CHAPTER II
FORCED CONVECTIVE NONLINEAR LAMINAR BOUNDARY LAYER FLOW OF A NANOFLUID OVER A FLAT PLATE

2.1 Introduction

Forced convective heat transfer plays a significant role in almost all industrial sectors. Examples include cooling of microelectronics, process intensification in the chemical industry, heat exchange/waste heat recovery in power plants and cooling of car engines, to name but a few. In special, a study of forced convective laminar boundary layer behaviour on a flat plate has attracted the attention of many researchers in the last several decades as the analysis of such flows finds applications in many areas such as aerospace, automotive and turbomachinery.

In recent days, an increasing demand of ultra high flux equipments for cooling along with the compactness requirements of devices in processes involving electronic chips and high power engines motivated researchers in the finding of a new and improved heat transfer media such as nanofluids. All these applications motivated the author to deal with the problem involving the laminar boundary layer flow of nanofluid over a flat plate in this chapter.

A basic understanding of flow characteristics over a flat plate is essential to a complete study of Aerodynamics. Till today, there have been many studies regarding heat transfer of fluids over a flat plate other than nanofluids. One of the classical problems in fluid dynamics is the flow over a flat plate and Blasius (1908) was the first person to investigate the steady laminar boundary layer flow over a flat plate. Using the similarity variables, he obtained the nonlinear third-order ordinary differential equation and finally solved it using power series method. Later Howarth (1938) found out the numerical solution of Blasius problem. Contributions to the theory of heat
transfer through a laminar boundary layer were discussed by Lighthill (1950). Sakiadis (1961) was the first person to discuss the laminar boundary layer flow of a viscous and incompressible fluid on a moving flat plate. An analysis was carried out by Sparrow and Lin (1965) to determine the distribution of surface temperature along a flat plate experiencing simultaneous convective heat transfer. The problem of convective heat transfer in a laminar incompressible flow around a flat plate of a finite thickness was analysed by Luikov (1974).

Subsequently, several investigators studied various aspects of the laminar, steady, convection boundary layer flow problems under various physical situations. Convection on a flat plate was studied by Afzal and Hussain (1984). It was Abu-Sitta (1994) who clarified the existence of solution for flow past a flat plate. Laminar boundary layer flow over a horizontal permeable flat plate was studied by Aydin and Kaya (2005) and the effect of Prandtl number on heat transfer was also investigated. Cortell (2005) was able to find a numerical solution for the classical Blasius flat plate problem. The effects of suction and injection on steady laminar mixed convection boundary layer flow over a permeable horizontal flat plate in a viscous and incompressible fluid were investigated by Deswita et al. (2010). Recently, Merkin and Pop (2011) studied the forced convection heat transfer resulting from the flow of a uniform stream over a flat surface.

Research carried out in recent years confirm the fact that nanofluids enhance the thermal conductivity and there is a three fold increase in critical heat flux and two fold increase in convection heat transfer. There is remarkable heat transfer enhancement when nanofluids are used and the main reason for enhanced convection is thermal expansion as the various studies indicate. In recent years, the ongoing miniaturization of electronics is accompanied by the reduction of available space for cooling as discussed by Peterson and Ortega (1990). Studies by Xuan and Roetzel (2000) and Xuan and Li (2003) have
proved that one attractive way of enhancing the convection heat transfer is by adding high thermal conductivity nanoparticles in the fluid.

Gosselin and Silva (2004) have analysed the combined “heat transfer and power dissipation” optimization of nanofluid flows. They discussed the importance of maximizing the thermal performance of nanofluid flows under the appropriate constraints considering the laminar and turbulent boundary layer flows in forced and natural convection. Roy et al. (2004) have established the importance of nanofluids as interesting alternatives to conventional coolants in their numerical study of heat transfer enhancement with the use of nanofluids in laminar radial flow cooling systems. Enhanced thermal conductivity of Titanium dioxide ($\text{TiO}_2$)-water based nanofluids was explained by Murshed et al. (2005) and the experimental results show that the thermal conductivity increases with an increase in volume fraction. Aminossadati and Ghasemi (2009) conducted a numerical study on natural convection cooling of a localized heat source at the bottom of a nanofluid-filled enclosure and the results indicate that adding nanoparticles into pure water improves its cooling performance. Recently, it was found by Alammar and Hu (2010) that all nanofluids exhibited increase in heat transfer enhancement with increasing volume fraction. All these and various other studies regarding the flow of nanofluids clearly show the particular importance of nanofluids which are to be used as coolants.

