CHAPTER 3

3D RECURSIVE NETWORK TOPOLOGY AND 3D ROUTING ALGORITHM

MAA of a 3D NoC concentrates on the two processes which are network topology and routing algorithm. Third dimension in 3D NoC provides opportunities to the chip designers to propose a lot of innovative interconnect architectures. The architectures require simple 3D routing algorithm to establish an efficient on chip-communications. In this chapter, the popular 2D mesh and 3D mesh topologies are discussed; the mesh topologies are ideal and natural choices for an NoC architecture. A 3D NoC topology and a 3D routing algorithm required for the topology are proposed. The mesh topologies are used to compare the performance of the proposed topology.

3.1 TWO AND THREE DIMENSIONAL NOC MESH TOPOLOGIES

Many interconnect architectures are designed for NoC topology such as Mesh, 2D WK-recursive network, Ring, Fat Tree, Butterfly-Fat Tree, Torus, Spidergon and Octagon. Among the various topologies, mesh topology is more popular since it has the soul properties required for any topology design. In this section, 2D and 3D mesh topologies for NoCs are presented.
3.1.1  2D Mesh Topology (2D MT)

Two dimensional mesh topology (2D MT) (Rahmati et al. 2007) with mesh size of 7 x 7 shown in Figure 3.1, is a very familiar topology for NoC interconnect architecture where the nodes are interconnected by using the horizontal links in a horizontal plane with an ID to identify them (Benini & De Michali 2002). In 2D MT, XY routing algorithm which is a distributed deterministic routing algorithm is used for data communication.

In the topology shown in Figure 3.1, each node is identified by its coordinates $x$ and $y$. The XY routing algorithm compares the current node ID $(C_x, C_y)$ to the destination node ID $(D_x, D_y)$ stored in the header flit. If the ID of the both current and destination nodes are same, the flits are consumed by the destination node. Otherwise, the flits are routed in $X$ direction first and then $Y$ direction to reach their destination node.

![2D mesh topology (2D MT)](image_url)

Figure 3.1  2D mesh topology (2D MT)
3.1.2 3D Fully Connected Mesh Topology (3D FMT)

3D Fully connected Mesh Topology (3D FMT) with the mesh size of $4 \times 4 \times 3$ is shown in Figure 3.2 in which one more dimension ‘Z’ is added to 2D MT (Shivam Tyagi & Shweta Bohare 2011). In Figure 3.2, the red color links are used to interconnect the switches/routers in the horizontal plane and the black color links are used to provide the vertical links between two adjacent layers.

![3D Fully Connected Mesh Topology](image)

**Figure 3.2** 3D fully connected mesh topology (3D FMT)

In 3D FMT, all the nodes of a layer are interconnected with the counterpart nodes of the neighboring layers by using the vertical links. In each layer, four clusters are formed by grouping each four nodes as a
cluster; one node is identified as cluster head (CH) in each cluster. The cluster heads and other nodes of the each cluster are identified with an ID of three digits ‘LCN’ where L represents a layer, C is used to identify a cluster and a particular node in a cluster is identified by using the digit N. In 3D FMT, ZXY routing algorithm is used for flits communication among the IP cores (Claudia Rusu et al. 2009).

The algorithm compares current node ID \((C_z, C_x, C_y)\) with destination node ID \((D_z, D_x, D_y)\) stored in header flit and the flits are moved to up or down layer if \(C_z\) is not equal to \(D_z\). Having reached destination layer, the flits are moved to \(x\) direction first, then \(y\) direction to reach the destination node (Muhammad Ali et al. 2005).

3.2 3D RECURSIVE NETWORK TOPOLOGY (3D RNT)

In this section, a recursive network based 2D NoC topology is first introduced and then a 3D NoC topology is constructed. 2D NoC topology is a graph based recursive topology. Degree of a particular node in the graph / network is the number of links incident to that node. If the degree of all the nodes is same, say \(\mu\) then the graph is called as \(\mu\)-regular graph.

![Figure 3.3](image)

**Figure 3.3** A complete graph with 4 nodes
If $\mu = n - 1$, then the graph is called as a complete graph, where $n$ is the number of nodes in the graph. A complete graph with four nodes is the basic module for 2D NoC which is shown in Figure 3.3 (Dharma et al. 1998).

