7.1 Introduction

Human joints function more or less as mechanical bearings under physiological circumstances, their surfaces being lubricated by a special bio-fluid known as synovial fluid and therefore they are known as synovial joints. The synovial joint is a remarkable tribological product of nature with low friction and high wear resistance during its life time of service.

The porous squeeze film hydrodynamic lubrication problem describes the action of a rigid body in normal motion on a confined lubricating film. This basic configuration has attracted the attention of many researchers because it is often confronted in industry and biomechanics (Megat et al. 1998; Hou et al. 1992; Naduvinamani et al. 2010).

The hip joint is a ball and socket synovial joint that allows movement in three planes. The schematic diagram of a hip joint is shown in Fig.1.9. The cartilage covered ball of the boney femur is closely fitted to the acetabulum of the pelvis. The 1 to 3 mm thick layer of articular cartilage covering the ball and socket provides frictionless movement and protects the joint from pain. The joint tissues include a strong fibrous articular capsule that helps to stabilize the femoral head in the acetabulum. Natural hip joints are remarkable bearings in engineering terms. The bearing materials are articular cartilage, supported by subchondral bones and lubricated by synovial fluid. The load experienced in hip joints during normal steady walking can reach up to 3-5 times body weight, while wide range of motions is encountered. The healthy hip joint can last more than 70 years with minimum friction and wear.
Human articulations synovial fluid contains long chain hyaluronic acid as natural additives (Fujimura et al. 2005; Momberger et al. 2005; Suguchi et al. 2005). The resulting nonlinear behavior significantly influences the characteristics of the fluid; the non-Newtonian effect must be taken into account. In addition to this non-Newtonian effect, the elastic deformation of a contiguous porous matrix (Feng et al. 2000) may be significant.

Stokes (1966) has formulated the theory of couple-stresses in fluids. The theory due to Stokes allows for polar effects such as the presence of couple-stresses and body couples. The theory has been applied to the study of some simple lubrication problems. According to the theory, couple-stresses are found to appear in noticeable magnitudes in fluids with large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluid motivated for modeling of the synovial fluid as a couple-stress fluid. Walicki and Walicka (1999) have studied the modeled synovial fluid as a couplestress fluid in human joints. Bujurke et al. (1991) have studied the theoretical modeling of poro-elastic slider bearings lubricated by couple-stress fluids with special reference to synovial joints. Lin (1997, 1998) studied the effects of couple-stress on the performance of short journal bearings and squeeze film characteristics of finite journal bearings. Recently, Naduvinamani and Patil (2009) studied the Numerical solution of finite modified Reynolds equation for couplestress squeeze film lubrication of porous journal bearings.

In this chapter, the squeeze film characteristics of finite poro-elastic journal bearings lubricated with couple-stress fluid is studied. The modified finite Reynolds equation derived for the squeeze film lubrication of poro-elastic partial journal
bearing is solved numerically by using finite difference technique. The load
 carrying capacity and time-height relation are compared with the classical
Newtonian case.

7.2 Mathematical Formulation of the Problem

The physical configuration of a simplified model of a hip joint is considered
as squeeze film lubrication finite poro-elastic partial journal bearing with no journal
rotation and is shown in the Fig. 7.1. The journal of radius \( R \) approaches the poro-
elastic bearing of wall thickness \( H \) at any circumferential section \( \theta \) with
velocity \( V \left( \frac{dh}{dt} \right) \). The lubricant in the film region and also in the poro-elastic
region is taken to be Stokes couple-stress fluid. The film thickness \( h \) is a function of
\( \theta \) is given by

\[
h = C(1 - \varepsilon \cos \theta)
\]

(7.2.1)

where \( \varepsilon (= e / C) \) is the eccentricity ratio, \( C \) is the radial clearance and \( e \) being the
eccentricity.
Poro-elastic region

Solid backing

Film region

Figure 7.1: A geometry of simplified model for hip joint
Governing Equations

Region-I (Fluid film region)

The constitutive equations governing the motion of an incompressible couple-stress fluid in the absence of body forces and body couples, derived by Stokes (1966) are as given in equations (1.10.1.7) - (1.10.1.10) in the form

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \\
\mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^2 u}{\partial y^2} &= \frac{\partial p}{\partial x} \\
\frac{\partial p}{\partial y} &= 0 \\
\mu \frac{\partial^2 w}{\partial y^2} - \eta \frac{\partial^2 w}{\partial y^2} &= \frac{\partial p}{\partial z}
\end{align*}
\] (7.2.2 - 7.2.5)

where \( u, v \) and \( w \) are velocity components in \( x, y \) and \( z \) directions respectively.