Most of the research investigated was for laminar forced convection for ordinary fluids whereas, there is very little research on the forced convection regarding nanofluids, in particular over flat plates. In view of the applications of nanofluids as excellent coolants, this work mainly deals with nanofluid flow over a flat plate.
2.2 Author’s Contribution

To the best of the knowledge of the author, no other numerical study on nonlinear steady laminar forced convection of nanofluids over a flat plate for the two cases of copper-water nanofluid and alumina-water nanofluid which has been reported so far. The present work is mainly dealt with nanofluid flows which are suspensions of solid nanoparticles (1-100 nm diameter) in conventional liquids like water and the suspended ultra fine particles change transport properties and heat transfer performance of the nanofluid, which exhibits a great potential in enhancing heat transfer.

The two-dimensional problem of steady, incompressible, viscous, forced convective, laminar, nonlinear boundary layer flow with heat transfer of copper-water and alumina-water nanofluid respectively over a flat plate is investigated in the present study. First of all, the numerical values of viscosity, specific heat capacity, density, thermal conductivity, kinematic viscosity and thermal diffusivity for copper-water and alumina-water nanofluid for different values of volume fraction are evaluated. To determine the effective thermal conductivity \( k_{\text{eff}} \) of the nanofluid, the model given by Patel et al. (2005) has been used with appropriate value of ‘c’, which has been calculated by matching the experimental results. Since no other model takes care of the temperature dependence of \( k_{\text{eff}} \) of the nanofluid, this model has been considered as the best option. The viscosity of nanofluid has been calculated using Brinkman model (1952).

The nonlinear partial differential equations governing the flow are transformed into nonlinear ordinary differential equations utilizing similarity transformations, which are then solved using the Nachtsheim-Swigert Shooting iteration technique (1965) along with the fourth order Runge-Kutta method. Numerical results for dimensionless velocity and temperature of the nanofluid
flows are obtained and computations are carried out for various values of volume fraction, density, viscosity, specific heat capacity, thermal conductivity as well as Prandtl number of nanofluids. The results are displayed graphically to show the interesting aspects of the nanofluids. Non-dimensional skin friction and non-dimensional rate of heat transfer are also obtained and tabulated.

2.3 Formulation of the problem

The problem of steady, incompressible, viscous, forced convective, laminar, two-dimensional boundary layer flow of copper-water and alumina-water nanofluid over a flat plate is investigated in the present work. The flat plate of infinite length is placed in the direction of a uniform velocity $U_\infty$ of the fluid flow. The temperature and velocity far away from the plate are denoted as $T_\infty$ and $U_\infty$. The problem considered is one of two dimensional motion which can be analysed by using the Prandtl boundary layer equations. Let the origin of the coordinates be at the leading edge of the plate, the x-axis be the direction of the uniform stream, and the y-axis, normal to the plate. Two types of nanofluids are considered for study under case (a) and case (b).

Case (a) Copper-water nanofluid

Water has been considered as the base fluid in considering the problem involving electronic chips and high power engines.
base temperature as 293 K, for the present study. Solid spherical copper nanoparticles of 100 nm diameter mixed with water has been considered as the nanofluid.

The viscosity, specific heat capacity, density and thermal conductivity of the nanofluids depend on the volume fraction $\phi$ of the nanoparticles used. The effective density of the nanofluid is given by

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s$$

(2.1)

and the specific heat capacity of the nanofluid is given by

$$\left(\frac{c_p}{\rho}\right)_{nf} = (1 - \phi)\left(\frac{c_p}{\rho}\right)_f + \phi\left(\frac{c_p}{\rho}\right)_s$$

(2.2)

as given by Xuan and Roetzel (2000), where $\rho_f$ and $\rho_s$ are densities, $(c_p)_f$ and $(c_p)_s$ are specific heat capacities of the base fluid and solid particle respectively as given in Table 2.1. The dynamic viscosity of the nanofluid as given by Brinkman (1952) is as follows:

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}$$

(2.3)

where $\mu_f$ is the dynamic viscosity of the base fluid the value of which is given by 0.001002 and $\phi$ is solid volume fraction.