A recursive network is denoted as $R(n,t)$, where $n$ is the number of nodes, and $t$ is the level, $t \geq 1$ and $n > 1$. A topology constructed using $R(n,t)$ is known as 2D Recursive Network Topology (2D RNT). In 2D RNT, the basic module $R(4,1)$ shown in Figure 3.3 is a 3-regular graph.

![Figure 3.4 2D recursive network topology](image)

The basic module with level $t = 1$ is shown in Figure 3.3. If the level $t$ is increased from one to two, the resultant topology is known as 2D RNT with $R(4,2)$ which is constructed and given in Figure 3.4. In the topology, the degree of the nodes is four excepting the four corner nodes and there are four copies of $R(4,1)$ which are interconnected in clock wise manner by using six horizontal links. A 3D Recursive Network Topology (3D RNT) is constructed recursively by stacking three copies of $R(4,2)$
which are interconnected by using eight vertical links as shown in Figure 3.5. The irregular topology is a 4-regular graph excepting the four corner nodes of the intermediate layer (Xu 2009).

Figure 3.5 3D recursive network topology (3D RNT) with substituting and flipping link labels

Node clustering is a fundamental property in an interconnect network topology and it has the following advantages:

- It improves network performance

- It is useful for encapsulating a group of IP cores to a specific task like parallel processing and processing a stream and block oriented data, etc.
- Communication scenario such as low power consumption, higher bandwidth, data security, etc., inside a cluster may be differentiated from other clusters.

- Resources which require high capacity interlayer communication can be placed into a particular cluster.

- A cluster can be used for integrating a specific technology into a NoC. For instance, an embedded memory or FPGA can be implemented in a cluster.

In the proposed topology 3D RNT, a node consists of a switch / router and an IP core. Node clustering is achieved in the topology such that each four nodes are grouped to create a cluster and each layer has four clusters thus total number of nodes in a layer is sixteen. In each cluster, a node is identified as Cluster Head (CH) and each CH and other nodes are identified with an ID of three digits LCN, first digit L represents a layer, second digit C represents a cluster and the digit N is used to represent a node in the cluster C and layer L. (Ville Rantala et al. 2008).

For instance, 000, 011, 022 and 033 are the IDs of the cluster heads in the layer 0 and the clusters 0, 1, 2 and 3 respectively. In the proposed topology, IP cores are connected to switches and in turn the switches are interconnected by bidirectional horizontal and vertical links where the vertical links are employed to establish inter layer communications.

Only 25% of the nodes in each layer are interconnected with the counterpart nodes of the neighboring layers using vertical links in order to minimize the number of vertical links. Three layers are considered in the proposed topology for experimental analysis (Dubois et al. 2013).
The properties of 3D RNT are given as follows (Jung-Sheng Fu 2005):

**Property 1**

- A node in 3D RNT is denoted by either \( L_i R(n,t) \) or \( L_i C_j R(n,t-1) \), where \( L_i = 0,1,2,\ldots,L-1 \) in which \( L \) is the total number of layer and \( C_j = 0,1,2,\ldots,n-1 \), where \( n \) is the number of nodes in a cluster.

- \( L_i R(n,t) \) is represented by \( L_i (a_{i-1} a_{j-2} \ldots a_1 a_0) \mid a_j \in \{0,1,\ldots,n-1\} \), where \( 0 \leq a \leq n-1 \)

**Property 2**

- The node adjacency is defined as follows: \( L_i (a_{j-1},a_{j-2},\ldots,a_1 a_0) \) is adjacent to (i) \( L_i (a_{j-1},a_{j-2},\ldots,a_1 b) \), where \( 0 \leq b \leq n-1 \) and \( b \neq a_0 \), (ii) \( L_i (a_{j-1},a_{j-2},\ldots,b_1 b_0) \), where \( a_i = b_0, a_0 = b_i \) and (iii) \( L_i (a_{j-1},a_{j-2},\ldots,b_1 b_0) \), where \( a_i = a_0 = b_i = b_0 \). The links of (i) are called as substituting links which are labeled as 0 in Figure 3.5. The links of (ii) and (iii) are called as flipping links labeled as \( \geq 1 \). The substituting links are those within the basic module \( R(n,t-1) \) and the flipping links are used to interconnect the modules \( R(n,t) \) to form the 3D NoC topology.
Property 3

- $(t-1)$ flipping links between module $L_i C_k R(n, t-1)$ and $L_i C_m R(n, t-1)$ which connect nodes $a(b)^{t-1}$ and $b(a)^{t-1}$, where $a \neq b$

Property 4

- $t$ flipping links between $L_i R(n, t)$ and $L_k R(n, t)$ which connect $(a)^t$ and $(b)^t$ where $a = b$, $L_i \neq L_k$.

