer
analysis etc.

1.6.1 TYPES OF FLOWS

STEADY AND UNSTEADY FLOWS

A flow in which the various parameters like velocity, pressure and density at any
point do not change with time is said to be a steady flow. For steady flow if \( u \) is the
velocity at a point then

\[
\frac{\partial u}{\partial t} = 0
\]

(1.5.1)

A flow in which these parameter depend on time is called unsteady flow.

LAMINAR AND TURBULENT FLOW

A flow in which each fluid particle traces out a definite curve and curves traced
out by any two different particle do not intersect, is said to be laminar. On the other hand,
a flow, in which each fluid particle does not trace out a definite curve and the curves traced out by fluid particle intersect, is said to be turbulent flow. The most of the flows, which occur in practical applications, are turbulent, and this term denotes a motion in which an irregular fluctuation (mixing, or eddying motion) is superimposed on the main stream.

**COMPRESSIBLE AND INCOMPRESSIBLE FLOW**

It is common practice to divide flows into two groups. Gases are compressible and their density changes with temperature and pressure. On the other hand, liquids are rather difficult to compress and for all practical purposes these may be considered as incompressible fluids.

**1.7 MAGNETOHYDRODYNAMICS**

When a conductor moves in magnetic field a current is induced in the conductor in a direction mutually at right angles to both the field and the direction of motion. Conversely when a conductor currying an electric current moves in a magnetic field it experience a force tending to move it at right angles to the electric field. These two statement first enunciated by Faraday

Electromagnetic forces will be generated which may be of the same order of magnitude as the hydrodynamical and inertial forces in the case when the conductor is either a liquid or a gas. Thus the equation of motion will have to take these electromagnetic forces into account as well as the other forces. The science that treats these phenomena is called magnetohydrodynamics (MHD). Other variants of nomenclature are hydromagnetics, magneto-fluid dynamics, magneto-gas dynamics etc.

As we know that MHD is relatively new but important branch of fluid dynamics. It is
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cconcerned with the interaction of electrically conducting fluid and electro magnetic fields, such interaction occurs both in nature and in new man-made device.

MHD flow occurs in the sun, the earth’s interior, the ionosphere, and the stars and atmosphere, to mention a few. Engineering level experiments have been made for electric power generation by passing an ionized gas between the poles of a strong electromagnet so that an electric current would be generated at right angles to the magnetic field and to the direction of flow of the plasma, the current being collected by two spaced electrodes at right angles to the direction of the current flow. At the present time MHD generators are not a practical possibility owing to the difficulties of producing suitably efficient and stable plasmas and sufficiently refractory material to withstand the high temperatures of the plasmas.

Both plasma and conducting fluids are related in common theory by assuming plasma as a continuous fluid for which the kinetic theory of gases still holds true. In MHD induced electric current produces mechanical force, which in turn modifies the motion of the fluid. Hence study of electrically conducting fluid flow in the presence of traverse magnetic field assures significance.

