8.1 Introduction

Synovial joints which are usually globular in appearance are covered with sponge like material called articular cartilage. The joint cavity is filled with sponge like material called synovial fluid. In the recent years considerable attention is paid by researchers on the studies of human locomotion such as knee joints and hip joints. The surfaces of synovial joints have a high degree of geometrical conformity. Their behavior is governed by articular cartilage which is soft glistening tissue with porous properties and synovial fluid which is dialysate of plasma with concentration of hyaluronic molecules, which do not normally pass through cartilage pores. In hydrodynamic lubrication the thickness of the fluid film is much larger than the height of surface roughness asperities. The pressure in the fluid film is because of the relative motion of surfaces and wedge action.

The squeeze film lubrication phenomenon is observed in several applications such as gears, bearings, machines tools, rolling elements and automotive engines (Naduvinamani and Santosh, 2010), dampers and human joints (Lin et al. 2004). The hip joint is a spherical joint between the femoral head and the acetabulum in the pelvis; it is a diarthrosis or synovial joint, since it is wrapped in a capsule that contains the synovial fluid, a biological lubricant that acts also like a shock absorber (Nordin and Frankel, 2001). The hip joint can transmit high dynamic loads (7-8 times body weight) and accommodate a wide range of movements. A number of lubrication theories have been proposed in the literature to account for the low coefficient of friction and low wear observed in healthy joints (Tepei 1979, Tsukamoto 1983). Normal joints exhibit coefficients of friction
of the order of 0.005-0.002 and undergo very little wear or degradation over several decades of use.

Articular cartilage is the bearing material that lines the ends of the bones of synovial joints. Its primary function is to reduce friction and wear at the articulations of the musculoskeletal system. The tribological properties of cartilage are intimately related to its structure and mechanical properties. The modes of lubrication in cartilage extend beyond the traditional mechanisms of fluid-film or boundary lubrication. Recently, Ateshian and Hung (2006) presented the properties of cartilage of natural joints. Cartilage is a white connective tissue which is synthesized and maintained by cells called chondrocyte. In human joints, the thickness of the articular cartilage layer varies from 0.5 to 1.5 mm in upper extremity joints, such as the hand and the shoulder and from 1 to 6 mm in lower extremity joints, such as the hip, knee, and ankle (Ateshian and Hung, 2006). Under normal conditions, articular cartilage provides low friction and wear over a life span. It is a highly hydrated tissue, with a porosity varying from 68 to 85 per cent in adult joints (Ateshian and Hung, 2006, Mow and Rik Huiskes, 2005).

Couple stresses are found to appear in noticeable magnitude in liquids with very large molecules. The additives stabilize the flow properties and minimize the sensitivity of the lubricant to changes in the shear rate. The long chain polysaccharide hyaluronic acid molecules present in the synovial fluid give us the motivation for modeling of the synovial fluid as a Stokes couple-stress fluid. Bujurke et al. (1990) have studied the effect of couple stresses in squeeze film poro-elastic bearings with special reference to synovial joints on the basis of Stokes (1966) couplestress fluid theory. Walicki and Walicka (2002) studied the inertia and

The experimental study of Sayles et al. (1979) on the measurement of the surface micrometry of articular cartilage revealed that, the cartilage surfaces are rough and roughness height distribution is Gaussian in nature. Hence, any realistic study of the joint lubrication must take in to account of surface roughness effects of articular cartilage. Christensen (1969-70) developed the stochastic theory to understand the effect of surface roughness in hydrodynamic lubrication of rough surfaces. Many researchers have used this theory to analyse the effect of surface roughness of various types of bearings (Walicka 2012, Bujurke et al. 2007, Prakash and Tiwari 1985). Recently, Naduvinamani and Savitramma (2012) studied the
micropolar fluid squeeze film lubrication between rough anisotropic poroelastic rectangular plates with a special reference to synovial joint lubrication.

The main objective of this chapter is to investigate the performance characteristics of finite rough poro-elastic partial journal bearing system lubricated with couplestress fluids in general and a hip joint lubrication in particular which has not been studied so far.

8.2 Mathematical Formulation of the Problem

A hip joint of human body is shown in Fig. 1.9. The physical configuration of a simplified model of hip joint lubrication i.e., squeeze film finite partial rough poro-elastic journal bearing with no journal rotation is shown in the Fig. 8.1. A journal of radius $R$ approaches the rough poro-elastic bearing of wall thickness $H_0$ at any circumferential section $\theta$ with a normal velocity $\left( \frac{dh}{dt} \right)$. The lubricant in the film region is considered to be an incompressible non-Newtonian Stokes couplestress fluid. To account for the surface roughness effects of the cartilage the film thickness is stochastic function consisting of two parts

$$H = h + h_s(\theta, z, \xi)$$  \hspace{1cm} (8.2.1)