Region-II (Poro-elastic region)

The quasi-steady coupled governing equations of motion for the deformable cartilage matrix and mobile portion of fluid contained in its pores may be written in a slightly modified form of those given by Torzilli and Mow (1976)

\[
\begin{align*}
\text{div} \sigma_s - \frac{1}{k'} \left( \frac{d \omega}{dt} - \dot{V}' \right) &= 0 \\
\text{div} \sigma_f + \frac{1}{k'} \left( \frac{d \omega}{dt} - \dot{V}' \right) &= 0
\end{align*}
\] (7.2.6a - 7.2.6b)

Where \( \sigma_s \) and \( \sigma_f \) are the apparent stress tensors for the solid and fluid phases, \( \dot{\omega} \) is the displacement vector of the cartilage, \( \dot{V}' \) is the fluid velocity and \( k' \) is the permeability of the cartilaginous matrix to the fluid. Equations (7.2.6a) and (7.2.6b)
represent force balances for the cartilage and the suspending medium of the synovial fluid components respectively. The second term in equations (7.2.6a) and (7.2.6b) represent diffusive drag arising from the relative velocities between solid and the fluid contents. Both are assumed to be incompressible. Omission of the inertia terms is justified because the diffusive drag coefficient $K$ is of the order $10^{-15}$ N$\text{sm}^{-4}$ even in the unloaded state. Further, under slow conditions, the diffusive drag coefficient $K$ is related to the permeability $k^*$ of the tissue by the relation Mow and Lai (1980).

$$k^* = \frac{1}{(1+\alpha)^2 K} \quad (7.2.7)$$

where $\alpha$ is the ratio of solidity to porosity of the tissue.

Again, introducing the assumptions mentioned above in the generalized constitutive equations of each phase, we obtain their modified forms as

$$\sigma_i = p^* I + A \text{tr} e, I + Ne_i \quad (7.2.8a)$$

$$\sigma_f = -p^* I + D \text{tr} e, I \quad (7.2.8b)$$

where $N, A$ and $D$ are the elastic parameters of the cartilage. $p^*$ is the hydrostatic pressure, $e_i$ is the strain tensor describing the deformation of the solid matrix. After neglecting the inertia terms addition of equations of (7.2.6a) and (7.2.6b) eliminate the pressure and fluid velocity and thereafter, taking the divergence of the results, yields the following Laplace equation

$$\nabla^2 (\text{tr}(e_i)) = 0 \quad (7.2.9)$$

where $\text{tr}(e_i)$ is known as the cartilage dilatation. Following Hori and Mockros (1976) the bulk modulus $E$ and $p^*$ can be related as

$$\text{tr}(e_i) = e_0 + \frac{p^*}{E} \quad (7.2.10)$$
From equations (7.2.9) and (7.2.10) we get

\[ \nabla^2 p' = 0 \]  \hspace{1cm} (7.2.11)

The relevant boundary conditions for the velocity fields are

\begin{align*}
    u = 0, & \quad v = 0, \quad w = 0, \quad \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{at} \quad y = 0 \hspace{1cm} (7.2.12a) \\
    u = -\sigma \frac{\partial u}{\partial y}, & \quad w = -\sigma \frac{\partial w}{\partial y}, \quad v = V - v', \quad \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{at} \quad y = h \hspace{1cm} (7.2.12b)
\end{align*}

where \( \sigma \) is the slip parameter, \( V(=\partial h/\partial t) \) is the velocity of approach and \( v' \) is the normal component of relative velocity.