LITERATURE REVIEW

The flow of an electrically conducting fluid caused solely by stretching of an elastic sheet in presence of a uniform transverse magnetic field was considered by Pavlov in [1974] and obtained a similarity solution of this problem, later Chakrabarati and Gupta [1979] extended Pavlov work to study temperature distribution in MHD boundary layer flow in the presence of uniform suction. Soundalgekar and Takhar [1980] have discussed the effects of physical parameters on flow and heat transfer characteristics by considering the
effects of uniform transverse magnetic field on forced and free convection flow past a semi-infinite plate taking into account of viscous dissipation and stress work. Rapits and Tazivanides [1983] carried out analytical investigations on free convective flow past an infinite vertical surface when the fluid is electrically conducting in presence of an external transverse uniform magnetic field. Hydromagnetic flow of Newtonian fluid and heat transfer over continuous moving flat surface with uniform suction has been studied by Vajravelu and Nayfeh [1993] Mahesh Kumari et al. [1990] studied the effects of induced magnetic field and source/sink on flow and heat transfer characteristics over a stretching surface. Andersson [1992] considered in his work MHD flow of a visco-elastic fluid past a stretching surface and contribute some important information of MHD. Gorla et al. [1993] investigated the effects of magnetic field strength on mixed convective flow arising from an infinitely long horizontal line source of heat when the ambient fluid considered was a non-Newtonian power-law fluid having moderately large values of Grashof number. Na and Pop [1996] investigated the boundary layer flow over a moving continuous flat plate in an electrically conducting ambient fluid with a step change in applied magnetic field. The governing equations were solved numerically using the Keller-box method. The results showed the decrease of the skin friction parameter with the increase of the magnetic parameter. Elbashbeshy [1997] investigated heat and mass transfer phenomena along a vertical plate under the combined buoyancy effects of thermal and species diffusion in presence of magnetic field. Vajravelu and Hadjinicolaou [1997] carried out the investigations of free convection and internal heat generation on flow and heat transfer characteristics in an electrically conducting fluid near an isothermal stretching sheet. Chiam [1997] presented an analytical solution of the energy
equations for a boundary layer flow of an electrically conducting fluid under the influence of transverse magnetic field over a linearly stretching non-isothermal flat sheet. Ali Chamkha [1997] obtained the similarity solutions of laminar boundary layer equations describing the steady hydromagnetic two-dimension flow and heat transfer in a stationary electrically conducting and heat generating fluid driven by a continuous moving porous surface immersed in a fluid saturated porous medium. Elbashbesgy [2000] studied the flow of a viscous incompressible fluid along a heated vertical plate, taking into account the variation of viscosity and thermal diffusion with temperature in the presence of magnetic field. Sujit Kumar Khan and Sanjayanand. [2004] Considered a problem on visco-elastic boundary layer MHD flows through a porous medium over a porous quadratic stretching sheet. Very recently Mahmoud and Mahmoud [2006] presented analytical solutions of hydromagnetic boundary layer flow of a non-Newtonian power-law fluid past a continuously moving surface.

The study of non-Newtonian fluid is more important in constructing electromagnetic flow meters, in understanding the principles of the method and its application to blood flow measurement. When the conducting non-Newtonian fluids flow in the presence of external magnetic field, the non-Newtonian and the magnetic force effects will be coupled with in the flow field. Thus it would be possible to influence the flow of these conducting fluids.

1.8 FLOW WITH HEAT AND MASS TRANSFER

TEMPERATURE

The word temperature indicates a physical property on which depends the sense-impression of hotness or coldness. Temperature has been defined as “the state of a
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substance or body with regard to sensible warmth referred to some standard of comparison”. Sense-impressions can give only a crude estimate, and temperature is usually measured by means of a Thermometer.

HEAT

The conception of heat (Caloric) which passes from the hotter to the colder body, and is thought of as bringing about the change of temperature. According to Max Planck “The conception of heat, like all other physical concepts, originates in the sense-perception, but it acquires its physical significance of the events which excite the sensation. So heat regarded physically, has no more to do with the sense of hotness then colour in the physical sense, and has to do with the perception of colour.

The terms Heat and Temperature in older philosophy drew little or no distinction between them, and we still use words like blood-heat and summer heat, which introduce the term Heat in connection with the idea of temperature. Joseph Black was the first to perceive clearly the necessity of removing this confusion, and he pointed out that we must distinguish between quantity and intensity of heat, quantity corresponding to the amount of heat and intensity to temperature.

As we know that the knowledge of heat transfer is very important now a days for construction and designing of a power plant, which will perform in the prescribed fashion, is the objective of the engineer. This clearly requires detailed knowledge of the principles governing heat transfer in the various components, which may be involved i.e., boilers, turbines, condensers, pumps and compressors. Some of the other industrial fields of heat transfer plays an important role like heating and air conditioning, chemical reactions and process.
A detailed heat transfer analysis is essential. The dimensions of boilers, heater analysis is essential. The dimensions of boilers, heaters, refrigerators and heat exchangers depend not only on the amount of heat to be transmitted but also on the rate at which heat is to be transferred under given conditions.

**TYPES OF HEAT TRANSFORMATION**

Heat transfer is a transmission of energy from one region to another as a result of temperature difference between them. There are three types of heat transfer process, they are

(i) Radiation

(ii) Conduction

(iii) Convection

**RADIATION**

Radiation heat transfer is concerned with the exchange of thermal radiation energy between two or more bodies. This mode of heat transfer is associated with the emission of energy in the form of electromagnetic waves and is not dependent on the presence of a transmitting medium. In the majority of engineering systems, radiation effects are not significant, the principal exception being in flames resulting from reacting mixtures.