Property 5

- Substituting link label $l=0$, where $L_i = L_1$, $a_1 = a_i = b_2 = b_i$ and $a_0 \neq b_0$

Property 6

- Flipping link label $l=1$, where $L_i = L_1$, $b_1 = a_0$ and $a_i = b_0$ and the flipping link label $l=2$, where $L_i \neq L_1$, $a_i = a_0 = b_1 = b_0$.

In Figure 3.5, the blue color links are referred to as substituting links; the flipping links with link label 1 is identified by the orange color links and the flipping links with link label 2 is identified by the violet color links.

3.2.1 Hamiltonian Connected 3D RNT

In this section, it is shown that 3D RNT is Hamiltonian connected. A Hamiltonian path of an interconnection topology is a path in an undirected graph that visits each node exactly once. A Hamiltonian cycle
is a closed Hamiltonian path that begins and ends with same node and the Hamiltonian path problem is to determine whether such path exists or not (Fu & Chen 2002).

A Hamiltonian connected interconnect topology has an advantage that it can have longest linear array between any two distinct nodes with dilation, congestion, load, and expansions equal to one. The longest linear array supports for interconnecting more number of IP cores and it requires simple routing algorithm which reduces communication cost.

The following are the node ID representation of the 3D RNT:

(i) For sub network level \( t-1 \), \( R(n,t-1) \leftrightarrow N \) where \( N \) is the ID of the nodes; \( N = 0,1,2,...n-1 \), where \( n \) is a number of nodes in a cluster

(ii) For 2D RNT in the network level \( t \),
\[
R(n,t) = C_m R(n,t-1) \leftrightarrow C_m N, \quad \text{where} \quad C_m = 0,1,2,..C-1,
\]
where \( C \) is a number of clusters in a layer

(iii) For 3D RNT in the network level \( t \),
\[
L_k R(n,t) = L_k C_m R(n,t-1) \leftrightarrow L_k C_m N, \quad \text{where} \quad L_k = 0,1,2,...L-1,
\]
where \( L \) is a number of layers in 3D RNT

The 3D recursive topology \( L_k R(n,t) \) is Hamiltonian connected if there is a Hamiltonian path between any two arbitrary nodes. According to Dirac's theorem, if \( G \) is a simple graph with \( \delta \) vertices where \( \delta \geq 3 \) and \( \text{Deg}(v) \geq \delta / 2 \) for each vertex \( v \), then \( G \) is Hamiltonian connected. Consider the basic module \( R(4,1) \) of the 3D RNT shown in Figure 3.3
where there are four nodes and the degree of each node is three, hence the basic module of 3D RNT is Hamiltonian connected.

![Diagram of Hamiltonian path between two arbitrary nodes S and D]

**Figure 3.6  Hamiltonian path between two arbitrary nodes S and D**

In Figure 3.6, it is assumed that \( C_i = N_i = V_i \); \( V_i = V_0, V_1, V_2, \ldots V_{n-1} = 0, 1, 2, 3, \ldots n - 1 \) and the Hamiltonian path is constructed between two arbitrary nodes S and D which are located in the first cluster of the layer \( L_0 \) and the last cluster of the layer \( L_k \) respectively as shown in Figure 3.6 in which the sign ‘\( \longrightarrow \)’ denotes a Hamiltonian path for each sub network of the level \( t-1 \); the sign ‘\( \rightarrow \)’ denotes \( t-1 \) flipping link and the sign ‘\( \rightarrow \)’ denotes \( t \) flipping link. Table 3.1 shows the Hamiltonian path between two nodes CH 000 and CH 233 of the proposed 3D RNT. Similarly, the Hamiltonian path between any two arbitrary nodes is established.
Table 3.1  Hamiltonian path between the nodes CH 000 and CH 233

<table>
<thead>
<tr>
<th>Layer ID</th>
<th>Cluster and Node IDs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0→</td>
<td>00→01→02→03→30→33→32→31→13→10→11→12→21→20→23→22→</td>
</tr>
<tr>
<td>1→</td>
<td>22→23→20→21→12→11→10→13→31→32→33→30→03→02→01→00→</td>
</tr>
<tr>
<td>2→</td>
<td>00→03→02→01→10→11→13→12→21→20→22→23→32→31→30→33→</td>
</tr>
</tbody>
</table>

