2.1 Introduction

The study of mechanism of synovial joints has recently become an active area of scientific research. The human joint is a dynamically loaded bearing which employs articular cartilage as the bearing and synovial fluid as the lubricant. Once a fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The loaded bearing synovial joints of the human body are the, shoulder, hip, knee and ankle joints such joints have a lower friction co-efficient and negligible wear. Synovial fluid is a clear viscous fluid, a dialysate of plasma containing mucopolysaccharides. Synovial fluid usually exhibits a non-Newtonian shear thinning behavior. However, under high shear rates; the viscosity of synovial fluid approaches a constant value not much higher than that of water (Cooke, 1978). Therefore a Newtonian lubricant model has often been used for synovial fluid in lubrication modeling (Dowson and Jin, 1986). In this study the synovial fluid is modeled as non-Newtonian micropolar fluid.

Articular cartilage is a poro-elastic or biphasic consisting of both fluid and solid phases. The importance of the unique biphasic load carrying characteristics of articular cartilage and fluid flow inside have been recognized in the lubrication of synovial joints such as weeping and boosted lubrication theories. A more general biphasic lubrication theory was subsequently proposed by Mow and Lai (1980). However, it was not until in the 1990’s that the relation between friction and interstitial fluid pressurization was comprehensively studied (Forster and Fisher, 1996; Ateshian, 1997; Soltz and Ateshian, 1998; Graindorge et al. 2006). A number of friction studies have been carried out under a wide range of tribological
conditions to investigate the biphasic lubrication of articular cartilage. Under both start up and reciprocating motion of a cartilage plug against a metallic counter face friction was found to increase with loading time (Forster and Fisher, 1996). The transient friction behavior observed was a direct result of the interstitial fluid pressurization and fluid load support, directly measured experimentally (Soltz and Ateshian, 1998; Morrell et al. 2005). However, for a similar configuration but under cyclic loading, friction was found to be similar or even at a higher level (Krishnan et al. 2005). The importance of the biphasic lubrication has also been studied by enzymatic treatment of articular cartilage alter the biphasic properties and fluid pressurization such as chondroitinase (Picard et al. 1998; Kumar et al. 2001; Basalo et al. 2006; Katta et al. 2008). However, the results obtained have been contradictory. Pickard et al. (1998) found no major differences in friction levels following chondroitinase treatment. While Kumar et al. (2001) and Basalo et al. (2006) showed a significant reduction.

All these studies were confined to the smooth cartilage surfaces of human knee. But, Sayles et al. (1979) revealed experimentally that cartilage surfaces are rough, and roughness height distribution is Gaussian in nature. This has motivated us to investigate the influence of roughness of cartilage surfaces in lubrication aspects of synovial joint. Christensen (1969) developed the stochastic theory to understand the influence of surface roughness in hydrodynamic lubrication of bearings. Many researchers have used this theory to analyze the effect of surface roughness of various types of bearings. Naduvnamani et al. (2004) have studied the problem of squeeze film lubrication between rough rectangular plates with
couple stress fluid as lubricant. These investigations have not incorporated the poro-elasticity of the bearing surface.

The squeeze film lubrication characteristics of micropolar fluid have been extensively studied in the literature. Agarwal (1972) studied the squeeze film and externally pressurized bearings lubricated with micropolar fluids and formed that the time of approach is more for the micropolar fluids as compared to the corresponding Newtonian fluids. The analytical solution of the problem of squeeze film lubrication of micropolar fluid between two parallel plates (one dimensional) has been given by Bujurke et al. (1987).

In this chapter, a theoretical study of combined effects of surface roughness and micropolar fluid in squeeze film lubrication between poro-elastic rectangular plates is presented. For mathematical simplicity, the average of three layers of the cartilage is modeled as a single poro-elastic layer.

2.2 Mathematical Formulation of the Problem

The geometry and co-ordinates of the problem are as shown in the Fig.2.1. The squeezing flow of micropolar fluid between two rectangular surfaces is considered. The upper rough articular surface is approaching the lower smooth poro-elastic matrix normally with a constant velocity \( V = \frac{\partial h}{\partial t} \). The lubricant in the joint cavity is taken to be Eringen (1966) micropolar fluid. As the load bearing area of the synovial knee joint is small, the two surfaces may be considered to be parallel under high loading conditions. The moving boundary is characterized by
\[ h = h(t) + h_s(x, z, \xi) \]  

(2.2.1)

Where \( h(t) \) represents the nominal smooth part of the film geometry and \( h_s \) is part due the surface asperities measured from the nominal level and is a randomly varying quantity of zero mean and \( \xi \) is an index parameter determining a definite roughness parameter.