where $h(= C - e \cos \theta)$ denotes the nominal smooth part of the film geometry, while $h_s(\theta, z, \xi)$ is the part due to the surface asperities measured from the nominal level and is regarded as a randomly varying quantity of zero mean, $\xi$ is an index determining a definite roughness arrangement and $\epsilon(= e / C)$ is the eccentricity ratio parameter. Further $C$ is the radial clearance and $\theta(= x / R)$ circumferential co-ordinate with $R$ being the radius of the journal.
Figure 8.1: A geometry of simplified model for hip joint
Governing Equations

i) Region-I: Fluid film region

The basic equations for the flow of couplestressed fluid in the fluid film region are as given in equations (7.2.2)-(7.2.5)

ii) Region-II: Poro-elastic region

The quasi-steady coupled governing equations of motion for the deformable cartilage matrix and mobile portion of fluid contained in its pores may be written in a slightly modified form of those given by Torzilli and Mow (1976)

\[ \text{div}\sigma_s - \frac{1}{k^*} \left( \frac{d\omega}{dt} - \dot{V} \right) = 0 \]  \hspace{1cm} (8.2.2a)

\[ \text{div}\sigma_f + \frac{1}{k^*} \left( \frac{d\omega}{dt} - \dot{V} \right) = 0 \]  \hspace{1cm} (8.2.2b)

Where \( \sigma_s \) and \( \sigma_f \) are the apparent stress tensors for the solid and fluid phases, \( \dot{\omega} \) is the displacement vector of the cartilage, \( \dot{V} \) is the fluid velocity and \( k^* \) is the permeability of the cartilaginous matrix to the fluid. Equations (8.2.2a) and (8.2.2b) represent force balances for the cartilage and the suspending medium of the synovial fluid components respectively. The second term in equations (8.2.2a) and (8.2.2b) represent diffusive drag arising from the relative velocities between solid and the fluid contents. Both are assumed to be incompressible. Omission of the inertia terms is justified because the diffusive drag coefficient \( K \) is of the order \( 10^{-15} \) Nsm\(^{-4}\) even in the unloaded state. Further, under slow conditions, the diffusive drag coefficient \( K \) is related to the permeability \( k^* \) of the tissue by the relation (Mow and Lai, 1980)

\[ k^* = \frac{1}{(1 + \alpha)^2 K} \]  \hspace{1cm} (8.2.3)
Where $\alpha$ is the ratio of solidity to porosity of the tissue.

Again, introducing the assumptions mentioned above in the generalized constitutive equations of each phase, their modified forms are obtained as

$$\sigma_s = p^* I + A \text{tr} e_s I + Ne_s$$  \hspace{1cm} (8.2.4a)

$$\sigma_f = -p^* I + D \text{tr} e_f I$$ \hspace{1cm} (8.2.4b)

Where $N$, $A$ and $D$ are the elastic parameters of the cartilage, $p^*$ is the hydrostatic pressure, $e_s$ is the strain tensor describing the deformation of the solid matrix. After neglecting the inertia terms addition of equations of (8.2.2a) and (8.2.2b) eliminates the pressure and fluid velocity and thereafter, taking the divergence of the results, yields the following Laplace equation

$$\nabla^2 \text{tr}(e_s) = 0$$ \hspace{1cm} (8.2.5)

where $\text{tr}(e_s)$ is known as the cartilage dilatation. Following Hori and Mockros (1976) the bulk modulus $E$ and $p^*$ can be related as

$$\text{tr}(e_s) = e_o + \frac{p^*}{E}$$ \hspace{1cm} (8.2.6)

From equations (8.2.5) and (8.2.6) we get

$$\nabla^2 p^* = 0$$ \hspace{1cm} (8.2.7)

The relevant boundary conditions for the velocity fields are

$$u = 0, \quad v = 0, \quad w = 0, \quad \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{at} \quad y = 0$$ \hspace{1cm} (8.2.8a)

$$u = -\sigma \frac{\partial u}{\partial y}, \quad w = -\sigma \frac{\partial w}{\partial y}, \quad v = V - v^*, \quad \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{at} \quad y = H$$ \hspace{1cm} (8.2.8b)

where $\sigma$ is the slip parameter, $V$ is the velocity of approach and $v^*$ is the normal component of relative velocity.
8.3 Solution of the Problem

The solution of equations (7.2.3) and (7.2.5) subject to the boundary conditions (8.2.8a) and (8.2.8b) is

\[ u = \frac{1}{2\mu} \frac{\partial p}{\partial x} \left\{ y \left[ (y-h) - \xi h + 2l \xi \tanh(h/2l) \right] + 2l^2 \left[ 1 - \frac{\cosh \left( (2y-h)/2l \right)}{\cosh(h/2l)} \right] \right\} \]

(8.3.1)

\[ w = \frac{1}{2\mu} \frac{\partial p}{\partial z} \left\{ y \left[ (y-h) - \xi h + 2l \xi \tanh(h/2l) \right] + 2l^2 \left[ 1 - \frac{\cosh \left( (2y-h)/2l \right)}{\cosh(h/2l)} \right] \right\} \]

(8.3.2)

where \( l = \sqrt{\frac{\eta}{\mu}} \), \( \xi = \frac{\sigma}{h+\sigma} \), \( \sigma = \frac{\alpha}{\sqrt{k}} \)