**CONDUCTION**

Heat transfer by conduction is effected on a molecular scale, with no mass movement of the conducting medium. In solid medium, heat conduction is attributable to a combination of molecular collisions and internal radiation. In liquids, the molecules bear greater mobility which assists the transfer of heat by conduction.
In gases, the molecules possess even greater mobility but the longer mean-free-path between molecular collision results in a lower conductivity than that of a liquid or solid.

**CONVECTION**

The heat transfer process between a heated wall and a fluid is (except in the immediate vicinity of the wall) a combination of the conduction and convection processes. The convection process is a heat transfer process involving bodily movement of the fluid. The process may be of the “forced convection” type, in which the fluid motion is induced by a fan, compressor, or pump, or of the “free convection” type, in which fluid motion occurs entirely as a result of density gradients resulting from the temperature gradients in the fluid.

Natural convection or buoyancy-driven convection is one of the more complex fluid phenomena and in order to assess its effect properly, it is essential to be aware of what is known about it at present. Convection flows are abound in nature and industrial processes, which are arising due to variation of density, resulting from temperature and/or concentration differences.

In general there are two types of convection based on the nature of flow generation. The fluid motion generated by buoyancy due to density variations, resulting from a temperature difference is referred to as natural convection or free convection and in case the fluid motion is artificially induced, say with a pump or fan which forces the fluid to flow, then the convection is said to be forced convection. But in some practical situations these two effects are of comparable order. A flow in which both these effects are significant is commonly referred to as mixed or combined convection. In the case of forced convection, the externally imposed flow is generally known, whereas in free
convection, the flow results from the interaction of the density differences with the gravitational field. Hence the flow and temperature/concentration fields are coupled. As such the flow field cannot be combined independent of temperature and/or concentration fields and they must be considered simultaneously. But in the forced convection the flow and temperature and/or concentration fields are decoupled and hence the flow field can be obtained independent of the temperature and/or concentration fields.

Buoyancy driven flows are of two basic configurations. The first one, referred to as stable configuration, is one in which a density gradient (due to thermal or concentration) is normal to the gravity. This kind of convection, where the flow is instantaneous, is known as OberBeck convection. In such case (conventional convection) the flow results immediately and the transport may or may not be affected. The second configuration is an unstable one in which the density gradient is parallel but opposes the gravitational field. This type of convection in a horizontal layer heated from below and cooled from above is known as Rayleigh-Benard convection. Under such a situation, the fluid remains in a state of unstable equilibrium. Due to heavier fluid being above the lighter fluid, until a critical density gradient is exceeded. A spontaneous flow then ensues, which quickly becomes steady. This motion usually takes the form of cells or vortex rolls and, hence, causes more mixing than laminar convectional convection. Thus, the significance of fluid motion, resulting from buoyancy forces, in many engineering applications has prompted many researchers to carry out numerous studies to improve their understanding of governing processes in these applications, Rayleigh-Benard and
OberBeck convections have been extensively investigated due to their importance in science and technology.

**THERMAL CONDUCTIVITY**

The concept of thermal conductivity is that “The quantity of heat passing in unit time through each unit of area when there is a difference of temperature of one degree between the inside and outside face of a wall of unit thickness”.

To be more specific about discussion of thermal conductivity we consider two parallel layers of fluid, at a distance d apart are kept at different temperatures \( T_1 \) and \( T_2 \) (one of the layers may be a solid surface). Fourier noticed that a flow of heat is set up through the layer such that the quantity of heat \( q \) transferred through unit area in unit time is directly proportional to the difference of the temperature between the layers and inversely proportional to the distance d. Thus he found

\[
q = K \frac{T_1 - T_2}{d}
\]

Where \( K \) is the constant of proportionality and is known as the coefficient of thermal conductivity.

If the distance d between the two layers of fluid is infinitesimal the above law can be written in the differential form as

\[
q = -K \frac{dT}{dy}
\]

where the negative sign indicates that the heat flows in the direction of decreasing temperature.