---

**Figure 2.1:** A geometry of simplified model for knee joint
The following hydrodynamic lubrication approximations are used in the present analysis;

1. Body forces are neglected, i.e. there are no extra fields of forces acting on the fluid.

2. The pressure is constant through the fluid film thickness. As the film thickness is only one or two thousandths of an inch, it is always true.

3. The curvature of the surface is large compared with film thickness. Surface velocities need not be considered as varying in direction.

4. There is no slip at the solid boundaries. The velocity of the lubricant layer adjacent to the solid boundary of the bearings is the same as that of the boundary.

5. The lubricant flow in the film is laminar

6. Fluid inertia is neglected.

7. The viscosity is constant throughout the film thickness.

**Governing equations**

**Region-I (Fluid film region)**

Under the usual assumptions of hydrodynamic lubrication and in the absence of body couples, the basic equations for the flow of micropolar fluid in the film region given in Eqns. (1.7.2.13) to (1.7.2.18)
Region-II (Poro-elastic region):

Following Torzilli and Mow (1976) and Collins (1982) the coupled equations of motion for deformable cartilage matrix and the mobile portion of the fluid contained in it can be written in the form.

Matrix:

\[
\rho_m \frac{\partial^2 \mathbf{U}}{\partial t^2} = \text{div} \tau_m - \frac{1}{k'} \left( \frac{\partial \mathbf{U}}{\partial t} - \mathbf{V} \right)
\]  

(2.2.2a)

Fluid:

\[
\rho_f \frac{D \mathbf{V}}{D t} = \text{div} \mathbf{f} + \frac{1}{k'} \left( \frac{\partial \mathbf{U}}{\partial t} - \mathbf{V} \right)
\]  

(2.2.2b)

Where \( \rho_m \) and \( \rho_f \) denote the densities of solid matrix and fluid respectively, \( \mathbf{U} \) is the corresponding displacement vector, \( \mathbf{V} \) is the fluid velocity vector, \( k' \) is the permeability of the cartilage, and \( \frac{D}{D t} \) denotes the material derivative. Equations (2.2.2a) and (2.2.2b) represent the force balance for the linear elastic solid and viscous fluid components, of the cartilage, respectively. In these equations, left hand terms denote the local forces (mass X acceleration) which are counter balanced by right porous media driving force respectively.

In fact these two equations may be viewed simply as a generalized form of Darcy’s law for unsteady flow in a deformable porous medium in terms of the
relative velocity \( \left( \frac{dU}{dt} - V \right) \) between the moving cartilage and the fluid contained in its pores.

The classical stress tensor \( \tau \) for a continuous homogeneous medium may be expressed for the matrix and fluid, respectively as

\[
\tau_m = p_1 I + N^1e + AeI
\]

(2.2.3a)

\[
\tau_f = -p_1 I + EeI
\]

(2.2.3b)

Where \( N^1, A \) and \( E \) are the elastic parameters of the cartilage. After neglecting the inertia terms, addition of equations (2.2.2a) and (2.2.2b) eliminate the pressure and fluid velocity and thereafter, taking the divergence of the results, yields the following Laplace equation.

\[
\nabla^2 e = 0
\]

(2.2.4)

where \( e = \text{div}U \) is known as the cartilage dilatation. Following Hori and Mockers (1976) we characterize the cartilage dilatation by a sample similar linear equation in terms of corresponding average bulk modulus \( K \), in the following form

\[
e = e_0 + \frac{p_1}{k^*}
\]

(2.2.5)

From equations (2.2.4) and (2.2.5) we get

\[
\nabla^2 p_1 = 0
\]

(2.2.6)
To solve the governing equations of the model, the relevant boundary conditions are

\[ u = w = 0, \quad v = v_n, \quad v_1 = v_2 = 0 \quad \text{at} \quad y=0 \]  

