6.1 Introduction

The human joint is a self-acting and dynamic load bearing structure, which uses a porous and elastic biomaterial as well as highly non-Newtonian lubricant for its functioning. A simplified scheme of a human hip joint is presented in Fig.1.9. The hip joint is a ball and socket joint, formed by the head of the femur and acetabulum of the pelvis. The dome shaped head of the femur forms the ball, which fits snugly into the concave socket of the acetabulum. The hip joint is a very sturdy joint, due to the tight fitting of the bones and the strong surrounding ligaments and muscles. The femur is the longest bone in the body which forms the thigh. The head of the femur is a round, dome shaped protrusion which fits into the pelvis to enable degrees of movement. The surface of the femur is covered with a thin layer of hyaline cartilage which acts to allow smooth movement of the joint. The thickness of the cartilage varies with each joint, and may sometimes be uneven. The primary function of the cartilage layer is to minimize contact stresses generated during joint loading contribute to lubrication mechanisms in the joints (Mow and Ateshian, 1997; Mow and Guo, 2002) cartilage serves as a kind of damper under dynamic loads.

Studies of the lubrication of natural synovial joints have advanced considerably in the last thirty years (Mow and Mak, 1986; Dowson, 1990). In particular, there have been two predominant fluid film lubrication mechanisms proposed for synovial joints: elasto-hydrodynamic lubrication in which the elasticity of cartilage is assumed to be important and biphasic lubrication, in which the porosity of articular cartilage is assumed to be mainly responsible for fluid film generation (Mansour and Mow, 1977). Hou et al. (1992) and Jin et al. (1992) examined the squeeze film lubrication of normal human hip joints incorporating the
biphasic elasticity equation for articular cartilage. It was found that under physiological walking conditions, as experienced in hip joints, the film thickness was slightly reduced, but the elasticity was not significantly affected.

The present analysis models the lubricant by Stokes (1966) couple stress fluid and tries to explain its efficiency in the lubrication of hip joints. No fluid model available so far includes all the rheological characters of synovial fluid. The viscosity of this fluid (synovial fluid) depends on both content and the molecular size of the hyaluronic acid component. The long-chain hyaluronic acid molecules found as additives in synovial fluid have been responsible for modeling the synovial fluid by a polar fluid. Since couple stresses are found to appear in noticeable magnitudes in liquids with very large molecules, the theory of couple stress fluid was used (Bujurke et al. 2007; Lin, 1997; Jurczak, 2006) to analyze the hydrodynamic lubrication of various bearing configuration. Lin (1966) investigated the couple stress effect on the squeeze film characteristics of hemispherical bearings with special reference to synovial joints. Recently, Nabhani et al. (2013) studied the non-Newtonian couple stress poro-elastic squeeze film.

Sayles et al. (1979) revealed experimentally that cartilage surfaces are rough and roughness height distribution is Gaussian in nature. This has motivated us to investigate the effect of surface roughness on cartilage surface. Christensen (1970) developed the stochastic theory to understand the effect of surface roughness in hydrodynamic lubrication of rough bearings. Siddanagouda et al. (2013) studied the combined effects of surface roughness and viscosity variation due to additives on long journal bearing. They concluded that the combined effect is to increase the load carrying capacity and to decrease the coefficient of friction, which improves the performance of the bearing.
In this chapter, the Christensen stochastic theory for rough surfaces is used to analyze the effect of surface roughness on the squeeze film characteristics of long poro-elastic partial journal bearings with couplestress fluids. Two types of one-dimensional surface roughness (longitudinal and transverse) patterns are considered. The modified stochastic Reynolds type equation governing the mean film pressure in the presence of couple stress fluids are derived for the two types of roughness patterns. The closed form expressions for the mean film pressure, the mean load carrying capacity and squeeze film time are obtained.