The dimensions of the coefficient of thermal conductivity can be determined as follows
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\[ K = \frac{\text{Heat flux}}{\text{Temperature gradient}} \]

**THERMAL DIFFUSIVITY**

The effect of conductivity on the temperature field is determined by the ratio of \( k \) to the product of density \( \rho \) and specific heat \( c_p \) rather than \( k \) alone. This ratio is known as the thermal diffusivity and it is usually denoted by

\[ a = \frac{k}{\rho c_p} \]

**LITERATURE REVIEW**

Study of heat transfer, mass transfer and momentum transfer in a laminar boundary layer over a moving stretching surface has gained considerable practical relevance in the field of electrochemistry and polymer processing (Gorla [1978], Erickson et al [1960]). The important studies of these transport processes have so far been devoted to flows induced by surface moving with constant velocity. Pioneering work was carried out by Sakiadis [1961] and that was extended by Crane [1970]. Crane [1970] considered a laminar boundary layer flow of a Newtonian fluid caused by a flat elastic sheet. Due to the increasing applications of non-Newtonian fluids in industry, the same problem was extended to fluid obeying non-Newtonian constitutive equation (Siddappa and Abel [1985] Andersson [1992]).

Several researchers (Carrager and Crane [1982], and Soundalgekar and Murthy [1980]) studied the heat transfer problem associated with the Newtonian and Non-Newtonian boundary layer flow past a stretching sheet. In these studies there exists mathematical equivalence of the heat transfer problem with the mass transfer in the boundary layer. Hence, the results obtained for heat transfer characteristics can be carried
directly to the mass transfer by replacing Prandtl number by Schmidt number. Diffusion and chemical reaction is an isothermal laminar flow along a soluble flat plate was studied and an appropriate mass transfer analog to the flow along a flat plate that contains a species A slightly soluble in the fluid B has been discussed by Fairbanks [1950]. Andersson et al. [1994] studied the diffusion of a chemically reactive species from a stretching sheet. K A. Yih [1999] presented a problem on free convection effects on MHD coupled heat and mass transfer of a moving vertical surface. Takhar et al. [2000] considered the flow and mass transfer on stretching sheet with magnetic field and chemically reactive species. Anjali Devi and Kandaswamy [2000] studied the effects of chemical reaction, heat and mass transfer on MHD flow past a semi-infinite plate.


**LITERATURE REVIEW ON NEWTONIAN FLUIDS**

Sakiadis (1961 a, b, c) was the first to study a two-dimensional boundary layer flow over a moving plate with constant velocity. Both exact and approximate solutions were presented for a laminar flow with the latter being obtained by the integral method. Due to the entrainment of the ambient liquid, this situation represents a different
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class of boundary layer problems (Blasius, 1908) and has a solution substantially different from that of boundary layer flow over a semi-infinite flat plate. Erickson et. al. (1966) extended the work of Sakiadis to study the effect of mass transfer at the moving surface.

Crane (1970) pointed out that in the polymer industry it is sometimes necessary to consider a stretching plastic sheet. An analytical study was presented by Crane (1970) for the steady boundary layer flow of an incompressible viscous liquid caused solely by the linear stretching of an elastic flat sheet which moves in its own plane with a velocity varying linearly with distance from a fixed point.

Gupta and Gupta (1977) investigated the heat and mass transfer in the flow over a stretching surface (with suction or blowing) issuing from a slit. A non-isothermal moving sheet was dealt with and the temperature and concentration distribution profiles for that situation were obtained.

Banks (1983) examined a class of similarity solutions of the boundary layer equations for the flow due to a stretching surface. The ordinary differential equation that arises admits a one-parameter family of solutions in much the same way the Falkner-Skan equation does. The equation is integrated numerically for a number of parameter values and the results are presented. Analytical solution is also presented for a couple of values of the parameter and these, together with perturbation solutions, support the numerical ones.

Grubka and Bobba (1985) carried out heat transfer studies by considering the power law variation of surface temperature. Dutta et. al. (1985) analyzed the temperature distribution in a flow over a stretching sheet with uniform heat flux. The governing
differential equation transformed to a confluent hyper-geometric differential equation and solution was obtained in terms of incomplete gamma function. It was shown that temperature at a point decreased with the increase of Prandtl number. Dutta and Gupta (1987) solved the coupled heat transfer problem involving a stretching sheet. Variation of the sheet temperature with distance from the slit was found for several values of the Prandtl number and stretching speeds. It was shown that for a fixed Prandtl number, the surface temperature decreases with an increase in the stretching speed.

Dutta (1988) presented an analytical solution of the heat transfer problem for cooling of a thin stretching sheet in a viscous flow in the presence of suction or blowing. The local velocity of the sheet was assumed to be proportional to the distance from the slit. The convergence criteria of the solution were also established. Chen and Char (1988) investigated the effects of both power-law surface temperature and power-law heat flux variations on the heat transfer characteristics of a continuous, linearly stretching sheet subjected to suction/blowing.