(2.2.7a)

\[ u = w = 0, \quad v = -\frac{\partial h}{\partial t}, \quad v_1 = v_2 = 0 \quad \text{at} \quad y=h \]  

(2.2.7b)

### 2.3 Solution of the Problem

The Solution of equations (1.7.2.13)-(1.7.2.16) subject to the corresponding boundary conditions given in the equations (2.2.7a) to (2.2.7b) we get

\[
u = \frac{1}{\mu} \left[ \frac{y^2}{2} \frac{\partial p}{\partial x} + C_{11} y \right] - \frac{2N^2}{m} \left[ C_{21} \sinh(my) + C_{31} \cosh(my) \right] + C_{41} \]  

(2.3.1)

\[
w = \frac{1}{\mu} \left[ \frac{y^2}{2} \frac{\partial p}{\partial z} + C_{12} y \right] - \frac{2N^2}{m} \left[ C_{22} \sinh(my) + C_{32} \cosh(my) \right] + C_{42} \]  

(2.3.2)

\[
v_1 = -\frac{1}{2\mu} \left[ \frac{y}{2} \frac{\partial p}{\partial z} + C_{12} \right] + \left[ C_{22} \cosh(my) + C_{32} \sinh(my) \right] \]  

(2.3.3)

\[
v_3 = -\frac{1}{2\mu} \left[ \frac{y}{2} \frac{\partial p}{\partial x} + C_{11} \right] + \left[ C_{21} \cosh(my) + C_{31} \sinh(my) \right] \]  

(2.3.4)

where for \( i =1, 2 \).

\[ C_{ii} = 2\mu C_{2i} \]

\[ C_{3i} \sinh(mh) = \frac{h}{2\mu} \frac{\partial p}{\partial x_i}, \quad C_{2i} = \frac{C_{3i} \cosh(mh)}{1 - \cosh(mh)} \]
\[ C_s = \frac{h}{2\mu} \left( \frac{h}{2} (\cosh mh - 1) + h - \frac{N^2}{m} \sinh mh \right) \frac{1}{C_s}, \]

\[ C_{si} = \frac{2N^2}{m} C_s, \]

\[ C_s = h \left( \sinh(mh) - \frac{2N^2}{mh} (\cosh mh - 1) \right), \]

and \( m = \frac{N}{l}, N = \left( \frac{\chi}{2\mu + \chi} \right)^{\frac{1}{2}}, l = \left( \frac{\gamma}{4\mu} \right)^{\frac{1}{2}} \)

By neglecting inertia terms, equation (2.2.2b) may be arranged in terms of relative velocity in the form

\[ \left( \frac{\vec{r}}{V} - \frac{dU}{dt} \right) = -k' (\nabla p_i - EV e) \quad (2.3.5) \]

and elimination of \( e \) through equation (2.2.6) and (2.3.5) gives

\[ \left( \frac{\vec{r}}{V} - \frac{dU}{dt} \right) = -k' \nabla p_i \left( 1 - \frac{E}{K} \right) \quad (2.3.6) \]

The normal component of the relative fluid velocity at the cartilage surface is

\[ v_n = \left[ -k' \left( \frac{E}{K} - 1 \right) \frac{\partial p_i}{\partial y} \right]_{y=0} \quad (2.3.7) \]

Integrating equation (1.7.2.18) across the fluid film region and using the boundary conditions for \( v \) given in equations (2.2.7a) and (2.2.7b) and also using
the expression (2.3.1) and (2.3.2) for \( u \) and \( w \), the modified Reynolds equation is obtained in the form

\[
\frac{\partial}{\partial x} \left[ f(N, l, h) \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ f(N, l, h) \frac{\partial p}{\partial z} \right] = -12\mu \frac{\partial h}{\partial t} + 12\mu k^{\prime} \left( 1 - \frac{E}{K} \right) \frac{\partial^2 p}{\partial y^2} \right]_{y=0} \quad (2.3.8)
\]

Where

\[
f(N, l, h) = h^3 + 12l^2h - 6Nh^2 \cot \frac{Nh}{2l}
\]

**Stochastic Reynolds Equation**

In the context of rough surfaces, there are two types of roughness pattern which are of special interest. The one-dimensional longitudinal structure where the roughness has the form of long narrow ridges and valleys running in the \( x \)-direction and the one-dimensional transverse structure where roughness striations are running in the \( z \)-direction in the form of long narrow ridges and valleys. However, the present study is restricted to one-dimensional longitudinal roughness since the one roughness structure can be obtained from other by just rotation of co-ordinate axes.