### 7.3 Solution of the Problem

The solution of equation (7.2.3) and (7.2.5) subject to the boundary conditions (7.2.12a) and (7.2.12b) is

\begin{align*}
    u &= \frac{1}{2\mu} \frac{\partial p}{\partial x} \left\{ y[(y-h) - \xi h + 2l\xi \tanh(h/2l)] + 2l^2 \left[ 1 - \frac{\cosh\left(2y-h\right)/2l\right]}{\cosh(h/2l)} \right\} \hspace{1cm} (7.3.1) \\
    w &= \frac{1}{2\mu} \frac{\partial p}{\partial z} \left\{ y[(y-h) - \xi h + 2l\xi \tanh(h/2l)] + 2l^2 \left[ 1 - \frac{\cosh\left(2y-h\right)/2l\right]}{\cosh(h/2l)} \right\} \hspace{1cm} (7.3.2)
\end{align*}

Where \( l = \sqrt{\eta/\mu}, \quad \xi = \frac{\sigma}{h+\sigma}, \quad \sigma = \frac{\alpha}{\sqrt{k}} \)

Integrate the continuity equation (7.2.2) with respect to \( y \) over the film thickness \( h \) and using the expression (7.3.1) and (7.3.2) the modified Reynolds equation is obtained in the form

\[ \frac{\partial}{\partial x} \left[ f(h,l,\xi) \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ f(h,l,\xi) \frac{\partial p}{\partial z} \right] = 12\mu \frac{\partial h}{\partial t} - 12\mu (v')_{y=h} \hspace{1cm} (7.3.4) \]
Integrating equation (7.2.11) with respect to \( y \) in the interval \((h, h+H)\) and also using Morgan Cameron approximation we obtain

\[
\left[ \frac{\partial p^*}{\partial y} \right]_{y=h} = H \left[ \frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial z^2} \right] \tag{7.3.5}
\]

By neglecting the inertia terms, equation (7.2.6b) may be arranged in terms of relative velocity in the form

\[
\left( \frac{r}{V'} - \frac{d\omega}{dt} \right) = -k' (\nabla p^* - D \nabla e_r) \tag{7.3.6}
\]

and elimination of \( e_r \) through equation (7.2.11) and (7.3.6) gives

\[
\left( \frac{r}{V'} - \frac{d\omega}{dt} \right) = -k' \nabla p^* \left( 1 - \frac{D}{E} \right) \tag{7.3.7}
\]

The normal component of the relative fluid velocity at the cartilage surface is given by

\[
v^* = \left[ -k' \left( 1 - \frac{D}{E} \right) \frac{\partial p^*}{\partial y} \right]_{y=h} \tag{7.3.8}
\]

By using equation (7.3.5), equation (7.3.7) can be written as

\[
v^* \big|_{y=h} = -k' \left( 1 - \frac{D}{E} \right) H \left[ \frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial z^2} \right] \tag{7.3.9}
\]

Integrating equation (7.2.2) across the fluid film and using the boundary conditions for \( v \) given in equations (7.2.12a) and (7.2.12b) and also using the expression (7.3.1), (7.3.2) and (7.3.9) the modified Reynolds equation is obtained in the form

\[
\frac{\partial}{\partial x} \left[ \left\{ f(h,l,\xi) - 12\mu k' H \left( 1 - \frac{D}{E} \right) \right\} \frac{\partial p^*}{\partial x} \right]

+ \frac{\partial}{\partial z} \left[ \left\{ f(h,l,\xi) - 12\mu k' H \left( 1 - \frac{D}{E} \right) \right\} \frac{\partial p^*}{\partial z} \right] = 12\mu \frac{\partial h}{\partial t} \tag{7.3.10}
\]

where
\[ f(h, l, \xi) = h^3(1 + 3\xi) - 6\xi lh^2 \tanh(h / 2l) - 12l^3 h + 24l^3 \tanh(h / 2l) \]

and \( V = \frac{dh}{dt} = -C \frac{d\varepsilon}{dt} \cos \theta \)

Using the following non-dimensional quantities

\[
\bar{z} = \frac{z}{L}, \bar{T} = \frac{t}{C}, \psi = \frac{k^* H}{C^3}, \quad \theta = \frac{x}{R}, \quad \bar{h} = \frac{h}{C} = 1 - \varepsilon \cos \theta, \quad \bar{k} = \frac{k^*}{C^2}, \quad \bar{H} = \frac{H}{C},
\]

\[
\bar{p} = \frac{p^* C^2}{\mu R^2 (d\varepsilon / dt)} , \quad \varepsilon = \frac{e}{C}, \quad \bar{\sigma} = \frac{\sigma}{C}, \quad f(h, l, \xi) = C^3 f(\bar{h}, \bar{T}, \bar{\xi}), \quad \lambda = \frac{L}{2R}
\]

in to the equation (7.3.10), the non-dimensional modified Reynolds equation is obtained in the form

\[
\frac{\partial}{\partial \theta} \left[ \left( f(\bar{h}, \bar{T}, \bar{\xi}) - 12\psi^3 \left( 1 - \frac{D}{E} \right) \frac{\partial \bar{p}}{\partial \psi} \right) \right] + \frac{1}{4\lambda^2} \frac{\partial}{\partial \bar{z}} \left[ \left( f(\bar{h}, \bar{T}, \bar{\xi}) - 12\psi^3 \left( 1 - \frac{D}{E} \right) \frac{\partial \bar{p}}{\partial \bar{z}} \right) \right] = -12 \cos \theta
\]  