Soewono et.al.(1992) analyzed the existence of solutions of a nonlinear boundary value problem, arising in flow and heat transfer over a stretching sheet with variable thermal conductivity and temperature-dependent heat sources/sinks. Karahalios (1992) obtained an exact similarity solution of the time-dependent Navier-stokes equation when a flat surface stretches radially. The velocity components were expressed in a power series in time up to the second-order of approximation. Vajravelu (1994) carried out analysis of convective flow and heat transfer in a viscous heat generating liquid near an infinite vertical stretching surface. The effects of free convection and suction/injection
on the flow and heat transfer were considered. The equations of conservation of momentum, mass and energy, governing dynamics of the fluid were solved numerically by using a variable order, variable step size finite difference method. The numerical results obtained for the flow and heat transfer characteristics revealed many interesting behaviors.

Kumaran and Ramanaiah (1996) for the first time studied the viscous boundary layer flow over a quadratically stretching sheet. The plot of skin friction and streamline pattern as a function of the stretching parameters was discussed.

Magyari and Keller (1999) examined both analytically and numerically the heat and mass transfer in the boundary layer on an exponentially stretching continuous surface with an exponential temperature distribution. Magyari and Keller (2000) studied the steady boundary layer flow induced by permeable stretching surfaces with variable temperature distribution under Reynolds analogy. Reynolds analogy makes use of the advantage of all the exact analytic solutions of the momentum and energy equations.

Magyari and Keller (2001) analyzed the free laminar jets of classical hydrodynamics that may be identified with certain boundary-layer flows induced by continuous surfaces immersed in quiescent incompressible liquids and stretched with well-defined velocities. Two cases were considered: (i) Schlichting’s round jet of momentum flow and (ii) Schlichting-Bickley plane jet of momentum flow. By presenting an analytic solution of the flow problem, it was shown that in the limiting case of a vanishing lateral mass flux, this stretching-induced flow goes over, by an adequate scaling transformation, to the well known wall jet.
Magyari et.al. (2002, 2003) examined the self-similar boundary layer flow of a Newtonian liquid over a permeable continuous plane surface stretching with inverse linear velocity. It was shown that in order to obtain from pseudo-similarity the correct similarity problem, in this case the usual expression of the stream function a logarithmic term in the wall coordinate x must be added. The new analytical solution of a well-known boundary value problem shows that the hyperbolic-tangent solution of this problem belongs to a one-parameter family of multiple solutions that can be expressed in terms of Airy’s function.

Mahapatra and Gupta (2003) examined an exact similarity solution of the Navier-Stokes equation. The solution represents steady asymmetric stagnation-point flow towards a stretching surface. It is shown that the flow displays a boundary layer structure when the stretching velocity of the surface is less than the free stream velocity. On the other hand, an inverted boundary layer is formed when the surface stretching velocity exceeds the free stream velocity. Temperature distribution in the flow is found when the surface is held at a constant temperature. It turns out that when the surface temperature exceeds the ambient temperature, heat flows from the surface to the liquid near the stagnation point but further away from the stagnation point heat flows from the liquid to the stretching surface.

Partha et.al (2004) have examined the mixed convection flow and heat transfer from an exponentially stretching vertical surface in a quiescent liquid using a similarity solution. They found that the wall temperature and stretching velocity can have a specific exponential form. The influence of buoyancy force along with viscous dissipation on the
convective transport in the boundary layer region was analyzed in both aiding and opposing flow situations.

Liao (2003, 2005), Liao and Pop (2004) and Xu (2005) have used the homotopy analysis method for nonlinear problems arising due to stretching sheet. Two rules, the rule of solution expression and rule of coefficient ergodicity, were proposed which play an important role in the frame of the homotopy analysis method. An explicitly analytic solution is given for the first time, with recursive formulae for coefficients.

All the above investigators, however, restricted their analysis to flow of a Newtonian liquid. In what follows we review literature on the stretching sheet problem involving a liquid with asymmetric stress i.e., liquids whose stress-strain relationship is non-linear.