For the one-dimensional longitudinal roughness pattern, the film thickness assumes the form

\[
h = h(t) + h_s(x, \xi) \quad (2.3.9)
\]

Taking the expectation on both sides of equation (2.3.8) and simplifying, the stochastic modified Reynolds type equation is obtained in the form
\[
\frac{\partial^2 E(p)}{\partial x^2} + R \frac{\partial^2 E(p)}{\partial z^2} = \frac{1}{E[f(N,l,h)]} \left[ -12\mu \frac{\partial h}{\partial t} + 12\mu k \left( 1 - \frac{E}{K} \frac{\partial p}{\partial y} \right) \right]_{\gamma=0}
\]  
(2.3.10)

where

\[
R = \left( E[f(N,l,h)] \times E \left[ \frac{1}{f(N,l,h)} \right] \right)^{-1}
\]

\[
E[f(N,l,h)] = \frac{35}{32c^3} \int_{-c}^{c} f(N,l,h) \left( c^2 - h_i^2 \right) dh_i,
\]

\[
E \left[ \frac{1}{f(N,l,h)} \right] = \frac{35}{32c^3} \int_{-c}^{c} \frac{c^2 - h_i^2}{f(N,l,h)} dh_i.
\]

\( E(g) \) denotes the expectancy operator defined by

\[
E(g) = \int_{-\infty}^{\infty} (g)f(h_i)dh_i
\]

\( f(h_i) \) is the probability density function of the stochastic film thickness variable \( h_i \).

According to the Sayles et al. (1979), the cartilage surfaces are rough and roughness height distribution is Gaussian in nature. Therefore, polynomial form which approximates the Gaussian distribution is chosen in the present study. Such a probability density function is

\[
f(h_i) = \begin{cases} 
\frac{35}{32c^3} \left( c^2 - h_i^2 \right)^3, & -c \leq h_i \leq c \\
0, & \text{elsewhere}
\end{cases}
\]

(2.3.12)

where \( c = \pm 3\sigma \) and \( \sigma \) is the standard deviation.
The relevant boundary conditions for $E(p)$ and $p_1$ are

(i). for the fluid film region

$$E\left[p(x, z)\right] = 0 \text{ at } x = 0, a \text{ and } z = \pm \frac{b}{2} \quad (2.3.13)$$

(ii). for the poro-elastic region

$$p_1(x, y, z) = 0 \text{ at } x = 0, a \text{ and } z = \pm \frac{b}{2} \quad (2.3.14a)$$

$$\frac{\partial p_1}{\partial y} = 0 \text{ at } y = -\delta \quad (2.3.14b)$$

(iii). at the interface

$$p_1(x, y, z) = E[p(x, z)] \text{ at } y = 0 \quad (2.3.15)$$

where $\delta$ is the cartilage layer thickness

The solution of Laplace equation (2.2.6) with boundary conditions (2.3.14a) and (2.3.14b) is

$$p_1(x, y, z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \cosh\left(\gamma_{mn} (y - \delta)\right) \sin(\alpha_{mn} x)\cos(\beta_{mn} z) \quad (2.3.16)$$

where

$$\alpha_{mn} = \frac{m\pi}{a}, \beta_{mn} = \frac{n\pi}{b}, \gamma_{mn} = \left(\alpha_{mn}^2 + \beta_{mn}^2\right)^{\frac{1}{2}}$$

and the constants $C_{mn}$ are the Fourier coefficients to be determined.
Using the mean pressure continuity condition (2.3.15) in equation (2.3.16), we get

\[ E[p(x, z)] = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin(\alpha_m x) \cos(\beta_n z) \]  

(2.3.17)

On substituting equation (2.3.16) and (2.3.17) into equation (2.3.10) and using the orthogonality of Eigen functions \( \sin(\alpha_m x) \) and \( \cos(\beta_n z) \), the Fourier coefficients \( C_{mn} \) are obtained in the form

\[ C_{mn} = \frac{-192 \mu}{ab \alpha_m \beta_n E[f(N,l,h)]} \left( \alpha_m^2 + R \beta_n^2 \right) - \frac{12k^1 \delta \left( 1 - \frac{E}{K} \right) \gamma_{mn} \tanh(\gamma_{mn} \delta)}{E[f(N,l,h)]} \]  