(7.3.11)

Where

\[
f(\bar{h}, \bar{T}, \bar{\xi}) = \bar{h}^3(1 + 3\bar{\xi}) - 6\bar{\xi} \bar{h}^2 \tanh(\bar{h} / 2\bar{T}) - 12\bar{T}^3 \bar{h} + 24\bar{T}^3 \tanh(\bar{h} / 2\bar{T})
\]

As the permeability \( k^* \to 0 \) equation (7.3.11) reduces to the corresponding solid case.

For 180° partial poroelastic journal bearing the boundary conditions for the fluid pressure are

\[
\bar{p} = 0 \quad \text{at} \quad \theta = \pm \frac{\pi}{2} \quad (7.3.12a)
\]

\[
\bar{p} = 0 \quad \text{at} \quad \bar{z} = \pm \frac{1}{2} \quad (7.3.12b)
\]

The non-dimensional modified Reynolds equation is solved numerically by using a finite difference scheme. The film domain under consideration is divided by the
grid spacing shown in Fig.7.2. In finite increment format, the terms of equation (7.3.11) can be expressed as

\[
\frac{\partial}{\partial \theta} \left[ \tilde{f} - 12\psi \left( 1 - \frac{D}{E} \right) \frac{\partial \tilde{p}}{\partial \theta} \right] =
\]

\[
\frac{1}{\Delta \theta} \left[ \left( \tilde{f}_{i+\frac{1}{2},j} - 12\psi \left( 1 - \frac{D}{E} \right) \right) \left( \tilde{p}_{i+1,j} - \tilde{p}_{i,j} \right) - \left( \tilde{f}_{i-\frac{1}{2},j} - 12\psi \left( 1 - \frac{D}{E} \right) \right) \left( \tilde{p}_{i,j} - \tilde{p}_{i,j-1} \right) \right]
\]

(7.3.13)

\[
\frac{1}{4\lambda^2} \times \frac{\partial}{\partial \zeta} \left[ \left( \tilde{f} - 12\psi \left( 1 - \frac{D}{E} \right) \right) \frac{\partial \tilde{p}}{\partial \zeta} \right] =
\]

\[
\frac{1}{4\lambda^2} \times \frac{1}{\Delta \xi} \left[ \left( \tilde{f}_{i,j+1} - 12\psi \left( 1 - \frac{D}{E} \right) \right) \left( \tilde{p}_{i,j+1} - \tilde{p}_{i,j} \right) - \left( \tilde{f}_{i,j-1} - 12\psi \left( 1 - \frac{D}{E} \right) \right) \left( \tilde{p}_{i,j} - \tilde{p}_{i,j-1} \right) \right]
\]

(7.3.14)

Figure 7.2: Grid point notation for film domain

Substituting these expressions (7.3.13) and (7.3.14) into the modified Reynolds equation (7.3.11) we get

\[
\tilde{p}_{i,j} = c_1 \tilde{p}_{i+1,j} + c_2 \tilde{p}_{i-1,j} + c_3 \tilde{p}_{i,j+1} + c_4 \tilde{p}_{i,j-1} + c_5
\]

(7.3.15)
Where

\[
c_i = \frac{4\lambda^2 r^2 \left( \bar{f}_{i, i+\frac{1}{2}} - 12\psi \left( 1 - \frac{D}{E} \right) \right)}{c_0}
\]

\[
c_2 = \frac{4\lambda^2 r^2 \left( \bar{f}_{i, i+\frac{1}{2}} - 12\psi \left( 1 - \frac{D}{E} \right) \right)}{c_0}
\]

\[
c_3 = \frac{ \left( \bar{f}_{i, i+\frac{1}{2}} - 12\psi \left( 1 - \frac{D}{E} \right) \right)}{c_0}
\]

\[
c_4 = \frac{ \left( \bar{f}_{i, i+\frac{1}{2}} - 12\psi \left( 1 - \frac{D}{E} \right) \right)}{c_0}
\]

\[
c_5 = \frac{48\lambda^2 \cos \theta \left( \Delta z \right)^2}{c_0}, \quad r = \frac{\Delta z}{\Delta \theta}
\]

The pressure, \( \bar{p} \) is calculated by using the numerical method with grid spacing of \( \Delta \theta = 9 \) and \( \Delta z = 0.05 \).