6.2 Mathematical Formulation of the Problem

The physical configuration of a squeeze film long rough poro-elastic partial journal bearing with no journal rotation is shown in the Fig.6.1. A journal of radius \( R \) approaches the rough poro-elastic bearing of wall thickness \( H_0 \) at any circumferential section \( \theta \) with velocity \( V \). The lubricant in the system is taken to be an incompressible non-Newtonian Stokes couple stress fluid. The stochastic film thickness is made up of two parts

\[
H = h + h_\gamma(\theta, z, \xi)
\]  

(6.2.1)

where \( h (= C - e \cos \theta) \) denotes the nominal smooth part of the film geometry, while \( h_\gamma(\theta, z, \xi) \) is the part due to the surface asperities measured from the nominal level and is regarded as a randomly varying quantity of zero mean, \( \xi \) is an index determining a definite roughness arrangement and \( \varepsilon (= e / C) \) is the eccentricity ratio parameter. Further \( C \) is the radial clearance and \( \theta (= x / R) \) the angular coordinate with \( R \) with being the radius of the journal.
Figure 6.1: A geometry of simplified model for hip joint
Governing Equations

Region-I (Fluid film region)

The basic equations for the flow of couplestress fluid in the fluid film region are as given in equations (5.2.2)-(5.2.4).

Region-II (Poro-elastic region)

The basic equations in the poro-elastic region are as given in equations (5.2.5a) and (5.2.5b) for the matrix and fluid respectively.

To solve the fluid film equations (5.2.2) to (5.2.4) the appropriate boundary conditions are

\[ u = 0, \quad v = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = 0 \]  

(6.2.2a)

\[ u = -\sigma \frac{\partial u}{\partial y}, \quad v = V - v^*, \quad \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = H \]  

(6.2.2b)

where \( \sigma \) is the slip parameter, \( V \) is the velocity of approach and \( v^* \) is the normal component of relative velocity.

6.3 Solution of the Problem

The solution of equation (5.2.3) subject to the boundary conditions (6.2.2a) and (6.2.2b) is

\[ u = \frac{1}{2\mu} \frac{\partial p}{\partial x} \left\{ y \left[ (y - h) - \xi h + 2l \xi \tanh(h/2l) \right] + 2l^2 \left[ 1 - \frac{\cosh \left\{ (2y - h)/2l \right\}}{\cosh(h/2l)} \right] \right\} \]  

(6.3.1)

Where \( l = \sqrt{\frac{\eta}{\mu}}, \quad \xi = \frac{\sigma}{h + \sigma}, \quad \sigma = \frac{\alpha}{\sqrt{k}} \)
Integrating equation (5.2.2) across the fluid film and using the boundary conditions for \( v \) given in equations (6.2.2a) and (6.2.2b) and also using the expression (6.3.1) for \( u \) the modified Reynolds equation is obtained in the form

\[
\frac{\partial}{\partial x} \left[ f(H,l,\xi) - 12\mu k' H_o \left( 1 - \frac{D}{E} \right) \frac{\partial p^*}{\partial x} \right] = 12\mu \frac{\partial H}{\partial t} \tag{6.3.2}
\]

where

\[
f(H,l,\xi) = H^3 (1 + 3\xi) - 6\xi lH^2 \tanh(H / 2l) - 12l^2 H + 24l^3 \tanh(H / 2l)
\]

and \( V = \frac{dH}{dt} = -C \frac{dE}{dt} \cos \theta \)

For loading roughness features of the cartilage surface, taking expectation of equation (6.3.2), we get the stochastic Reynolds equation

\[
\frac{\partial}{\partial x} \left[ E[f(H,l,\xi)] - 12\mu k' H_o \left( 1 - \frac{D}{E} \right) \frac{\partial E(p^*)}{\partial x} \right] = 12\mu \frac{\partial H}{\partial t} \tag{6.3.3}
\]

where expectancy operator \( E(g) \) is defined by

\[
E(g) = \int_{-\infty}^{\infty} (g(h)) f(h) dh
\]

Let \( f(h_i) \) is the probability density function of the stochastic film thickness \( h_i \).

According to Sayles et al. (1979), the cartilage surfaces are rough and roughness height distribution is Gaussian. Therefore, the polynomial which approximates the Gaussian is chosen in the present study. Such a probability density function is

\[
f(h) = \begin{cases} 
\frac{35}{32c^3} (c^2 - h^2)^3, & -c \leq h \leq c \\
0 & \text{elsewhere} 
\end{cases}
\]

(6.3.5)
where $\sigma = c/3$ is the standard deviation.