3.3 3D ROUTING ALGORITHM

A hierarchical, cluster based 3D routing algorithm is developed for on-chip communication among the IP cores of 3D RNT. Advantages of hierarchical routing are scalability, higher performance, easy maintainability and manageability. The following assumptions are required to develop the 3D routing algorithm (Verdoscia et al. 1999, Palesi et al. 2009):

- Physical channels are used to interconnect switches
- Each physical channel is divided into group of virtual channels
- A node can generate flits at any rate subject to the constraints of available channel bandwidth, switching speed and buffer space of switches
- Once a channel queue accepts header flit of a message, it must accept all the remaining flits of the message before accepting any other flits from another message
• A route taken by a header flit is determined based on only destination node ID

• Deterministic routing is used and the header flit only contains source and destination node ID

The routing algorithm follows 3 hierarchical steps.

Step 1 : Finding destination layer

Step 2 : Finding destination cluster in the destination layer

Step 3 : Finding destination node in the destination cluster and layer

**Pseudo code of the 3D routing algorithm**

1. @ L_iC_jN_k : ID of a source node; @ L_iC_mN_n : ID of a destination node

2. @ L : Layer; @ C : Cluster; @ N : Node

3. If (L_i = = L_j) and L_iC_jN_j ↔ ID of a CH

4. Both source and destination nodes are located in a particular layer L_i and the source node is CH of cluster C_j

5. Else

6. Move flits to CH of cluster C_j

7. End if
8. If \( L_i < L_{t} \) then

9. Route flits to down layer CH of cluster \( C_j \)

10. Else if \( L_i > L_{t} \) then

11. Route flits to upper layer CH of cluster \( C_j \)

12. End if

13. End if

14. If \( C_j = C_m \) then

15. \( C_m \) is destination cluster

16. Else

17. Move flits to cluster \( C_m \)

18. End if

19. If \( N_k = N_n \) then

20. Node \( N_n \) in cluster \( C_m \) and layer \( L_i \) consumes flits

21. Else

22. Move flits to node \( N_n \) in cluster \( C_m \)

23. Node \( N_n \) in cluster \( C_m \) and layer \( L_i \) consumes flits
24. End if

For 3D RNT, the hierarchical routing path is established between a source node 231 and destination node 001 as follows:

Source node → 231 → layer 2, cluster 3, node 1

↔ 233 → layer 2, cluster 3, node 3

↔ 133 → layer 1, cluster 3, node 3

↔ 033 → layer 0, cluster 3, node 3

↔ 030 → layer 0, cluster 3, node 0

↔ 003 → layer 0, cluster 0, node 3

Destination node → 001 → layer 0, cluster 0, node 1

3.3.1 Deadlock Freedom of the 3D Routing Algorithm

Deadlock freedom is an important property for every routing algorithm. One of the major problems in wormhole routing is preventing or avoiding deadlock. The deadlock is occurred at a situation in which some existing messages are blocked which would never clear in a network. As a result, the header flits of a message cannot advance towards their destination. Hence the header flits will continue to hold the occupied channel and blocked for an indefinite long period.

In this section, it is proved that the 3D routing algorithm developed for the proposed 3D RNT is deadlock free. Deadlock is cataclysmic to an interconnection network. A few network resources either
channels or buffers are occupied by deadlocked packets which causes block of other packets on the resources, paralyzing entire network operation. Deadlock can be avoided by insisting packets to follow an order in acquiring resources.

3D RNT can be considered as a strongly connected undirected graph \( G = (N, C) \), where \( N \) is the set of nodes and \( C \) is the set of channels. Each channel \( c \) from the \( C \) connects its starting node \( s(c) \) to its ending node \( e(c) \). It can be written as \( c = (s(c), e(c)) \), where \( c \) is an output channel from \( s(c) \) and an input channel to \( e(c) \).