Integrate the continuity equation (7.2.2) with respect to \( y \) over the film thickness \( H \) and using the expression (8.3.1) and (8.3.2) the modified Reynolds equation is obtained in the form

\[ \frac{\partial}{\partial x} \left[ f(H,l,\xi) \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ f(H,l,\xi) \frac{\partial p}{\partial z} \right] = 12\mu \frac{\partial H}{\partial t} - 12\mu(v')_{y-n} \]

(8.3.3)

Integrating equation (8.2.7) with respect to \( y \) in the interval \( (H,H+H_0) \) and also using Morgan Cameron approximation we obtain

\[ \left[ \frac{\partial p'}{\partial y} \right]_{y-n} = H_0 \left[ \frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial z^2} \right] \]

(8.3.4)

By neglecting the inertia terms, equation (8.2.2b) may be arranged in terms of relative velocity in the form

\[ \left( \frac{r}{V'} - \frac{d\omega}{dt} \right) = -k'(\nabla p' - D \nabla e) \]

(8.3.5)
and elimination of \( e \), through equation (8.2.7) and (8.3.5) gives

\[
\left( \frac{r}{V} - \frac{d\omega}{dt} \right) = -k' \nabla p' \left( 1 - \frac{D}{E} \right) \tag{8.3.6}
\]

The normal component of the relative fluid velocity at the cartilage surface is given by

\[
v' = \left[ -k' \left( 1 - \frac{D}{E} \right) \frac{\partial p'}{\partial y} \right]_{y=H} \tag{8.3.7}
\]

By using equation (8.3.4) and (8.3.6) can be written as

\[
v' \bigg|_{y=H} = -k' \left( 1 - \frac{D}{E} \right) H_o \left[ \frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial z^2} \right] \tag{8.3.8}
\]

Integrating equation (7.2.2) across the fluid film and using the boundary conditions for \( v \) given in equations (8.2.8a) and (8.2.8b) and also using the expression (8.3.1), (8.3.2) and (8.3.8) the modified Reynolds type equation is obtained in the form

\[
\frac{\partial}{\partial x} \left[ f(H,l,\xi) - 12 \mu k' H_o \left( 1 - \frac{D}{E} \right) \frac{\partial p'}{\partial x} \right] + \frac{\partial}{\partial z} \left[ f(H,l,\xi) - 12 \mu k' H_o \left( 1 - \frac{D}{E} \right) \frac{\partial p'}{\partial z} \right] = 12 \mu \frac{\partial H}{\partial t} \tag{8.3.9}
\]

where

\[
f(H,l,\xi) = H^3 (1 + 3\xi) - 6\xi l H^2 \tanh(H/2l) - 12 l^3 H + 24 l^3 \tanh(H/2l)
\]

and \( V = \frac{dH}{dt} = -C \frac{d\xi}{dt} \cos \theta \)

Let \( f(h_s) \) be the probability density function of the stochastic film thickness \( h_s \) taking the stochastic average of equation (8.3.9) with respect to \( f(h_s) \), we obtain
\[
\frac{\partial}{\partial x} \left[ E[f(H,l,\xi)] - 12\mu k^* H_0 \left( 1 - \frac{D}{E} \right) \frac{\partial E(p^*)}{\partial x} \right] \\
+ \frac{\partial}{\partial z} \left[ E[f(H,l,\xi)] - 12\mu k^* H_0 \left( 1 - \frac{D}{E} \right) \frac{\partial (p^*)}{\partial z} \right] = 12\mu \frac{\partial H}{\partial t} 
\] (8.3.10)

Where expectancy operator \( E(g) \) is defined by

\[
E(g) = \int_{-\infty}^{\infty} (g(h)) \, dh 
\] (8.3.11)

In accordance with Christensen, we assume that

\[
f(h_l) = \begin{cases} 
\frac{35}{32c^2} (c^2 - h_l^2)^3, & -c \leq h_l \leq c \\
0 & \text{elsewhere}
\end{cases} 
\] (8.3.12)

where \( \sigma = c/3 \) is the standard deviation.

Using the following non-dimensional quantities

\[
\bar{z} = \frac{z}{L}, \bar{T} = \frac{t}{C}, \psi = \frac{k^* H_0}{C^2}, \theta = \frac{x}{R}, \bar{H} = \frac{H}{C} = h + \bar{h}_l, \quad \bar{h}_l = \frac{h_l}{C}, \\
\bar{k} = \frac{k^*}{C^2}, \bar{H}_0 = \frac{H_0}{C}, \bar{p} = \frac{p^* C^2}{\mu R^2 (dE/dt)}, \quad \bar{\varepsilon} = \frac{E}{C}, \bar{\sigma} = \frac{\sigma}{C}, \quad f(h,l,\xi) = Cf(h_0,\bar{h},\bar{z}), \\
\lambda = \frac{L}{2R}, \bar{C} = \frac{C}{C}
\]