Using the following non-dimensional quantities

\[
\bar{l} = \frac{l}{C}, \quad \bar{\psi} = \frac{k' H_o}{C^3}, \quad \bar{\theta} = \frac{\chi}{R}, \quad \bar{H} = \frac{H}{C} = \bar{h} + \bar{h}_r, \quad \bar{h} = \frac{\bar{h}_r}{C} = 1 - \varepsilon \cos \theta, \quad \bar{h}_r = \frac{h_r}{C},
\]

\[
\bar{k} = \frac{k'}{C^2}, \quad \bar{H}_0 = \frac{H_0}{C}, \quad \bar{p} = \frac{E(p')C^2}{\mu R^2 (d \varepsilon / d t)}, \quad \varepsilon = \frac{e}{C}, \quad \bar{\sigma} = \frac{\sigma}{C},
\]

\[
f(H, l, \xi) = C^3 f(\bar{H}, \bar{l}, \bar{\xi}), \quad \bar{C} = \frac{C}{C}
\]

The modified stochastic Reynolds type equation can be written in non-dimensional form as

\[
\frac{\partial}{\partial \theta} \left[ E \left[ f(\bar{H}, \bar{l}, \bar{\xi}) \right] - 12 \bar{\psi} \left(1 - \frac{D}{E}\right) \frac{\partial \bar{p}}{\partial \theta} \right] = -12 \cos \theta \tag{6.3.6}
\]

where

\[
f(\bar{H}, \bar{l}, \bar{\xi}) = \bar{H}'(1 + 3\bar{\xi}) - 6\bar{\xi} \bar{l} \bar{H}' \tanh(\bar{H}/2\bar{l}) - 12\bar{\xi}^2 \bar{H}' + 24\bar{\xi} \bar{H}' \tanh(\bar{H}/2\bar{l})
\]

According to Christensen stochastic theory for rough surfaces, the following two types of roughness patterns are of special interest.

**Longitudinal Roughness**

For the longitudinal model, the roughness on the cartilage surface is assumed to have the form of long narrow ridges and valleys running in the $x$-direction. Therefore, the lubricant film thickness can be expressed as

\[
\bar{H} = \bar{h} + \bar{h}_r(\varepsilon, \xi) \tag{6.3.7}
\]

Then, equation (6.3.6) becomes
\[
\frac{\partial}{\partial \theta} \left[ E \left[ f(H, \bar{T}, \bar{z}) \right] - 12\psi \left( 1 - \frac{D}{E} \right) \frac{\partial \bar{p}}{\partial \theta} \right] = -12 \cos \theta
\] (6.3.8)

**Transverse Roughness**

For the transverse model, the roughness is assumed to have the form of long narrow ridges and valleys running in the z-direction. Therefore, the lubricant film thickness can be expressed as

\[
\bar{H} = h + \bar{h}_t(\theta, \bar{z})
\] (6.3.9)

Then, equation (6.3.6) takes the form

\[
\frac{\partial}{\partial \theta} \left[ \alpha(H, \bar{T}, \bar{z}, \bar{c}, \psi) - 12\psi \left( 1 - \frac{D}{E} \right) \frac{\partial \bar{p}}{\partial \theta} \right] = -12 \cos \theta
\] (6.3.10)

The modified Reynolds equations (6.3.8) and (6.3.10) for longitudinal and transverse types of directional structures can be expressed as

\[
\frac{\partial}{\partial \theta} \left[ \alpha(H, \bar{T}, \bar{z}, \bar{c}, \psi) - 12\psi \left( 1 - \frac{D}{E} \right) \frac{\partial \bar{p}}{\partial \theta} \right] = -12 \cos \theta
\] (6.3.11)

where

\[
\alpha(H, \bar{T}, \bar{z}, \bar{c}, \psi) = \begin{cases} 
E \left[ f(H, \bar{T}, \bar{z}) \right] & \text{longitudinal roughness} \\
\frac{1}{E \left[ 1/f(H, \bar{T}, \bar{z}) \right]} & \text{transverse roughness}
\end{cases}
\]