The effective thermal conductivity of the nanofluid has been determined by the model proposed by Patel et al. (2005). Here $k_{eff}$ denotes $k_{nf}$. For the two component entity of spherical particle suspension, the thermal conductivity $k_{eff}$ is given by

$$\frac{k_{eff}}{k_f} = 1 + \frac{k_p A_p}{k_f A_f} + \frac{c k_p Pe A_p}{k_f A_f}$$

(2.4)

where

$$\frac{A_p}{A_f} = \frac{d_f}{d_p} \frac{\phi}{(1 - \phi)}$$

(2.5)
\[ Pe = \frac{u_p d_p}{\alpha_f} \]  \hspace{1cm} (2.6)

where \( u_p \) is the Brownian motion velocity of the particle which is given by

\[ u_p = \frac{2T k_b}{\pi \mu_f d_f^2} \]  \hspace{1cm} (2.7)

where \( k_b \) is the Boltzmann constant \((k_b=1.3806\times10^{-24} \text{ J/K})\) and \( T \) is the temperature. Here \( A_p \) and \( A_f \) denote heat transfer areas, \( k_p \) and \( k_f \) represent the thermal conductivities of the paricle and the fluid respectively, \( d_p \) is the particle diameter and \( d_f \) is the molecular size of the liquid. Here, for water \( d_f \) is 2 Angstroms \((2 \text{Å})\). Also \( \alpha_f \) is the thermal diffusivity of the fluid. Santra et al. (2009) calculated from the experimental data available for copper-water nanofluid the value of the constant \( 'c' \) and the value is 36000. The calculation of the effective thermal conductivity can be obtained from the equation (2.4).

Case (b) Alumina-water nanofluid

Spherical nanoparticles of alumina (Aluminium oxide) are mixed with water and values of the thermal properties are calculated using the specified equations given by Gosselin and Silva (2004).

The effective density of alumina-water nanofluid is given by

\[ \rho_{nf} = (1-\phi)\rho_f + \phi \rho_s \]  \hspace{1cm} (2.8)

and the specific heat capacity of the nanofluid is given by

\[ \left( \frac{c_p}{c_p} \right)_{nf} = (1-\phi)(c_p)_{f} + \phi (c_p)_{s} \]  \hspace{1cm} (2.9)

The dynamic viscosity of the nanofluid as given by Brinkman (1952) is as follows:

\[ \mu_{nf} = \frac{\mu_f}{(1-\phi)^{\frac{5}{3}}} \]  \hspace{1cm} (2.10)
The thermal conductivity of the nanofluid is calculated from the following formula:

\[
k_{nf} = k_f \left\{ \frac{(k_s / k_f) + (n - 1) - (n - 1)\phi(1 - (k_s / k_f))}{(k_s / k_f) + (n - 1) - \phi(1 - (k_s / k_f))} \right\}
\]

(2.11)

Here \(k_s\) and \(k_f\) denote the thermal conductivities of the solid particle and the fluid respectively. \((n=3/\Psi, \Psi=1\) for sphere, where \(\Psi\) is the spherocity and \(n\) is the empirical shape factor).

**Table 2.1 Physical properties of base fluid water, copper, alumina at 20° C (293 K)**

<table>
<thead>
<tr>
<th></th>
<th>(\rho) (Kg/m(^3))</th>
<th>(c_p) (J/Kg.K)</th>
<th>(k) (W/m.K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1000.52</td>
<td>4181.8</td>
<td>0.597</td>
</tr>
<tr>
<td>Copper</td>
<td>8954</td>
<td>383.1</td>
<td>386</td>
</tr>
<tr>
<td>Alumina</td>
<td>3970</td>
<td>769</td>
<td>36</td>
</tr>
</tbody>
</table>

**Governing equations of the flow**

The governing Prandtl boundary layer equations for the steady two dimensional, nonlinear, laminar, forced convective, incompressible, viscous nanofluid flow over the flat plate are as follows:

**Equation of continuity**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(2.12)

**Equation of momentum**

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_f \frac{\partial^2 u}{\partial y^2}
\]

(2.13)
Energy equation

\[
\frac{u \partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{\rho_{nf} \left( \frac{c_p}{\eta} \right)_{nf}} \left( \frac{\partial^2 T}{\partial y^2} \right) \quad (2.14)
\]

where \( u \) and \( v \) are the velocity components in x and y directions respectively, \( T \) is the temperature, \( \nu_{nf} \) is the kinematic viscosity, \( k_{nf} \) is the thermal conductivity, \( (c_p)_{nf} \) is the specific heat and \( \rho_{nf} \) is the density of the nanofluid respectively. In the energy equation the viscous dissipation effects are neglected. The boundary conditions for the velocity and temperature of this problem are

\[
\begin{align*}
At \quad y=0, \quad u=v=0, \quad T=T_w \\
As \quad y \to \infty, \quad u=U_\infty, \quad T=T_\infty
\end{align*}
\]

\[ (2.15) \]