The modified stochastic Reynolds equation can be written in non-dimensional form as

\[
\frac{\partial}{\partial \theta} \left[ E[f(\bar{H},\bar{T},\bar{z})] - 12\psi \left( 1 - \frac{D}{E} \right) \frac{\partial \bar{p}}{\partial \theta} \right] \\
+ \frac{1}{4\lambda^2} \frac{\partial}{\partial \xi} \left[ E[f(\bar{H},\bar{T},\bar{z})] - 12\psi \left( 1 - \frac{D}{E} \right) \frac{\partial \bar{p}}{\partial \xi} \right] = -12\cos \theta 
\] (8.3.13)

where

\[
f(\bar{H},\bar{T},\bar{z}) = \bar{H}^3 (1 + 3\bar{z}) - 6\bar{z} \bar{H}^2 \tanh(\bar{H}/2\bar{T}) - 12\bar{T}^2 \bar{H} + 24\bar{T}^3 \tanh(\bar{H}/2\bar{T})
\]
The left hand side of the stochastic Reynolds equation (8.3.13) will depend upon the structures of surface roughness and the following two types of one directional roughness structures are of special theoretical interest.

**Longitudinal Roughness**

For the longitudinal model, the roughness on the cartilage surface is assumed to have the form of long narrow ridges and valleys running in the $x$-direction. Therefore, the lubricant film thickness can be expressed as

$$\bar{H} = \bar{h} + \bar{h}_l (z, \xi)$$  \hspace{1cm} (8.3.14)

and the Reynolds type equation (8.3.13) can be reduced to

$$\frac{\partial}{\partial \theta} \left[ E \left[ f(\bar{H}, \bar{T}, \bar{\xi}) \right] - 12 \psi \left( 1 - \frac{D}{E} \right) \frac{\partial p}{\partial \theta} \right] + \frac{1}{4 \lambda^2} \frac{\partial}{\partial \xi} \left[ \frac{1}{E \left[ 1/f(\bar{H}, \bar{T}, \bar{\xi}) \right]} - 12 \psi \left( 1 - \frac{D}{E} \right) \frac{\partial p}{\partial \xi} \right] = -12 \cos \theta$$  \hspace{1cm} (8.3.15)

**Transverse Roughness**

For the transverse model, the roughness is assumed to have the form of long narrow ridges and valleys running in the $z$-direction. Therefore, the lubricant film thickness can be expressed as

$$\bar{H} = \bar{h} + \bar{h}_t (\theta, \xi)$$  \hspace{1cm} (8.3.16)

and the Reynolds type equation (8.3.13) can be reduced to

$$\frac{\partial}{\partial \theta} \left[ \frac{1}{E \left[ 1/f(\bar{H}, \bar{T}, \bar{\xi}) \right]} - 12 \psi \left( 1 - \frac{D}{E} \right) \frac{\partial p}{\partial \theta} \right] + \frac{1}{4 \lambda^2} \frac{\partial}{\partial \xi} \left[ E \left[ f(\bar{H}, \bar{T}, \bar{\xi}) \right] - 12 \psi \left( 1 - \frac{D}{E} \right) \frac{\partial p}{\partial \xi} \right] = -12 \cos \theta$$  \hspace{1cm} (8.3.17)
After simplification, the modified Reynolds type equations for longitudinal and transverse types of directional structures can be expressed as

\[
\frac{\partial}{\partial \theta} \left[ \alpha(\bar{H}, \bar{T}, \zeta) - 12 \psi \left( 1 - \frac{D}{E} \right) \hat{\bar{p}} \right] + \frac{1}{4 \lambda^2} \frac{\partial^2}{\partial \zeta^2} \left[ \beta(\bar{H}, \bar{T}, \zeta) - 12 \psi \left( 1 - \frac{D}{E} \right) \hat{\bar{p}} \right] = -12 \cos \theta
\]  

(8.3.18)

where

\[
\alpha(\bar{H}, \bar{T}, \zeta) = \begin{cases} 
E \left[ f(\bar{H}, \bar{T}, \zeta) \right] & \text{longitudinal roughness} \\
\frac{1}{E \left[ 1/f(\bar{H}, \bar{T}, \zeta) \right]} & \text{transverse roughness}
\end{cases}
\]

\[
\beta(\bar{H}, \bar{T}, \zeta) = \begin{cases} 
E \left[ 1/f(\bar{H}, \bar{T}, \zeta) \right] & \text{longitudinal roughness} \\
E \left[ f(\bar{H}, \bar{T}, \zeta) \right] & \text{transverse roughness}
\end{cases}
\]

\[
E \left[ f(\bar{H}, \bar{T}, \zeta) \right] = \bar{H} \left( 1 + 3 \zeta \right) + \frac{H c^2}{3} - 12 \bar{T}^2 \bar{H} + (24 \bar{T}^3 - 6 \zeta \bar{H} \bar{T}) \times g
\]

\[
g = \frac{35}{32 c^2} \int_{-c}^{c} (c^2 - h_i^2)^3 \times \tanh \left( \frac{\bar{H}}{2T} \right) \, d\bar{h}_i
\]

\[
\frac{1}{E \left[ 1/f(\bar{H}, \bar{T}, \zeta) \right]} = \frac{35}{32 c^2} \int_{-c}^{c} \frac{(c^2 - h_i^2)^3}{\bar{H} \left( 1 + 3 \zeta \right) - 6 \zeta \bar{H} \bar{T}^2 - 12 \bar{T}^2 \bar{H} + 24 \bar{T}^3 + \tanh \left( \frac{\bar{H}}{2T} \right)} \, d\bar{h}_i
\]