\[
E \left[ f(H, \bar{T}, \bar{z}) \right] = \frac{35}{32c} \int \left( c^2 - h_t^2 \right)^{\psi} f(H, \bar{T}, \bar{z}) \, dh_t
\]
For the $180^\circ$ partial poro-elastic journal bearing the boundary conditions for the mean fluid film pressure are:

$$\bar{p} = 0 \quad \text{at} \quad \theta = \pm \frac{\pi}{2} \quad (6.3.12a)$$

$$\frac{dE(\bar{p})}{d\theta} = 0 \quad \text{at} \quad \theta = 0 \quad (6.3.12b)$$

Integration of equation (6.3.11) with respect to $\theta$ and using the boundary conditions (6.3.12a) and (6.3.12b) yield the non-dimensional mean fluid film pressure as

$$\bar{p} = -12 \int_{\frac{\alpha - \xi}{2}}^{\frac{\alpha + \xi}{2}} \frac{\sin \theta}{\alpha(\bar{H}, \bar{I}, \bar{\zeta}, \bar{\psi}) - 12\psi \left(1 - \frac{D}{E}\right)} \, d\theta \quad (6.3.13)$$

The mean load carrying capacity of the $180^\circ$ poro-elastic partial rough journal bearing is evaluated by integrating the mean fluid film pressure field acting on the journal

$$E(W) = \int_{\frac{\alpha - \xi}{2}}^{\frac{\alpha + \xi}{2}} E(p) \cos \theta \cdot R \, d\theta \quad (6.3.14)$$

where $W$ represents the load-carrying capacity per unit length of the bearing generated by the squeeze film pressure.

The non-dimensional form of (6.3.14) is
\[
\tilde{W} = \frac{E(W)C^2}{\mu R^3(d\varepsilon/dt)} = -12 \int_{0}^{\pi/2} \int_{0}^{\pi/2} \sin \theta \left[ \alpha(H, \bar{T} , \bar{c}, \psi) - 12\psi \left(1 - \frac{D}{E}\right) \right] d\theta \cos \theta d\theta
\]

(6.3.15)

Time-height relation is calculated by considering the time taken by the journal centre to move from \( \varepsilon = 0 \) to \( \varepsilon = \varepsilon_i \) can be obtained from equation (6.3.15) as

\[
\frac{d\varepsilon}{dt} = \frac{1}{g(\varepsilon, \bar{T}, \psi)}
\]

(6.3.16)

where \( \tau = \frac{WC^2}{\mu R^3 t} \) is the non-dimensional response time.

The first order non-linear differential equation (6.3.16) is solved numerically by using the fourth order Runge-Kutta method with the initial condition from \( \varepsilon(0) = 0 \).

### 6.4 Results and Discussion

The effect of surface roughness pattern on the squeeze film characteristics of a long poro-elastic partial journal bearings lubricated with couple stress fluids is predicted. The effect of couple stresses is characterized by the non-dimensional couple stress parameter \( \bar{T}(=l/C) \), the effect of permeability is characterized by the permeability parameter \( \psi \left(=\frac{k'W}{C^3}\right) \), the slip parameter \( \sigma \) and the effect of surface roughness is characterized by the roughness parameter \( \bar{c}(=c/C) \). The following ranges of values for these parameters are used in the numerical computations of the results.

\[
\bar{T} = 0.0, 0.1, 0.2, 0.3; \quad \sigma = 0.1, 0.2, 0.3; \quad \bar{c} = 0.0, 0.2, 0.4
\]
6.4.1 Squeeze Film Pressure

The variation of non-dimensional pressure $\overline{p}$ with circumferential coordinate $\theta$ for different values of couple stress parameter $\overline{I}$ is depicted in the Fig.6.2 with the parameter values of $\varepsilon = 0.1$, $\bar{\sigma} = 0.1$, $\bar{c} = 0.2$, $D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$ for both the types of roughness pattern. The effect of couple stress is to increase the pressure distribution for both types of roughness pattern compared to Newtonian case $\overline{I} \rightarrow 0$. Further the increase in $\overline{p}$ is more pronounced for the transverse roughness pattern as compared to the longitudinal roughness pattern. It is observed that the effect of transverse roughness pattern is to increase the fluid film pressure.