(2.3.18)

where

\[ E[f(N,l,h)] = \frac{35}{32c^3} \int_{-c}^{c} f(N,l,h)(c^2 - h^2) dh \]

The load carrying capacity of the squeeze film is obtained by integrating the averaged pressure field over the surface of the top plate

\[ E(W) = \int_{0}^{b} \int_{0}^{b} E[p(x, z)] dx dz \]  

(2.3.19)

The non-dimensional mean instantaneous load carrying capacity of the squeeze film is given by
\[
W = -\frac{E(W)h_i}{\mu a b^3 h} = \frac{768\delta}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ m^2 n^2 \left[ \frac{3}{2} \left( m^2 \pi^2 + Rn^2 \pi^2 \lambda^2 \right) E\left[ f(N, T, h) \right] - 12\psi \left( 1 - \frac{E}{K} \right) \tanh \left( \frac{\overline{p} \delta}{\sqrt{\lambda}} \right) \right] \right\}^{-1}
\]

(2.3.20)

where

\[
\overline{T} = \frac{1}{h_0}, \overline{h} = \frac{h}{h_0}, \overline{c} = \frac{c}{h_0}, \overline{\lambda} = \frac{\lambda}{b}, \overline{\delta} = \frac{\delta}{\sqrt{ab}}, \overline{\gamma}_{mn} = \gamma_{mn} a, \psi = \frac{k^* \delta}{h_0}, \overline{k} = \frac{k^*}{h_0}
\]

\[
E\left[ f(N, \overline{T}, \overline{h}) \right] = \frac{1}{h_0^2} E\left[ f(N, t, h) \right]
\]

The film thickness at any time \(t\) can be obtained by solving for the load as a function of time.

\[
t = \int_0^t E(W(T))dT = \frac{-768\mu a b^3 \delta}{h_0^2 \pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} I_{mn}
\]

(2.3.21)

where

\[
I_{mn} = \int \left\{ m^2 n^2 \left[ \frac{3}{2} \left( m^2 \pi^2 + Rn^2 \pi^2 \lambda^2 \right) E\left[ f(N, T, h) \right] - 12\psi \left( 1 - \frac{E}{K} \right) \tanh \left( \frac{\overline{p} \delta}{\sqrt{\lambda}} \right) \right] \right\}^{-1} d\overline{h}
\]

The non-dimensional squeeze film time is obtained as

\[
\overline{T} = -\frac{E(W(t))h_i}{\mu a b^3} = \frac{768\delta}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} I_{mn}
\]

(2.3.22)
As \( \bar{k} \to 0 \) (i.e. \( \psi \to 0 \)) and equations (2.3.20) and (2.3.22) reduce to the corresponding solid case studied by Prawal Sinha and Chandan Singh (1982) and the results of Wu (1972) can be recovered in the limiting case of \( N, \bar{T} \to 0 \).

### 2.4 Results and Discussion

A simplified mathematical model has been developed for understanding combined effects of surface roughness, poro-elasticity and micropolar fluid on lubrication aspects of synovial joints. The governing equations along with the appropriate constitutive relationships and boundary conditions have been formulated for modeling the roughness structure of cartilage with viscous fluid in the lubricant region in synovial joint lubrication. The load capacity \( \bar{W} \) and time height relation \( \bar{T} \) are functions of non-dimensional parameters \( \bar{\varphi} \left( = \frac{c}{h_0} \right) \),

\[
\psi \left( = \frac{k \cdot \delta}{h_0} \right), \quad \bar{h} \left( = \frac{h}{h_0} \right).
\]

The values of parameters \( E/K \ (= 0.1, 0.2, 0.3) \), \( \bar{k} \left( = 7.65 \times 10^{-5}, 4.3 \times 10^{-5} \right) \), \( \bar{\delta} \left( = 300, 200 \right) \) are taken from Torzilli, which are associated with healthy human articular cartilage during normal functioning.