In order to solve the stochastic generalized Reynolds equation (8.3.18) to obtain the film pressure distribution of the journal bearing system. The Reynolds boundary conditions are used

\[
\bar{p} = 0 \quad \text{at} \quad \theta = \pm \frac{\pi}{2} \quad \text{(8.3.19a)}
\]

\[
\bar{p} = 0 \quad \text{at} \quad \zeta = \pm \frac{1}{2} \quad \text{(8.3.19b)}
\]
Numerical Formulation

Since the modified Reynolds equation (8.3.18) is too complicated to be solved analytically, a finite difference scheme is adopted. First, the film domain under consideration is divided by the grid spacing shown in Fig. 8.2. Then the mesh for the film extent is constructed. To avoid the divergence of the finite difference scheme, the conservative form of finite increment formats is applied in this case the terms of equation (8.3.18) are given by

\[
\frac{\partial}{\partial \theta} \left[ \alpha(H, T, \xi) \frac{\partial \bar{p}}{\partial \theta} \right] =
\frac{1}{\Delta \theta} \left[ \left( \bar{T}_{i - \frac{1}{2}, j} - 12 \psi \left( 1 - \frac{D}{E} \right) \left( \bar{p}_{i-1,j} - \bar{p}_{i,j} \right) \right) - \left( \bar{T}_{i + \frac{1}{2}, j} - 12 \psi \left( 1 - \frac{D}{E} \right) \left( \bar{p}_{i,j} - \bar{p}_{i+1,j} \right) \right) \right]
\]

(8.3.20)
\[
\frac{1}{4\lambda^2} \times \frac{\partial}{\partial \zeta} \left( \beta(\vec{\Phi}, \vec{T}, \vec{z}) \right) \frac{\partial \vec{P}}{\partial \zeta} = \\
\frac{1}{4\lambda^2} \times \frac{1}{\Delta \zeta} \left[ \left( \bar{f}_{i,j}^{1/2} - 12\psi \left(1 - \frac{D}{E}\right) \right) \left( \bar{p}_{i,j+1} - \bar{p}_{i,j} \right) - \left( \bar{f}_{i,j}^{1/2} - 12\psi \left(1 - \frac{D}{E}\right) \right) \left( \bar{p}_{i,j+1} - \bar{p}_{i,j} \right) \right] \\
\]  
(8.3.21)

Substituting these expressions (8.3.20) and (8.3.21) into the average Reynolds equation (8.3.18) we get

\[
\bar{p}_{i,j} = c_1 \bar{p}_{i+1,j} + c_2 \bar{p}_{i-1,j} + c_3 \bar{p}_{i,j+1} + c_4 \bar{p}_{i,j-1} + c_5
\]  
(8.3.22)

where

\[
C_0 = 4\lambda^2 r^2 \left\{ \left( \bar{f}_{i,j}^{1/2} - 12\psi \left(1 - \frac{D}{E}\right) \right) \left( \bar{p}_{i,j+1} - \bar{p}_{i,j} \right) + \left( \bar{f}_{i,j}^{1/2} - 12\psi \left(1 - \frac{D}{E}\right) \right) \left( \bar{p}_{i,j+1} - \bar{p}_{i,j} \right) \right\} \\
C_1 = \frac{4\lambda^2 r^2 \left( \bar{f}_{i,j}^{1/2} - 12\psi \left(1 - \frac{D}{E}\right) \right)}{C_0} \\
C_2 = \frac{4\lambda^2 r^2 \left( \bar{f}_{i,j}^{1/2} - 12\psi \left(1 - \frac{D}{E}\right) \right)}{C_0} \\
C_3 = \frac{\left( \bar{f}_{i,j}^{1/2} - 12\psi \left(1 - \frac{D}{E}\right) \right)}{C_0} \\
C_4 = \frac{\left( \bar{f}_{i,j}^{1/2} - 12\psi \left(1 - \frac{D}{E}\right) \right)}{C_0} \\
C_5 = \frac{48\lambda^2 \cos \theta (\Delta \zeta)^2}{C_0}, \quad r = \frac{\Delta \zeta}{\Delta \theta}
\]
The pressure, $\bar{p}$, is calculated by using the numerical method with grid spacing of $\Delta \theta = 9^\circ$ and $\Delta \bar{z} = 0.05$. The load carrying capacity of the bearing, $W$, generated by the film pressure is obtained by

$$W = -LR \int_{\theta=\frac{\pi}{2}}^{\theta=\frac{\pi}{2} + \frac{\pi}{2}} \int_{\bar{z}=\frac{z}{2}}^{\bar{z}=\frac{z}{2} + \frac{z}{2}} E(p^*) \cos \theta \ d\theta \ dz$$