2.4 Method of Solution

To seek the solution of the problem, the following dimensionless variables are introduced:

\[
\psi(x,y) = (U_\infty \nu_{nf} x)^{1/2} f(\eta), \quad \eta = y \left( \frac{U_\infty}{\nu_{nf} x} \right)^{1/2}, \quad \theta = \frac{T-T_\infty}{T_w-T_\infty} \quad (2.16)
\]

where \( \psi(x,y) \) is the stream function that satisfies equation (2.12) with \( u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \) and \( \theta \) is the dimensionless temperature. In terms of these new variables, the velocity components can be expressed as

\[
u = \frac{1}{2} \left( \frac{\nu_{nf} U_\infty}{x} \right)^{1/2} \left( \eta f'(\eta) - f(\eta) \right) \quad (2.18)
\]

The momentum and energy equations together with the boundary conditions represented by equations (2.12), (2.13) and (2.14) using transformations can be written as
\begin{align*}
f''' + \frac{1}{2} f' f'' &= 0 \quad (2.19) \\
\frac{1}{(Pr)_{nf}} \theta'' + \frac{1}{2} f' \theta' &= 0 \quad (2.20)
\end{align*}

with the boundary conditions

\begin{align*}
\eta = 0, \ f = 0, \ f' = 0, \ \theta = 1 \\
\eta \to \infty, \ f' = 1, \ \theta = 0
\end{align*} \quad (2.21)

where

\[(Pr)_{nf} = \frac{\nu_{nf}}{\alpha_{nf}}.\]

Here \(\nu_{nf}\) and \(\alpha_{nf}\) are the kinematic viscosity and thermal diffusivity of the nanofluid respectively. The kinematic viscosity and thermal diffusivity of the nanofluid respectively are given by

\[
\nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}}
\]

and

\[
\alpha_{nf} = \frac{k_{nf}}{\rho_{nf}(c_p)_{nf}}
\]

### 2.5 Numerical Solution

In order to evaluate the numerical values of the density, viscosity, specific heat capacity, thermal conductivity, kinematic viscosity and thermal diffusivity for copper-water and alumina-water nanofluid for different values of volume fraction, FORTRAN programming is used. The set of nonlinear differential equations (2.19) and (2.20) subject to the boundary conditions (2.21) constitute a two-point boundary value problem. In order to solve this system of nonlinear differential equations together with the boundary conditions, ordinary methods fail. The equations (2.19) and (2.20) are solved numerically using Nachtsheim-Swigert Shooting iteration technique (1965)
along with Runge-Kutta fourth order method. The most important thing to be considered here is that the initial guesses for the values of \( f''(0) \) and \( \theta'(0) \) to initiate the shooting process are to be made. The success of the procedure depends very much on how good these guesses are. Initial guesses are made taking into account of convergency and numerical solutions for velocity and temperature are obtained for several values of the physical parameter.

Further, quantities of practical interest in this study are the non-dimensional skin friction coefficient \( C_f \) and Nusselt number \( Nu \), which are defined as

\[
C_f = \frac{\tau_w}{\rho f U_\infty^2}, \quad \text{where} \quad \tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right)_{y=0}
\]

(2.22)

\[
Nu = \frac{x q_w}{k_f (T_w - T_\infty)}, \quad \text{where} \quad q_w = -k_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0}
\]

(2.23)

Substituting equations (2.16) into equations (2.22) and (2.23),

\[
C_f (Re_x)_{nf}^{1/2} = \frac{1}{(1-\phi)2.5} f''(0)
\]

\[
Nu (Re_x)_{nf}^{-1/2} = \frac{-k_{nf}}{k_f} \theta'(0) \quad \text{where} \quad (Re_x)_{nf} = \frac{U_\infty x}{v_{nf}}
\]

and \((Re_x)_{nf}\) is the local Reynolds number.

2.6 Results and discussion

Forced convective heat transfer problem associated with steady, nonlinear, laminar flow of the nanofluids over a horizontal flat plate has been studied. The effect of volume fraction on density, viscosity, specific heat capacity, thermal conductivity, kinematic viscosity and thermal diffusivity respectively for copper-water and alumina-water nanofluids are analysed.
Though in the energy equation, the expression $(Pr)_{nf}$ is used to notate the Prandtl number of the nanofluids, for convenience sake, ordinary notation $Pr$ is used in graphs which notate the Prandtl number of nanofluids. The notations in all the following chapters with regard to all the physical parameters involved are dealt in the similar manner.

The effect of volume fraction on density for copper-water and alumina-water nanofluids is depicted in Fig. 2.2. When volume fraction increases, density of both types of nanofluids increases and the increase is more for copper-water nanofluid than the alumina-water nanofluid.

Fig. 2.3 reveals the fact that there is no change in viscosity for different types of nanofluids for a particular volume fraction and viscosity of the nanofluids increases with volume fraction for both the fluids.