### 2.4.1 Load Carrying Capacity

The variations of non-dimensional load carrying capacity \( \bar{W} \) with aspect ratio \( \lambda \) is depicted in the Fig.2.2 for different values of the coupling number \( N \) two values of permeability parameter \( \bar{k} \). The curves corresponding to \( N=0 \) represent the Newtonian case. It is observed that the non-dimensional load carrying capacity increases for the increasing value of a coupling number \( N \). Fig.2.3 shows the
variations of $\bar{W}$ with $\log_{10} \lambda$ for different values of $\bar{I}$ with two values of permeability parameter $\bar{k}$. It is observed that $\bar{W}$ increases for increasing values of $\bar{I}$ further, it is also observed that the maximum load carrying capacity is attained for the square plates ($\lambda = 1$).

The effect of elastic parameter $E/K$ on variations of $\bar{W}$ with $\log_{10} \lambda$ is shown in the Fig.2.4 for two values of permeability parameter $\bar{k}$. It is observed that $\bar{W}$ increases with $\log_{10} \lambda$ and decreases for increasing values of $E/K$.

The effect of roughness parameter $\bar{c}$ on the variations of $\bar{W}$ with $\log_{10} \lambda$ is depicted in the Fig.2.5 for two value of permeability parameter $\bar{k}$. It is observed that $\bar{W}$ increases for the increasing values of $\bar{c}$ . Further, it is interesting to note that the maximum load carrying capacity $\bar{W}_{\text{max}}$ is a function of $\lambda$ and is obtained for the rectangular plates.

2.4.2 Squeeze Film Time

The variations of non-dimensional squeeze film time $\bar{T}$ with the aspect ratio $\lambda$ for different values of coupling number $N$ is depicted in the Fig.2.6 for two values of permeability parameter $\bar{k}$. The curves corresponding to $N=0$ represent the Newtonian case. It is observed that $\bar{T}$ increases for increasing value of $N$.

Fig.2.7 shows the variation of squeeze film time $\bar{T}$ with $\log_{10} \lambda$ for different values of $\bar{T}$ with two values of permeability parameter $\bar{k}$. The curves corresponding to $\bar{T} = 0$ represent the Newtonian case. It is observed that $\bar{T}$ increases for the
increases values of $\bar{T}$. Hence the squeeze film bearings lubricated with micropolar fluid carries larger load for a longer time as compared to the corresponding Newtonian fluids, by which improves the performance of the bearings.

The effect of elastic parameter $E/K$ on variations of $\bar{T}$ with $\log_{10} \lambda$ is shown in the Fig.2.8 for two values of permeability parameter $\bar{k}$. It is observed that $\bar{T}$ increases with $\log_{10} \lambda$ and decreases for increasing values of $E/K$. The effect of roughness parameter $\bar{c}$ on the variations of $\bar{T}$ with $\log_{10} \lambda$ is depicted in the Fig.2.9. It is observed that $\bar{T}$ increases for the increasing value of $\bar{c}$.

2.5 Conclusions

The effect of surface roughness on the squeeze film characteristics of rough poro-elastic rectangular plates is presented. On the basis of Eringen’s micropolar fluid theory and the Christensen stochastic theory for the study of rough surfaces, the modified form of stochastic Reynolds equation is derived for one dimensional longitudinal roughness pattern. As the micropolar fluid parameter $N \to 0$ and $\bar{T} \to 0$, the squeeze film characteristics reduce to corresponding Newtonian case and as $\bar{k} \to 0$ these characteristics reduce to the smooth case. On the basis of the results presented, the following conclusions are drawn:

1) The effect of micropolar fluid provides an increased load carrying capacity and squeeze film time as compared to the corresponding Newtonian case.

2) The effect of surface roughness on the cartilage surface increases the load carrying capacity and squeeze film time as compared to smooth case.
Hence in a practical situation the required shape of the bearing may be rectangular, in which case a specific choice of $\bar{k}, N, \bar{T}$ and $E/K$ will yield larger load carrying capacity and longer squeeze film time as compared to the corresponding Newtonian fluids, by which improves the performance of the bearings.
Fig. 2.2 Variation of non-dimensional load $\bar{W}$ with $\log_{10}(\lambda)$ for different values of $N$ with $\bar{l} = 0.2$, $E/K=0.3$ and $\bar{c} = 0.2$. 