The load carrying capacity of the bearing, \( W \) generated by the film pressure is obtained by

\[
W = -LR \int_{\rho=\frac{1}{2}}^{\rho=\frac{1}{2}} \int_{\psi=\frac{1}{2}}^{\psi=\frac{1}{2}} p^* \cos \theta \ d\theta \ dz
\]

(7.3.16)

The non-dimensional load carrying capacity, \( \bar{W} \) of the 180° poro-elastic partial journal bearing is obtained in the form
\[
\bar{W} = \frac{WC^2}{\mu LR^2} \left( \frac{d\varepsilon}{dt} \right) = -\int_{\theta-\frac{\pi}{2}}^{\theta} \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} p \cos \theta \, d\theta \, d\bar{z} \tag{7.3.17}
\]

\[
\approx \sum_{i=0}^{M} \sum_{j=0}^{N} \bar{p}_{i,j} \cos \theta \Delta \theta \cdot \Delta \bar{z} = g(\varepsilon, \bar{\ell}, \psi) \tag{7.3.18}
\]

where \(M\) and \(N\) are the grid point numbers in the \(x\) and \(y\) directions respectively.

Time-height relation is calculated by considering the time taken by the journal to move from \(\varepsilon = 0\) to \(\varepsilon = \varepsilon_1\), can be obtained from equation (7.3.18)

\[
\frac{d\varepsilon}{d\tau} = \frac{1}{g(\varepsilon, \bar{\ell}, \psi)} \tag{7.3.19}
\]

where \(\tau = \frac{WC^2}{\mu LR^2} t\) is the non-dimensional response time.

The first order non-linear differential equation (7.3.19) is solved numerically by using the fourth order Runge-Kutta method with the initial conditions \(\varepsilon = 0\) at \(\tau = 0\).

### 7.4 Results and Discussion

To solve squeeze film pressure in the equation (7.3.15) the mesh of the film domain has 20 equal intervals along the bearing length and circumference. The coefficient matrix of the system of algebraic equations is of pentadiagonal form. These equations have been solved by using scilab tools.

On the basis of Stokes (1969) microcontinuum theory, for couplestress fluids, the effect of couple-stresses on the squeeze film characteristics of finite poro-elastic partial journal bearings has been investigated theoretically. The modified Reynolds equation is derived by using the Stokes constitutive equations to
account for the couple-stress effects due to the lubricant blended with additives. The effect of couple-stresses on the squeeze film characteristics of a finite poroelastic bearing is observed through the non-dimensional couple stress parameter $\overline{T}(=l/C)$, the effect of permeability is characterized by the permeability parameter $\varphi\left(=\frac{k}{C^3}\right)$ and the slip parameter $\sigma$ and it is to be noted that as $\varphi \to 0$ the problem reduce to the corresponding solid case and as $T \to 0$ it reduces to the corresponding Newtonian case.

### 7.4.1 Squeeze Film Pressure

The variation of non-dimensional squeeze film pressure $\overline{p}$ with the circumferential co-ordinate $\theta$ for various values of $\overline{T}$ is depicted in Fig.7.3 with the parametric values of $\varepsilon = 0.1$, $\bar{\sigma} = 0.1$ and $D/E = 0.3$, $\bar{k} = 7.65 \times 10^{-5}$. It is observed that $\overline{p}$ increases for increasing values of $\overline{T}$. Increases in $\overline{p}$ is more pronounced for larger values of $\overline{T}$. The two material constants $\mu$ and $\eta$ which are present in couple-stress fluid as polar additives are responsible for long chain hyaluronic acid (HA) molecules. The water and low molecular weight substances present in the lubricant are forced into poroelastic cartilage by the squeeze film action of the impinging cartilaginous surfaces. This leads to an increase of the concentration of the polymer additives on the cartilage surfaces, producing larger pressure due to increased viscosity of the lubricant. The effect of elastic parameter $D/E$ on variations of $\overline{p}$ with $\theta$ is shown in the Fig.7.4. It is observed that $\overline{p}$ increases with $\theta$ and decreases for increasing values of $D/E$. Further, squeeze film
pressure $\bar{p}$ decreases as intra articular gap between two articular surfaces decreases for all values of elastic parameter $D/E$.