1.10 LITERATURE REVIEW MHD STRETCHING SHEET PROBLEM INVOLVING NEWTONIAN AND NON-NEWTONIAN FLUIDS

Pavlov (1974) presented an exact similarity solution of the MHD boundary layer equations for the steady two-dimensional flow of an electrical conducting incompressible liquid due to the stretching of an elastic sheet in the presence of a uniform, transverse magnetic field. Chakrabarti and Gupta (1979) extended the work of Pavlov (1974) to study the temperature distribution in the MHD boundary layer flow, in the presence of uniform suction. A similarity solution for the velocity and heat transfer characteristics in the flow with uniform suction at the wall was considered. The equation was solved in terms of incomplete gamma function. Kumari et al. (1990) studied the effects of induced magnetic field and source/sink on flow and heat transfer characteristics over a stretching surface.

Anderson (1992) investigated MHD flow of a Walters’ liquid B past a stretching surface. An analytical solution was obtained for the governing nonlinear boundary layer equation. An expression for the boundary layer thickness was also presented.

Vajravelu and Nayfeh (1993) studied MHD convective flow and heat transfer in a viscous heat –generating liquid near an infinite vertical stretching surface. The effects of
free convection and heat generation/absorption on the flow and heat transfer characteristics were considered. The equations of conservation of momentum, mass and energy, which govern the flow and heat transfer, were solved numerically by using variable order, variable stepsize finite-difference method.

Char (1994) studied the heat and mass transfer in a MHD flow of a Walters’ liquid B over a stretching sheet. Exact solutions were obtained. The solutions for the heat and mass transfer characteristics were evaluated numerically for different values of modified Prandtl number, magnetic parameter, the surface temperature index and modified Schmidt number.

Dandapat et al. (1994) investigated the stability of MHD flow of a Walters’ liquid B past a stretching sheet. A three-dimensional linear stability analysis was performed by means of the method of weighted residuals. It was found that the magnetic field exerts a stabilizing influence on the flow. It was observed that high wave-number disturbances were more effectively damped than low wave number modes and also that disturbances of low wave-number modes were practically uninfluenced by the viscoelasticity, whereas the influence of the magnetic field was prominent.

Andresson (1995) presented an MHD flow past a stretching sheet and obtained a similarity solution for velocity and pressure of the steady two-dimensional Navier–Stokes equations. The solution for the velocity field turned out to be identical with the solution of Pavlov (1974).

Chiam (1995) examined the MHD boundary layer flow due to a sheet stretching with a power-law velocity distribution. A special form of the magnetic field was chosen so as to yield similarity equation. First, an analytical solution for the case of large magnetic parameters was studied. Then, an accurate expression for the skin coefficient was derived using Crocco’s transformation. This was followed by a direct numerical solution of the resulting boundary value problem using shooting method.

Chiam (1997) presented a solution of the energy equation for the boundary layer flow of an electrically conducting liquid under the influence of a constant transverse magnetic field (suction/blowing) over a linearly stretching non-isothermal flat sheet. Effects due to dissipation, stress work and heat generation were considered. Analytical solution of the resulting linear non-homogeneous boundary value problem, expressed in
terms of Kummer’s functions, were presented for the case of PST as well as PHF, both of which were assumed to be quadratic functions of distance.

Vajravelu and Hadjinicolaou (1977) carried out the investigations of free convection and internal heat generation on flow and heat transfer characteristics in an electrically conducting liquid near an isothermal stretching sheet.

Kumari and Nath (1999) considered the effect of the magnetic field on the stagnation point flow and heat transfer of a viscous electrically conduction liquid on a linearly stretching sheet, when the velocity of the sheet and the free stream velocity are not equal. The problem may be regarded as a combination of two problems, namely, two-dimensional stagnation-point flow and flow over a stretching sheet in an ambient liquid. Exact solutions of the Navier-Stokes equation were obtained.

Kelly et. al. (1999) investigated the behavior of the heat and mass transfer characteristics of an incompressible and electrically conducting viscoelastic liquid past a flat elastic sheet. Analytical solution of the resulting linear non-homogeneous boundary value problems, expressed in terms of Kummer’s functions, were presented for the case of PST as well as PHF, both of which are assumed to be functions of distance. They also considered the asymptotic limit of the solution for small and large Prandtl numbers.

Takhar et. al. (2000) considered Newtonian flow and mass transfer on a stretching sheet with magnetic field and chemically reactive species. Using an implicit finite difference scheme the partial differential equations governing the boundary layer flow and mass transfer were solved. The magnetic field was shown to significantly increase the surface skin friction, and to slightly reduce the surface mass transfer. The surface mass transfer was found to depend strongly on the Schmidt number and the reaction rate. The surface mass transfer for the first–order reaction was found to be more than that for second or third order reaction.