(8.3.23)

The non-dimensional load carrying capacity, $\bar{W}$, of the 180° poro-elastic partial journal bearing is obtained in the form

$$\bar{W} = \frac{E(W)C^2}{\mu LR \left( \frac{d\varepsilon}{dt} \right)} = -\int_{\theta=\frac{\pi}{2}}^{\theta=\frac{\pi}{2} + \frac{\pi}{2}} \int_{\bar{z}=\frac{z}{2}}^{\bar{z}=\frac{z}{2} + \frac{z}{2}} \bar{p} \cos \theta \ d\theta \ dz$$

(8.3.24)

$$\approx \sum_{i=0}^{M} \sum_{j=0}^{N} \bar{p}_{i,j} \cos \theta \Delta \theta \cdot \Delta \bar{z} = g(\varepsilon, \bar{I}, \mu \varepsilon)$$

(8.3.25)

where $M+1$ and $N+1$ are the grid point numbers in the $x$ and $y$ directions respectively.

Time-height relation is calculated by considering the time taken by the journal to move from $\varepsilon = 0$ to $\varepsilon = \varepsilon$, can be obtained from equation (8.3.24)

$$\frac{d\varepsilon}{d\tau} = \frac{1}{g(\varepsilon, \bar{I}, \mu \varepsilon)}$$

(8.3.26)

where $\tau = \frac{WC^2}{\mu LR \varepsilon}$ is the non-dimensional response time.

The first order non-linear differential equation (8.3.26) is solved numerically by using the fourth order Runge-Kutta method with the initial conditions $\varepsilon = 0$ at $\tau = 0$. 

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8.4 Results and Discussion

A simplified model has been developed for analyzing the combined effects of surface roughness, couplestress fluid and poro-elasticity on lubrication characteristics of hip joints. All the bearing characteristics are functions of non-dimensional parameters $\bar{T}, \bar{c}, \psi \left(= \frac{k^2 H_0}{C^3} \right)$ and $\bar{\sigma}$. The parameter $\bar{T}$ arises due to the presence of polar additives in the lubricant. The dimension of ratio $\eta/\mu$ is length square and this length may be regarded as the chain length of polar additives in a non-polar lubricant. Hence, the parameter $\bar{T}$ gives the influence of couple stresses and their interactions with cartilage surfaces. In the limiting cases, $\bar{T} \to 0, \bar{c} \to 0$ and $\psi \to 0$ analysis corresponds to classical case.

8.4.1 Squeeze Film Pressure

The variation of non-dimensional pressure $\bar{p}$ with circumferential coordinate $\theta$ for different values of couple-stress parameter $\bar{T}$ is depicted in the Fig. 8.3 with the parameter values of $\varepsilon = 0.3, \bar{\sigma} = 0.3, \lambda = 1.5, \bar{c} = 0.2, D/E = 0.3$, and $\bar{k} = 7.65 \times 10^{-3}$ for both the types of roughness pattern. The effect of couplestress is to increase the pressure distribution for both types of roughness pattern compared to Newtonian case $\bar{T} \to 0$. Further the increase in $\bar{p}$ is more pronounced for the longitudinal roughness pattern as compared to the transverse roughness pattern.

The variation of non-dimensional pressure $\bar{p}$ with circumferential coordinate $\theta$ for different values of slip parameter $\bar{\sigma}$ is shown in the Fig. 8.4 with
the parametric values of \( \varepsilon = 0.3, \bar{c} = 0.2, \lambda = 1.5, \bar{t} = 0.4, D/E = 0.3 \) and \( \bar{k} = 7.65 \times 10^{-3} \), for both types of roughness patterns. It is observed that \( \bar{p} \) increases with \( \theta \) and decreases for increasing values of slip parameter \( \bar{\sigma} \) for both the types of roughness pattern. Thus the existing slip velocity and the porous boundary contribute to the easy functioning of human joints, particularly when the cartilage surfaces approach one another.

The variation of non-dimensional pressure \( \bar{p} \) with circumferential coordinate \( \theta \) for different values of roughness parameter \( \bar{c} \) is shown in the Fig.8.5 with the parametric values of \( \varepsilon = 0.3, \bar{\sigma} = 0.3, \lambda = 1.5, \bar{t} = 0.4, D/E = 0.3 \) and \( \bar{k} = 7.65 \times 10^{-3} \), for both types of roughness patterns. It is observed that, the fluid film pressure distribution increases (decreases) with increasing roughness parameter \( \bar{c} \) for longitudinal (transverse) roughness pattern as compared to the corresponding smooth case \( (\bar{c} = 0) \). Because of the presence of hyaluronic acid complex molecules, water and other low-molecular weight substances in the synovial fluid, a thick dense substance is being formed on the cartilage surfaces during the squeezing process. Also the presence of surface asperities on the articular cartilage reduces the fluid flow and the large fluid is retained in the lubricant region, which enhances pressure built up.