Figure 7.5 shows the variation of $\bar{p}$ with $\theta$ for a different values of the slip parameter $\sigma$ with $\bar{k} = 7.65 \times 10^{-5}, \bar{I} = 0.3, D/E = 0.3, \lambda = 0.75$ and $\varepsilon = 0.1$. It is observed that $\bar{p}$ decreases for increasing values of slip parameter $\sigma$. Thus the existing slip velocity and the porous boundary contribute to the easy functioning of human joints, particularly when the cartilage surfaces approach one another. The variation of $\bar{p}$ with $\theta$ as a function of the permeability parameter $\bar{k}$ is shown in Fig.7.6 with the parametric values of $\bar{I} = 0.3, \sigma = 0.1, \lambda = 0.75$ and $D/E = 0.3$. It is observed that $\bar{p}$ increases for increasing values of cartilage permeability parameter $\bar{k}$. This is because, the large permeability value means, there are more voids available on the poroelastic region, which permits the quick escape of the fluid. The interstitial fluid which is present in the lubricant region is free to escape through the poroelastic region (cartilage).

7.4.2 Load Carrying Capacity

The variation of non-dimensional load carrying capacity $\tilde{W}$ as a function of eccentricity ratio parameter $\varepsilon$ is depicted in the Fig.7.7 for different values $\bar{I}$. It is observed that $\tilde{W}$ increases for increasing values of the couple stress parameter $\bar{I}$ and $\varepsilon$. This is due to the fact that as the couple-stress parameter increases the concentration of hyaluronic acid molecules increases. Further, it is observed that as eccentricity ratio increases, the clearance decreases and the lubricants sustain more loads. The effect of elastic parameter $D/E$ on variation of $\tilde{W}$ with eccentricity ratio
parameter $\varepsilon$ depicted in the Fig. 7.8. It is observed that $\bar{W}$ increases with $\varepsilon$ and decreases for increasing values of $D/E$. Further, the load carrying capacity for the joints with degenerate cartilage ($D/E=0.1$) is quite large compared with normal cartilage ($D/E=0.3$).

The variation of non-dimensional load carrying capacity $\bar{W}$ with $\varepsilon$ for different values of slip parameter $\bar{\sigma}$ is depicted in the Fig. 7.9. It is observed that $\bar{W}$ decreases for increasing values of $\bar{\sigma}$.

### 7.4.3 Time-Height Relation

The response time of squeeze film is an important factor in the design of squeeze film bearings. This is the time elapsed to reduce the initial film thickness to the minimum permissible squeeze film height. The variation of squeeze film height $\bar{h}_0$ with response time $\tau$ as a function of $\bar{T}$ is shown in the Fig. 7.10. It is observed that, the response time increases for increasing values of $\bar{T}$. Further, the bearings with couple stress fluid as lubricant have longer response time as compared to the corresponding Newtonian case. This is due to the fact that as the couple-stress parameter increases the concentration of hyaluronic acid molecules increases.

Fig. 7.11 depicts the variation of squeeze film height $\bar{h}_0$ with response time $\tau$ for different values of $D/E$ with two values of permeability parameter $\bar{k}$. It is observed that $\tau$ increases with $\bar{h}_0$ and decreases for increasing values of $D/E$. Further, the squeeze film time for the joints with degenerate cartilage ($D/E=0.1$) is quite large compared with normal cartilage ($D/E=0.3$). The variation of squeeze film height $\bar{h}_0$ with response time $\tau$ for different values of the slip parameter $\bar{\sigma}$ is depicted in
Fig. 7.12 for two values of permeability parameter $k$. It is observed that $\tau$ increases with $h_0$ and decreases for increasing values of $\bar{\sigma}$. This appears to be consistent with the earlier results that the slip velocity occurring at a porous boundary helps in maintaining normal functioning of human joints.