Mahapatra and Gupta (2001) presented a steady two-dimensional stagnation–point flow of an incompressible viscous electrically conducting liquid over a flat deformable sheet. The velocity at a point was shown to decrease /increase with increase in the magnetic field when the free stream velocity was less/ greater than the stretching velocity. The temperature distribution in the flow was obtained for a surface held at a
constant temperature. The results were obtained numerically by a finite difference method with the Thomas algorithm.

Abel et. al. (2001) analyzed the MHD Walters’ liquid B flow and heat transfer over a non–isothermal stretching surface embedded in a porous medium. They considered two different case of heat transfer namely (i) PST and (ii) PHF. The boundary layer equations for momentum and heat transfer, which are non-linear ordinary differential equations of momentum was solved exactly. The resulting non-linear ordinary differential equation of momentum with absorption and first order chemical reaction was also solved analytically.

Liao (2003) presented a powerful, easy –to- use analytic technique for nonlinear problems, namely the homotopy analysis method that gave an analytic solution of the MHD viscous flows of power –law liquids over a stretching sheet. For the so called second order and third order power –law liquids, the explicit analytic solutions are given by recursive formulae of constant coefficients. Besides, for real power-law indices and when magnetic field parameter is quite large, an analytic field tends to increase the skin friction and indicates that the flow is damped. This effect is more pronounced in shear-thinning as compared to shear-thickening liquids.

Datti et. al. (2004) analyzed MHD flow of a Walters’ B liquid over a non-isothermal stretching sheet with internal heat generation/ absorption and in the presence of radiation. Thermal conductivity was assumed to vary linearly with temperature. The governing partial differential equations were converted into ordinary differential equations by a similarity transformation. These equations were solved both by analytical and numerical methods.

Afify (2004) investigated MHD free convective flow and heat transfer over a stretching sheet with chemical reaction using a fourth-order Runge-Kutta scheme and the shooting method. Numerical results for the skin-friction coefficient, the local Nusselt number, Sherwood number, as well as the velocity temperature and concentration profiles were presented for liquids with a Prandtl number 0.71 and for various values of chemical reaction rate.

Liu (2005) studied the momentum, heat and mass transport of a hydrodynamic liquid past a stretching sheet in the presence of a uniform transverse magnetic field. The
mass transfer equation includes the chemical reaction of order one and heat transfer equation includes internal heat generation or absorption. The concentration and temperature boundary conditions were assumed to be a linear function of the distance. Analytical solution was derived in terms of the Kummer's function.

The flow of second grade fluid due to stretching sheet was discussed by McLeod and Rajgopal (1987) and for the Navier-Stokes case uniqueness of the solution was established. Rajgopal et.al (1984) studied the flow of a second grade fluid past a stretching sheet and gave an approximate solution to the problem. Troy et al. (1987) found one parameter family of solutions for the classical second grade fluid. Vajravelu Rollins (1991) accommodated the heat transfer analysis of the problem.


**LITERATURE SURVEY**

**1.11 STRETCHING SHEET PROBLEM INVOLVING NANOFLUIDS**

Layek et al[2007] considered the study of two-dimensional stagnation point flow of an incompressible viscous fluid towards a porous stretching surface embedded in a porous medium subject to suction/blowing with internal heat generation or absorption. The motion of this study is to explore the influence of suction/blowing on the control of
flow separation as well as heat transfer and also to investigate the effects of heat source or sink parameter on heat transfer. The momentum and thermal boundary layer equations are solved numerically using shooting method.

Anuar Ishak et al[2009] considered the steady two-dimensional MHD stagnation point flow towards a stretching sheet with variable surface temperature. In this paper the governing system of partial differential equations are transferred into ordinary differential equations, which are solved numerically using a finite-difference scheme known as the Keller-box method. The effects of the governing parameters on the flow field and heat transfer characteristics are obtained.

Mahapatra et.al[2009] studied the Analytical solution of magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching sheet. Here the governing equations of the flow heat and mass transfer are solved by Homotopy Analysis Method and studied effects of all governing parameters on flow, heat and mass transfer.