### 8.4.2 Load carrying capacity

The variation of non-dimensional load carrying capacity \( \bar{W} \) with the eccentricity ratio parameter \( \varepsilon \) for different values of \( \bar{t} \) is depicted in Fig.8.6 with the parametric values of \( \bar{\sigma} = 0.3, \lambda = 1.5, D/E = 0.3 \) and \( \bar{k} = 7.65 \times 10^{-3} \). It is
observed that, $\overline{W}$ increases for increasing values of $\overline{I}$. The effect of couplestressed fluid is to increase the load carrying capacity compared to Newtonian case ($\overline{I} = 0$) for both the types of roughness pattern. The couplestress will oppose the squeezing action and enhance the pressure in the film region, which results in increase of $\overline{W}$. This is an admissible prediction as the joints are capable of supporting load 3-4 times the body weight during normal functions and the load carried through the articulating surfaces can still be higher in energetic activities.

The variation of non-dimensional load carrying capacity $\overline{W}$ with the eccentricity ratio parameter $\varepsilon$ for different values of $\overline{\sigma}$ is shown in Fig.8.7 with the parametric values of $\overline{I} = 0.4, \lambda = 1.5, D/E = 0.3$ and $k = 7.65 \times 10^{-5}$. It is observed that $\overline{W}$ decreases for increasing values of $\overline{\sigma}$ for both longitudinal and transverse roughness pattern. Further, the increase in $\overline{W}$ is more pronounced for the longitudinal roughness pattern as compared to the transverse roughness pattern.

Figure 8.8 shows the variation of non-dimensional load carrying capacity $\overline{W}$ with the eccentricity ratio parameter $\varepsilon$ for different values of $D/E$ with the parametric values of $\overline{I} = 0.4, \lambda = 1.5, \overline{\sigma} = 0.3, \varepsilon = 0.3$ and $k = 7.65 \times 10^{-5}$. It is observed that $\overline{W}$ increases with $\varepsilon$ and decreases for increasing values of D/E for both the types of roughness pattern. Further, the increase in $\overline{W}$ is more pronounced for the longitudinal roughness pattern as compared to the transverse roughness pattern.

The effect of roughness parameter $\overline{c}$ on the variation of $\overline{W}$ with the eccentricity ratio parameter $\varepsilon$ is depicted in the Fig.8.9 with the parametric values
of $\bar{T} = 0.6$, $\lambda = 1.5$, $\varepsilon = 0.3$, $\bar{\sigma} = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$ for both types of roughness patterns. It is observed that $\bar{W}$ increase in $\bar{c}$ for longitudinal roughness whereas it decreases for transverse roughness compared to corresponding smooth case.

### 8.4.3 Time-Height Relation

The response time of the squeeze film is one of the significant factors in the design of bearings. The response time is the time that will elapse for a squeeze film reduces to some minimum permissible squeeze film height. The variation of squeeze film height $\bar{h}_0$ with response time $\tau$ as a function of $\bar{T}$ is shown in the Fig.8.10. It is observed that, the response time increases for increasing values of $\bar{T}$ for both the types of roughness patterns, hence the pressure of the microstructures in the lubricant the squeeze film time as compared to that of the Newtonian lubricants. Further, the bearings with couple stress fluid as lubricant have longer response time as compared to the corresponding Newtonian case. This is due to the fact that, as the couple-stress parameter increases the concentration of hyaluronic acid molecules increases. Fig.8.11 depicts the variation of squeeze film height $\bar{h}_0$ with response time $\tau$ for different values of $D/E$ with two values of permeability parameter $\bar{k}$. It is observed that $\tau$ increases with $\bar{h}_0$ and decreases for increasing values of $D/E$ for both the types of roughness pattern. Further, the squeeze film time for the joints with degenerate cartilage ($D/E=0.1$) is quite large compared with normal cartilage ($D/E=0.3$).

The variation of squeeze film height $\bar{h}_0$ with response time $\tau$ for different values of $\bar{\sigma}$ is depicted in the Fig.8.12 for two values of permeability parameter $\bar{k}$.
It is observed that $\tau$ increases with $\bar{h}$ and decreases for increasing values of slip parameter $\bar{\sigma}$ for both longitudinal and transverse roughness pattern. This appears to be consistent with the earlier results that the slip velocity occurring at a porous boundary helps in maintaining normal functioning of human joints.

The effect of roughness parameter, $\bar{c}$ on the variation of $\bar{h}$ with $\tau$ depicted in the Fig.8.13 with the parametric values of $\bar{T} = 0.6$, $\lambda = 1.5$, $\varepsilon = 0.3$, $\bar{\sigma} = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$ for both types of roughness patterns. It is interesting to note that the effect of $\bar{c}$ is to increase (decrease) the response time of the squeeze film for the longitudinal (transverse) roughness pattern as compared to the corresponding smooth case ($\bar{c} = 0.0$).