The variation of $\overline{p}$ with $\theta$ as a function of the eccentricity ratio parameter $\varepsilon$ is shown in Fig.6.3 with the parameter values of $\overline{I} = 0.2$, $\bar{\sigma} = 0.1$, $\bar{c} = 0.2$, $D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$ for both types of roughness pattern. It is observed that $\overline{p}$ increases for increasing values of $\varepsilon$ for both the types of roughness pattern. Further the increase in $\overline{p}$ is more pronounced for the transverse roughness pattern as compared to the longitudinal roughness pattern.

The effect of elastic parameter $D/E$ on variations of $\overline{p}$ with $\theta$ is shown in the Fig.6.4. It is observed that $\overline{p}$ increases with $\theta$ and decreases for increasing values of $D/E$ for both the longitudinal and transverse roughness pattern. Further, squeeze film pressure $\overline{p}$ decreases as intra articular gap between two articular surfaces decreases for all values of elastic parameter $D/E$. 

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The variation of non-dimensional pressure $\bar{p}$ with circumferential coordinate $\theta$ for different values of couple stress parameter $\bar{\sigma}$ is shown in the Fig.6.5 with the parametric values of $\varepsilon = 0.1, \bar{\varepsilon} = 0.2, \bar{\lambda} = 0.2, D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-3}$ for both types of roughness patterns. It is observed that $\bar{p}$ increases with $\theta$ and decreases for increasing values of slip parameter $\sigma$ for both the types of roughness pattern. Thus the existing slip velocity and the porous boundary contribute to the easy functioning of human joints, particularly when the cartilage surfaces approach one another.

The variation of non-dimensional pressure $\bar{p}$ with circumferential coordinate $\theta$ for different values of couple stress parameter $\bar{c}$ is shown in the Fig.6.6 with the parametric values of $\varepsilon = 0.3, \bar{\varepsilon} = 0.3, \bar{\lambda} = 1.5, D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-3}$, for both types of roughness patterns. It is observed that the fluid film pressure distribution increases (decreases) with increasing roughness parameter $\bar{c}$ for transverse (longitudinal) roughness pattern compared to the corresponding smooth case. Because of the presence of hyaluronic acid complex molecules, water and other low-molecular weight substances in the synovial fluid, a thick dense substance is being formed on the cartilage surfaces during the squeezing process. Also the presence of surface asperities on the articular cartilage reduces the fluid flow and the large fluid is retained in the lubricant region, which enhances pressure built up.
6.4.2 Load Carrying Capacity

The variation of non-dimensional load carrying capacity $\bar{W}$ as a function of couple stress parameter $\bar{T}$ is depicted in the Fig.6.7 for different values of $\varepsilon$. It is observed that $\bar{W}$ increases for increasing values of the couple stress parameter $\bar{T}$ and $\varepsilon$ for both the types of roughness pattern. This is due to the fact that as the couple-stress parameter increases the concentration of hyaluronic acid molecules increases. Further, it is observed that as eccentricity ratio increases, the clearance decreases and the lubricants sustain more loads. It is also observed that increases in $\bar{W}$ is more pronounced for the transverse roughness pattern as compared to the longitudinal roughness pattern.

The effect of elastic parameter $D/E$ on variation of $\bar{W}$ with couple stress parameter $\bar{T}$ depicted in the Fig.6.8. It is observed that $\bar{W}$ increases with $\bar{T}$ and decreases for increasing values of $D/E$ for both the types of roughness pattern. Further, $\bar{W}$ decreases with increases for cartilage permeability parameter. This is because, the large permeability value means, there are more voids available on the poro-elastic region, which permits the quick escape of the fluid. The interstitial fluid which is present in the lubricant region is free to escape through the poro-elastic region (cartilage).