$k = 7.65 \times 10^{-5}$ $\bar{k} = 4.3 \times 10^{-5}$ 

$\bar{\delta} = 300$ $\bar{\delta} = 200$

- - - N=0.0[Newtonian] 
- - - N=0.2 
- - - N=0.4 
- - - N=0.6
Fig. 2.3 Variation of non-dimensional load $W$ with $\log_{10}(\lambda)$ for different values of $\lambda$ with $N=0.2$, $E/K=0.3$ and $\bar{c}=0.2$. 

$\bar{k} = 7.65 \times 10^{-5}$, $\bar{k} = 4.3 \times 10^{-5}$

$\bar{\delta} = 300$, $\bar{\delta} = 200$
Fig. 2.4 Variation of non-dimensional load $\bar{W}$ with for different $\log_{10}(\lambda)$ values of $E/K$ with $N=0.2$, $\bar{l} = 0.2$ and $\bar{c} = 0.2$. 

$\bar{k} = 7.65 \times 10^{-5}$  $\bar{k} = 4.3 \times 10^{-5}$
$\bar{\delta} = 300$  $\bar{\delta} = 200$
Fig. 2.5 Variation of non dimensional load $\bar{W}$ with $\log_{10}(\lambda)$ for different values of $\bar{c}$ with $\bar{t} = 0.2$, $N = 0.2$ and $E/K = 0.3$. 

\[
\bar{k} = 7.65 \times 10^{-5} \quad \bar{k} = 4.3 \times 10^{-5} \\
\bar{\delta} = 300 \quad \bar{\delta} = 200 \\
- - - - - \bar{c} = 0.0 \quad - - - - - \bar{c} = 0.1 \quad - - - - - \bar{c} = 0.3 \quad - - - - - \bar{c} = 0.5
\]
Fig. 2.6 Variation of Squeeze film time $\bar{T}$ with $\log_{10}(\lambda)$ for different values of $N$ with $\bar{t} = 0.2$, $E/K = 0.3$ and $\bar{c} = 0.2$. 

$k = 7.65 \times 10^{-5} \quad k = 4.3 \times 10^{-3}$

$\bar{\delta} = 300 \quad \bar{\delta} = 200$

- - - - N=0.0[Newtonian]
- - - - N=0.2
- - - - N=0.4
- - - - N=0.6
Fig. 2.7 Variation of squeeze film time $\bar{T}$ with $\log_{10}(\lambda)$ for different values of $\bar{\ell}$ with $N=0.2$, $E/K=0.3$, $\bar{c} = 0.2$. 

$\bar{k} = 7.65 \times 10^{-5}$  $\bar{k} = 4.3 \times 10^{-5}$  
$\bar{\delta} = 300$  $\bar{\delta} = 200$  
- - - - $\bar{T}=0.0$ [Newtonian]  
- - - - $\bar{T}=0.2$  
- - - - $\bar{T}=0.4$  
- - - - $\bar{T}=0.6$
Fig. 2.8 Variation of non-dimensional Squeeze film time $\bar{T}$ with $\log_{10}(\lambda)$ for different values of $E/K$ with $N=0.2$, $\bar{t} = 0.2$ and $\bar{c} = 0.2$. 

$$\bar{k} = 7.65 \times 10^{-5} \quad \bar{k} = 4.3 \times 10^{-5}$$

$$\bar{\delta} = 300 \quad \bar{\delta} = 200$$

$- - E/K=0.1$

$- - - E/K=0.2$

$- - - E/K=0.3$
Fig. 2.9 Variation of non-dimensional squeeze film time $\bar{T}$ with $\log_{10}(\lambda)$ for different values $\bar{c}$ of with $N=0.6$, $\bar{l} = 0.2$ and $E/K=0.3$. 

$\bar{k} = 7.65 \times 10^{-5}$, $\bar{k} = 4.3 \times 10^{-5}$

$\bar{\delta} = 300$, $\bar{\delta} = 200$

$\cdot \cdot \cdot \cdot \bar{c}=0.0$

$\cdot \cdot \cdot \cdot \bar{c}=0.1$

$\cdot \cdot \cdot \cdot \bar{c}=0.3$

$\cdot \cdot \cdot \cdot \bar{c}=0.5$