### 7.5 Conclusions

The effect of couple-stresses on the squeeze film lubrication of finite poroelastic partial journal bearing is studied by using the Stokes (1966) couple-stress fluid model. The finite modified Reynolds equation is derived for the problem under consideration and is solved numerically by using the finite difference technique with grid spacing of $\Delta \theta = 9^\circ$ and $\Delta \bar{\sigma} = 0.05$. From the results obtained, the following conclusions are drawn

a) The effect of couplestress is to increases the squeeze film pressure and the load carrying capacity as compared to the corresponding Newtonian case.

b) The squeeze film time is lengthened for the couplestress lubricant as compared to the corresponding Newtonian case. This is due to the fact that as the couplestress parameter increases the concentration of hyaluronic acid molecules increases. This may be one of the reasons in the efficient lubrication and proper functioning of hip joints.
Figure 7.3: Variation of non-dimensional squeeze film pressure $\bar{p}$ with $\theta$ for different values of $\bar{I}$ with $\varepsilon = 0.1$, $\bar{\sigma} = 0.1$, $\bar{k} = 7.65 \times 10^{-5}$, $\lambda = 0.75$ and $D/E = 0.3$. 

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Figure 7.4: Variation of non-dimensional squeeze film pressure $\bar{p}$ with $\theta$ for different values of $D/E$ with $\varepsilon = 0.1, \sigma = 0.1, k = 7.65 \times 10^{-5}$, $\lambda = 0.75$ and $\bar{T} = 0.3$
Figure 7.5: Variation of non-dimensional squeeze film pressure $\bar{p}$ with $\theta$ for different values of $\bar{\sigma}$ with $\varepsilon=0.1, D/E=0.3, \bar{k} = 7.65 \times 10^{-5}$, $\lambda = 0.75$ and $\bar{t} = 0.3$. 
Figure 7.6: Variation of non-dimensional squeeze film pressure $\bar{p}$ with $\theta$ for different values of $\bar{k}$ with $\varepsilon = 0.1$, $D/E = 0.3$, $T = 0.3$, $\lambda = 0.75$ and $\bar{\sigma} = 0.3$. 
Fig. 7.7 Variation of non-dimensional load carrying capacity $\bar{W}$ with $\varepsilon$ for different values of $\bar{T}$ with $D/E = 0.3$, $\lambda = 0.75$ and $\bar{\sigma} = 0.2$. 

$\bar{k} = 7.65 \times 10^{-5}$  
$\bar{k} = 4.3 \times 10^{-5}$  
$\bar{H} = 300$  
$\bar{H} = 200$  

- - - -  $\bar{T} = 0.0$ [Newtonian]  
- - - -  $\bar{T} = 0.2$  
- - - -  $\bar{T} = 0.3$
Fig. 7.8 Variation of non-dimensional load carrying capacity \( \bar{W} \) with \( \varepsilon \) for different values of \( D/E \) with \( \bar{l} = 0.2 \), \( \lambda = 0.75 \) and \( \bar{\sigma} = 0.2 \).
Fig. 7.9 Variation of non-dimensional load carrying capacity $\tilde{W}$ with $\varepsilon$ for different values of $\bar{\sigma}$ with $\bar{l} = 0.3$, $\lambda = 0.75$ and $D/E = 0.3$. 

$\tilde{k} = 7.65 \times 10^{-5}$ $\tilde{k} = 4.3 \times 10^{-5}$ 

$\bar{H} = 300$ $\bar{H} = 200$ 

- - - - $\bar{\sigma} = 0.1$ 

- - - - $\bar{\sigma} = 0.2$ 

- - - - $\bar{\sigma} = 0.3$
Fig. 7.10 Variation of non-dimensional minimum film height $\bar{h}_0$ with $\tau$ for different values of $\bar{I}$ with $D/E = 0.3$, $\lambda = 0.75$ and $\bar{\sigma} = 0.2$. $\bar{k} = 7.65 \times 10^{-5}$, $\bar{k} = 4.3 \times 10^{-5}$, $\bar{H} = 300$, $\bar{H} = 200$, $\bar{I} = 0.0$ [Newtonian], $\bar{I} = 0.1$, $\bar{I} = 0.2$. 
Fig. 7.11 Variation of non-dimensional minimum film height $\bar{h}_0$ with $\tau$ for different values of $D/E$ with $\bar{I} = 0.2$, $\lambda = 0.75$ and $\bar{\sigma} = 0.2$.
Fig. 7.12 Variation of non-dimensional minimum film height $\bar{h}_0$ with $\tau$ for different values of $\bar{\sigma}$ with $\bar{T} = 0.2$, $\lambda = 0.75$ and $D/E = 0.3$.