A routing algorithm be supposed to establish a path from source to destination node without any node repetition to ensure deadlock freedom. For instance, if there are \( f_1, f_2, f_3, \ldots, f_x \) flits and \( c_1, c_2, c_3, \ldots, c_y \) channels in a given topology, deadlock exits when the flit \( f_x \) is transmitted over \( c_y \) and then \( c_1 \). Dally & Seitz (1987) have shown that:

- A deadlock-free routing algorithm can be developed for an arbitrary interconnection network by splitting the physical channels into groups of virtual channels
- A necessary and sufficient condition for deadlock-free routing is the absence of cycles in a channel dependency graph
- Nodes of an interconnection network are not repeated if channels in the network can be numbered such as every possible routing path uses strictly increasing or decreasing channels

If nodes are not repeated, then there is no cyclic dependency in channel dependency graph, hence there is no deadlock in a routing
(Somasundaram et al. 2012). The 3D routing algorithm developed for 3D RNT uses at least two virtual channels \( \pm V \) and \( \mp V \) in each physical channel. Two functions are defined to prove that the algorithm is deadlock free which are \( f_z(h_i) = f_z(v_i) = i \), where \( i > 0 \) and \( z \) is the level of the proposed topology. Planar and vertical channels are numbered using the function \( f_z(h_i) \) and \( f_z(v_i) \) respectively (Dubois et al. 2013).

Since virtual channels are totally separate and independent, proof for deadlock freedom is presented here for a case where channels contained in level \( z \) is numbered such that every possible routing path is only along strictly increasing channels.

At the level \( z = 0 \) (at the bottom of the 2D plane of the topology), planar channels are directly numbered with the corresponding function \( f_0(h_i) \) and the vertical channels are associated to a number strictly greater than all numbers in the 2D plane, i.e., \( f_0(v_i) = \max(f_0(h_i)) + 1 \) as shown in Figure 3.7. At the level \( z > 0 \), planar channels are numbered with the corresponding \( f_z(h_i) \) function augmented by a constant that is greater than any number found in the lower levels.

Vertical channels are associated to a number strictly greater than all the channels in a current level. If \( c \) is either a planar or vertical channel contained in the level \( k \) and \( k-1 \) then \( f_k(h_i) > f_{k-1}(v_i) \), where \( f_{k-1}(v_i) = \max(f_{k-1}(h_i)) + 1 \) and \( f_k(v_i) = \max(f_k(h_i)) + 1 \). It is proved by using the following two steps that the two functions are strictly increasing for every possible routing path.

Step 1: **Flits flows are in same level \( z \).** Consider \( c = 1, 2, 3, \ldots, i \) are the assigned channels for every
possible routing path in the level \( z \). The function 
\( f_k(h_i) \) can be defined such that 
\[ f_k(h_i) - f_k(h_{i-1}) = f_z(h_i) - f_z(h_{i-1}) = f_z(h_{i-1}) - f_z(h_{i-2}) > 0. \]

\[ f_k(h_{i-1}) = f_k(h_{i-2}) + 1 \]

\[ f_k(h_{i-2}) = f_{k-1}(v_i) + 1 \]

\[ f_{k-1}(v_i) = f_{k-2}(v_i) + 1 \]

\[ f_0(v_i) = \max(f_0(h_i) + 1) \]

\[ f_0(h_i) \]

**Figure 3.7** Assignment of channel numbers for a 3D routing path

Consequently, \( f_z(h_{i-2}) < f_z(h_{i-1}) < f_z(h_i) \); hence every possible routing path in a same level \( z \) has strictly increasing channels.

**Step 2** *Source and destination nodes of the flits flows are in different levels:* Consider \( f_k(h_{i-2}) \), \( f_{k-1}(v_i) \),
max \( f_{k-1}(h_i) \) respectively be the functions of first planar channel in level \( k \) after a level change, the descending vertical channel in level \( k-1 \) and the last planar channel in the level \( k-1 \) before a level change. The functions \( f(h_i) \) and \( f(v_i) \) can be defined as
\[
f_{k-1}(v_i) - \max(f_{k-1}(h_i)) = f_k(v_i) - \max(f_k(h_i)) = f_k(v_i) - \max(f_k(h_i)) > 0 \quad \text{and} \quad f_k(h_{i-2}) - f_{k-1}(v_i) = f_k(h_{i-2}) - \max(f_{k-1}(h_i)) + 1 > 0, \quad \text{which results}
\]
\[
f_{k-1}(h_i) < f_{k-1}(v_i) < f_k(h_{i-2}).
\]

The above two steps prove that the channels are numbered such that every possible routing path is along strictly increasing channels and hence it is proved that the 3D routing algorithm is deadlock free.

### 3.4 CONCLUSION

In this chapter, 3D RNT, an irregular 3D NoC topology, is proposed and it is showed that the topology has Hamiltonian connectedness. A 3D routing algorithm required for the topology is developed and it is proved that the algorithm is deadlock free. Further, the popular 2D mesh and 3D fully connected mesh topologies are discussed with ZXY routing algorithm.