The variation of specific heat capacity against the volume fraction for copper-water and alumina-water nanofluids respectively is displayed through Fig. 2.4. There is a decrease in specific heat capacity as the volume fraction increases. Also it is noted that alumina-water nanofluid has more specific heat capacity than copper-water nanofluid for the same volume fraction.

It is observed through Fig. 2.5 that the thermal conductivity of both types of nanofluids increases with volume fraction $\phi$. Further, copper-water nanofluid has more thermal conductivity than alumina-water nanofluid at the same volume fraction.

Fig. 2.6 exhibits the effect of volume fraction on Pr for copper-water and alumina-water nanofluids. It is obvious that as volume fraction increases Pr decreases in both the cases and the decrease of Pr is more for copper-water
nanofluid than alumina-water nanofluid. This is in accordance with the physical properties exhibited by copper particles suspended in water.

It is displayed in Fig.2.7 that the dimensionless velocity is in perfect agreement with Blasius solution in the case of water.

The comparison graph for the dimensionless temperature for ordinary fluid water and copper-water nanofluid is presented through Fig.2.8. It discloses the fact that the temperature distribution is enhanced in the case of copper-water nanofluid than water which is the realistic result.

Fig.2.9 shows the dimensionless temperature for copper-water nanofluid for different values of volume fraction. It is observed that as volume fraction increases, temperature increases. From the figure it is noted that the temperature is maximum at the wall and asymptotically decreases to zero as we move far away from the wall. Also the thermal boundary layer thickness increases for increasing volume fraction.

The dimensionless temperature for alumina-water nanofluid for various values of volume fraction is depicted in Fig.2.10. As the volume fraction increases, it is apparent that the temperature increases. However this effect is small in comparison to that of copper-water nanofluid.

Fig.2.11 displays the dimensionless temperature for copper-water nanofluid for different values of Prandtl number. It is observed that as Prandtl number increases, temperature decreases. This is in agreement with the physical fact that the thermal boundary layer thickness decreases with increasing Pr.
For alumina-water nanofluid, it is clear from Fig.2.12 that similar trends follow, but the effect of Pr is not as significant as in the case of copper-water nanofluid.

The dimensionless temperature for copper-water nanofluid (Pr = 2.37) and alumina-water nanofluid (Pr = 6.66) is exhibited in Fig.2.13. It is noted that as Prandtl number increases, temperature decreases.

Table 2.2 depicts the change in non-dimensional skin friction coefficient for different volume fraction $\phi$ for both copper-water nanofluid and alumina-water nanofluid. It is noted that as $\phi$ increases, non-dimensional skin friction coefficient increases. The variation in Nusselt number for copper-water and alumina-water nanofluid for different values of Prandtl number is portrayed in Table 2.3. It is elucidated from the table that as Pr increases, non-dimensional rate of heat transfer decreases for both copper-water nanofluid and alumina-water nanofluid the fact which is concurrent with the physical reality.

### 2.7 Conclusion

In this paper, two-dimensional, steady, laminar, nonlinear, forced convective flow with heat transfer of copper-water and alumina-water nanofluids over a horizontal flat plate is investigated. The results are presented for various values of physical parameters like Prandtl number and volume fraction. Comparative study elucidates the fact that the dimensionless velocity is in perfect agreement with Blasius solution in the case of water. Temperature distribution of copper-water nanofluid has the increasing effect in comparison to that of water.

A systematic study on the effect of heat transfer is carried out. Moreover the effect of volume fraction on density, viscosity, specific heat capacity, thermal conductivity as well as Prandtl number for copper-water and alumina-
water nanofluids is studied. In general, the dimensionless temperature increases with increase in volume fraction for both copper-water and alumina-water nanofluids.

From the numerical results and discussion, the following conclusions are arrived.

- Nanofluids are efficient coolants than ordinary base fluids as they can remove more heat than ordinary base fluids.

- The increase in volume fraction causes an increase in density and thermal conductivity for copper-water and alumina-water nanofluids.

- There is no change in viscosity for the two types of nanofluids for fixed value of volume fraction but increase in volume fraction causes an increase in viscosity accordingly.

- The increase in volume fraction causes a decrease in Prandtl number as well as on specific heat capacity for copper-water and alumina-water nanofluids.

- The increase in Prandtl number is to decrease the temperature for both types of nanofluids elucidating the fact that the thermal boundary layer thickness is reduced due to increasing Prandtl number.

- For copper-water nanofluid and alumina-water nanofluid, the non-dimensional skin friction coefficient increases with increase in the volume fraction.

- The non-dimensional heat transfer rate of nanofluids decreases with the increase of Prandtl number for both the copper-water and alumina-water nanofluids.