### 8.5 Conclusions

The effect of surface roughness on the squeeze film characteristics of finite partial rough poro-elastic journal bearing is presented. On the basis of Stokes couplstress fluid theory and the Christensen stochastic theory for the study of rough surfaces, the modified form of stochastic Reynold’s equation is derived for one-dimensional longitudinal and transverse roughness patterns. The governing equations along with the appropriate constitutive relationships and boundary conditions have been formulated for modeling the roughness structure of cartilage with couplstress fluid in the lubricant region in hip joint lubrication. The finite difference method is found to be accurate for the solution of modified Reynolds equation. As the coupl-stress fluid parameter $\bar{T} \to 0$ the squeeze film characteristics reduce to corresponding Newtonian case and as $\bar{c} \to 0$ these
characteristics reduce to the smooth case. On the basis of the results presented, the following conclusions are drawn.

1. The effect of couplestress fluid provides an increased load carrying capacity and squeeze film time as compared to the corresponding Newtonian case.

2. The Christensen surface roughness on the cartilage surface for longitudinal (transverse) pattern on the finite poro-elastic partial journal bearings increases (decreases) the load carrying capacity and the squeeze film time as compared to the corresponding smooth case.

3. The proposed rough poro-elastic and couple stress fluid model predicts some of the salient features of bearing characteristics which would enable us in the selection of suitable design parameters.
**Figure 8.3:** Variation of non-dimensional squeeze film Pressure $\bar{p}$ with $\theta$ for different values of $\bar{T}$ with $\varepsilon = 0.3$, $\bar{\sigma} = 0.3$, $\lambda = 1.5$, $\bar{c} = 0.2$, $D/E = 0.3$, and $k = 7.65 \times 10^{-5}$
Longitudinal case

Transverse case

**Figure 8.4:** Variation of non-dimensional squeeze film Pressure $\bar{p}$ with $\theta$ for different values of $\bar{\sigma}$ with $\varepsilon = 0.3, \bar{T} = 0.4, \lambda = 1.5, \bar{c} = 0.3, D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-3}$. 
Figure 8.5: Variation of non-dimensional squeeze film pressure $\bar{p}$ with $\theta$ for different values of $\bar{c}$ with $\varepsilon = 0.3, \bar{I} = 0.4, \lambda = 1.5, \bar{\sigma} = 0.3, D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-4}$. 

Longitudinal case  \hspace{1cm}  Transverse case
Fig. 8.6 Variation of non-dimensional load carrying capacity $\bar{W}$ with $\varepsilon$ for different values of $\bar{T}$ with $\varepsilon = 0.3, \bar{T} = 0.4, \lambda = 1.5, \bar{\sigma} = 0.3, D/E = 0.3, \bar{c} = 0.3$ and $k = 7.65 \times 10^{-5}$. 
Fig. 8.7 Variation of non-dimensional load carrying capacity $\bar{W}$ with $\varepsilon$ for different values of $\bar{\sigma}$ with $\bar{t} = 0.2$, $\lambda = 1.5$, $\bar{c} = 0.3$ $D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 8.8 Variation of non-dimensional load carrying capacity $\bar{W}$ with $\epsilon$ for different values of $D/E$ with $\bar{I} = 0.2$, $\lambda = 1.5$, $\bar{\sigma} = 0.3$, $\bar{c} = 0.3$ and $\bar{k} = 7.65 \times 10^{-8}$. 
Fig. 8.9 Variation of non-dimensional load carrying capacity $\overline{W}$ with $\varepsilon$ for different values of $\overline{\varepsilon}$ with $\overline{l} = 0.6$, $\overline{\lambda} = 1.5$, $\overline{\sigma} = 0.3$, $\overline{\tau} = 0.3$ and $\overline{\kappa} = 7.65 \times 10^{-5}$. 
Fig. 8.10 Variation of non-dimensional minimum film height $\overline{h}_0$ with $\tau$ for different values of $\overline{I}$ with $D/E = 0.3$, $\lambda = 1.5$, $\bar{\sigma} = 0.3$, $\bar{\varepsilon} = 0.3$, $\bar{\varphi} = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 8.11 Variation of non-dimensional minimum film height $\bar{h}_0$ with $\tau$ for different values of $D/E$ with $\bar{l} = 0.4$, $\lambda = 1.5$, $\sigma = 0.3$, $\varepsilon = 0.3$, $\bar{c} = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 8.12 Variation of non-dimensional minimum film height $\bar{h}_0$ with $\tau$ for different values of $\bar{\sigma}$ with $\bar{T} = 0.4$, $\bar{\lambda} = 1.5$, $D/E = 0.3$, $\varepsilon = 0.3$, $\bar{\sigma} = 0.3$ and $k = 7.65 \times 10^{-4}$. 
Fig. 8.13 Variation of non-dimensional minimum film height $\bar{h}_0$ with $\tau$ for different values of $\sigma$ with $\bar{T} = 0.4$, $\lambda = 1.5$, $D/E = 0.3$, $\varepsilon = 0.3$, $\sigma = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$. 