The variation of non-dimensional load carrying capacity $\bar{W}$ with $\bar{T}$ for different values of slip parameter $\bar{\sigma}$ is depicted in the Fig.6.9. It is observed that $\bar{W}$ decreases for increasing values of $\bar{\sigma}$ for both the types of roughness pattern. Further the increase in $\bar{W}$ is more pronounced for the transverse roughness pattern as compared to the longitudinal roughness pattern.
The effect of roughness parameter \( \bar{c} \) on the variation of \( \bar{W} \) with the couplestress parameter \( \bar{T} \) depicted in the Fig. 6.10 with the parametric values of \( \bar{T} = 0.6, \; \lambda = 1.5, \; \varepsilon = 0.3, \; \bar{\sigma} = 0.3 \) and \( \bar{k} = 7.65 \times 10^{-5} \) for both types of roughness patterns. It is observed that \( \bar{W} \) increase in \( \bar{c} \) for transverse roughness whereas it decreases for longitudinal roughness compared to corresponding smooth case.

### 6.4.3 Time-Height Relation

The response time of the squeeze film is one of the significant factors in the design of bearings. The response time is the time that will elapse for a squeeze film reduces to some minimum permissible squeeze film height. The variation of squeeze film height \( \bar{h}_0 \) with response time \( \tau \) as a function of \( \bar{T} \) is shown in the Fig.6.11. It is observed that, the response time increases for increasing values of \( \bar{T} \) for both the types of roughness patterns, hence the pressure of the microstructures in the lubricant the squeeze film time as compared to that of the Newtonian lubricants. Further, the bearings with couple stress fluid as lubricant have longer response time as compared to the corresponding Newtonian case. This is due to the fact that as the couple-stress parameter increases the concentration of hyaluronic acid molecules increases. Fig.6.12 depicts the variation of squeeze film height \( \bar{h}_0 \) with response time \( \tau \) for different values of \( D/E \) with two values of permeability parameter \( \bar{k} \). It is observed that \( \tau \) increases with \( \bar{h}_0 \) and decreases for increasing values of \( D/E \) for both the types of roughness pattern. Further, the squeeze film time for the joints with degenerate cartilage (\( D/E=0.1 \)) is quite large compared with normal cartilage (\( D/E=0.3 \)).
The variation of squeeze film height $\overline{h_0}$ with response time $\tau$ for different values of $\bar{\sigma}$ is depicted in the Fig.6.13 for two values of permeability parameter $k$. It is observed that $\tau$ increases with $\overline{h_0}$ and decreases for increasing values of slip parameter $\bar{\sigma}$ for both longitudinal and transverse roughness pattern. This appears to be consistent with the earlier results that the slip velocity occurring at a porous boundary helps in maintaining normal functioning of human joints.

The effect of roughness parameter, $\bar{c}$ on the variation of $\overline{h_0}$ with $\tau$ depicted in the Fig.6.14 with the parametric values of $T = 0.6$, $\lambda = 1.5$, $\varepsilon = 0.3$, $\bar{\sigma} = 0.3$ and $k = 7.65 \times 10^{-5}$ for both types of roughness patterns. It is interesting to note that the effect of $\bar{c}$ is to increase (decrease) the response time of the squeeze film for the longitudinal (transverse) roughness pattern as compared to the corresponding smooth case ($\bar{c} = 0.0$).

6.5 Conclusions

The effect of surface roughness on the squeeze film characteristics of long partial rough poro-elastic journal bearing is presented. On the basis of Stokes couplestress fluid theory and the Christensen stochastic theory for the study of rough surfaces, the modified form of stochastic Reynolds equation is derived for one-dimensional longitudinal and transverse roughness patterns. The governing equations along with the appropriate constitutive relationships and boundary conditions have been formulated for modeling the roughness structure of cartilage with couplestress fluid in the lubricant region in hip joint lubrication. As the couple-stress fluid parameter $T \rightarrow 0$ the squeeze film characteristics reduce to corresponding Newtonian case and as $\bar{c} \rightarrow 0$ these characteristics reduce to the
smooth case. On the basis of the results presented, the following conclusions are drawn.

1. The long chain of hyaluronic acid molecules existing in the synovial fluid gives the motivation for assuming the synovial fluid as a Stokes Couplestress fluid.

2. The effect of couplestress fluid provides an increased pressure, load carrying capacity and squeeze film time as compared to the corresponding Newtonian case.