Kuznetsov and Nield [2010] have studied the natural convection boundary layer flow, heat and mass transfer of nanofluid due to past a vertical plate. Here the governing equations of the flow heat and mass transfer are solved by analytical method and studied effects of all governing parameters on natural convection boundary layer flow, heat and mass transfer.

Hayat et. al [2010] studied the two-dimensional stagnation point flow of an incompressible fluid over a stretching sheet by taking into a account radiation effects using the Rosseland approximation to model the radiative heat transfer. Under suitable similarity variables, the partial differential equations are transformed into a system of
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non-linear ordinary differential equations which are solved analytically by the homotopy Analysis method (HAM).

Z. Abbas et al (2010) considered the steady mixed convection boundary layer flow of an incompressible Maxwell fluid near the two-dimensional stagnation-point flow over a vertical stretching surface and it is assumed that the stretching velocity and the surface temperature vary linearly with the distance from the stagnation point. The homotopy Analysis Method (HAM) and the influence of the various interesting parameters on the flow and heat transfer is analyzed.

Mustafa et al (2011) studied the flow heat and mass transfer of nanofluid due to stretching sheet. Here the governing equations of the flow heat and mass transfer are solved by Homotopy Analysis Method and studied effects of all governing parameters on flow, heat and mass transfer. Makinde and Aziz (2011) considered the study of the effect of a convective boundary condition on boundary layer flow, heat and mass transfer and nanoparticle fraction over a stretching surface in a nanofluid. The governing boundary layer equations have been transformed to a two-point boundary value problem and are solved numerically.

Hassani et al (2011) studied the boundary layer flow heat and mass transfer of nanofluid past a stretching sheet. Here the governing equations of the flow heat and mass transfer are solved by Homotopy Analysis Method and studied effects of all governing parameters on boundary layer flow, heat and mass transfer.

Kandasamy et al (2011) considered to study the boundary layer flow, heat transfer and nanoparticle volume fraction over a stretching surface in a nanofluid for various parameters using scaling group of transformation. Bhattacharyya and Vajravelu (2011)
investigated the boundary layer stagnation point flow and heat transfer over an exponentially shrinking sheet. Using an exponential form of similarity transformation, the governing mathematical equations for the flow and heat transfer are transformed into self-similar coupled, non-linear ordinary differential equations. Rohni et. al [2011] considered the flow and heat transfer over an unsteady shrinking surface with wall mass suction in a nanofluid by using an appropriate further similarity transformation, similarity equations are obtained and the shooting method is used to solve these equations for different values of the wall mass suction, unsteadiness nanofluid parameters.

Yacob et. al [2011] studied the boundary layer stagnation point flow of a micropolar fluid towards a horizontally linearly stretching/shrinking sheet. Here a mathematical model is devolved to study the heat transfer characteristics occurring during the melting process due to a stretching/shrinking sheet. The transformed non-linear ordinary differential equations governing the flow are solved numerically by the Runge-Kutta –Fehlberg method with shooting technique.

Norfiftah Bachok et.al [2012] studied the two-dimensional stagnation point flow of a water based nanofluid over an exponentially stretching/shrinking sheet. Noghrehabadi et. al [2012] to analyze the slip effects on the boundary layer flow and heat transfer over a stretching surface of nanoparticle fractions. Here the governing equations of slip effects on the boundary layer flow and heat transfer are solved by numerically. The effects of slip boundary condition in the presence of dynamic effects of nano particle have been investigated. Bhattacharyya [2012] studied the heat transfer in unsteady boundary layer stagnation point flow over a shrinking/stretching. The governing equations are transformed into self-similar ordinary differential equations by adopting
similarity transformations and then the converted equations are solved numerically by shooting method.

Turkyilmazoglu and Pop[2013] Considered the flow and heat transfer of a Jeffrey fluid near the stagnation point on a stretching/shrinking sheet with a parallel external flow. The main concern is to analytically investigate the structure of the solutions which might be unique or multiple. Heat transfer analysis is also carried out for a boundary heating process taking into consideration both a uniform wall temperature and a linearly increasing wall temperature. Ibrahim et. al [2013] studied the effect of magnetic field on stagnation point flow and heat transfer due to nanofluid towards a stretching sheet. The transport equations employed in the analysis include the effect of various parameters. The similarity transformation is used to convert the governing nonlinear boundary layer equations to coupled higher order nonlinear ordinary differential equations. These equations were numerically solved using Runge-Kutta fourth order method with shooting technique.