3. The Christensen surface roughness on the cartilage surface for longitudinal (transverse) pattern on the finite poro-elastic partial journal bearings increases (decreases) the load carrying capacity and the squeeze film time as compared to the corresponding smooth case.

4. The effect of large porosity parameter in articular cartilage causes reduction in load carrying capacity and time of approach to minimum film thickness as compared with small porous size.
Fig. 6.2 Variation of non-dimensional squeeze film pressure $\bar{p}$ with $\theta$ for different values of $\tilde{l}$ with $\varepsilon = 0.1, \bar{\sigma} = 0.1, \bar{c} = 0.2, \ D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 6.3 Variation of non-dimensional squeeze film pressure $\bar{p}$ with $\theta$ for different values of $\varepsilon$ with $\bar{l} = 0.2$, $\bar{\sigma} = 0.1$, $\bar{c} = 0.2$, $D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 6.4 Variation of non-dimensional squeeze film pressure $\bar{p}$ with $\theta$ for different values of $D/E$ with $\hat{l} = 0.2, \sigma = 0.1, \bar{c} = 0.2, \varepsilon = 0.1$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 6.5 Variation of non-dimensional squeeze film pressure $\bar{p}$ with $\theta$ for different values of $\bar{\sigma}$ with $\bar{l} = 0.2, D/E = 0.3, \bar{c} = 0.2, \varepsilon = 0.1$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 6.6 Variation of non-dimensional squeeze film pressure $\bar{p}$ with $\theta$ for different values of $c$ with $\bar{l} = 0.2$, $D/E = 0.3$, $\bar{\sigma} = 0.1$, $\varepsilon = 0.1$ and $k = 7.65 \times 10^{-5}$. 
Fig. 6.7 Variation of non-dimensional load carrying capacity $\overline{W}$ with $\overline{l}$ for different values of $D/E$ with $\overline{l} = 0.2$, $\overline{e} = 0.2$, $\overline{\sigma} = 0.1$, $\varnothing = 0.1$ and $\overline{k} = 7.65 \times 10^{-5}$. 
Fig. 6.8 Variation of non-dimensional load carrying capacity $\bar{W}$ with $\bar{l}$ for different values of $\varepsilon$ with $\bar{c} = 0.2$, $\bar{\sigma} = 0.1$, $D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 6.9 Variation of non-dimensional load carrying capacity $\bar{W}$ with $\bar{l}$ for different values of $\bar{c}$ with $\varepsilon = 0.1$, $\bar{\sigma} = 0.1$, $D/E = 0.3$ and $ar{k} = 7.65 \times 10^{-5}$. 
Fig. 6.10 Variation of non-dimensional load carrying capacity $\bar{W}$ with $\bar{l}$ for different values of $\bar{\sigma}$ with $\varepsilon = 0.1$, $\bar{\varepsilon} = 0.2$, $D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-4}$. 
Fig. 6.11  Variation of non-dimensional squeeze film time $\tau$ with $\bar{h}_0$ for different values of $\bar{l}$ with $\varepsilon = 0.1$, $\bar{\varepsilon} = 0.2$, $\bar{\sigma} = 0.1$, $D/E = 0.3$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 6.12 Variation of non-dimensional squeeze film time $\tau$ with $\bar{h}_0$ for different values of $D/E$ with $\varepsilon = 0.1$, $\bar{c} = 0.2$, $\bar{\sigma} = 0.1$, $\bar{l} = 0.2$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 6.13 Variation of non-dimensional squeeze film time $\tau$ with $\bar{h}_0$ for different values of $\bar{\sigma}$ with $\varepsilon = 0.1, \eta_0 = 0.2, D/E = 0.3, \bar{l} = 0.2$ and $\bar{k} = 7.65 \times 10^{-5}$. 
Fig. 6.14 Variation of non-dimensional squeeze film time $\tau$ with $\bar{h}_0$ for different values of $\bar{c}$ with $\epsilon = 0.1$, $\bar{\sigma} = 0.1$, $D/E = 0.3$, $\bar{l} = 0.2$ and $\bar{k} = 7.65 \times 10^{-6}$. 