CHAPTER III
CHAPTER III
MANAGERIAL DECISION MAKING FOR PROCUREMENT SYSTEM FOR PERISHABLE COMMODITY

3.1 INTRODUCTION
In chapter 2, optimum procurement quantity is determined for the procurement system having backorder procurement policy approach but this is not feasible for all the production process houses. Thus for such production process houses, it is required to determine optimum procurement quantity by considering the lost procurement policy approach.

In this chapter, optimum procurement quantity is determined for procurement system having lost procurement policy approach. In other words, the development of this model may be considered as the counterpart of the procurement model developed in chapter 2.
3.2 ASSUMPTIONS

The assumptions made for the development of this model are as under:

1. The demand is deterministic and uniform over the given cycle time. The demand rate is $D$ units per cycle time.
2. Procurement system adopts lost procurement policy approach.
3. Lead time ($t$) is a random variable with doubly truncated normal distribution. (Truncated below at the point $c$ and above at the point $d$ respectively, where $c$ is the minimum lead time and $d$ is the maximum possible lead time and they are assumed to be known, $0 \leq c < d \leq 1$).
4. Inventory holding cost is given by $C_1$ Rs. per unit per unit time.
5. Shortage cost is given by $C_2$ Rs. per unit.
6. Procurement rate is finite.
7. Procurement cost is given by functional form of the procurement quantity viz, $(a + bQ)$, where $a$ and $b$ are known cost coefficients.
8. Order is placed as soon as the inventory level drops to zero.
9. At the most one order is outstanding at any time.
10. Commodity gets perished at a fixed rate of perishability $\theta_1$ during the transaction and thereafter at a rate $\theta_2$ at the time when inventory is on hand.
11. No replacement or repair of perished units is done during the entire cycle.
3.3 NOTATIONS

The following notations are used in this chapter.

- \( N \) = Number of runs
- \( D^* \) = Expected number of units that are to be procured during the entire cycle time,
- \( c \) = Minimum lead time
- \( d \) = Maximum possible lead time
- \( P \) = Price per unit
- \( t_i \) = Lead time for \( i^{th} \) cycle \((i=1,2,...,N)\)
- \( \mu \) = Average lead time
- \( \sigma^2 \) = Variance of lead time
- \( Q_i \) = Inventory level at the beginning of the \( i^{th} \) cycle \((i=1,2,...,N)\)
- \( T_i \) = Inventory holding time for \( i^{th} \) cycle \((i=1,2,...,N)\)
- \( \theta_1 \) = The rate at which units are perished during the transaction of the commodity \((\theta_1 > 0)\)
- \( \theta_2 \) = The rate at which units are perished at the time when inventory is on hand \((\theta_2 > 0)\)
- \( Q \) = Procurement quantity
- \( Q^* \) = Optimum procurement quantity
- \( TC(Q) \) = Expected total cost
- \( TC(Q^*) \) = Optimum value of expected total cost
- \( I_1(Q) \) = Inventory holding cost
- \( I_2(Q) \) = Expected shortage cost
- \( I_3(Q) \) = Procurement cost
- \( I_4(Q) \) = Loss occurred due to perished units
3.4 MATHEMATICAL FORMULATION

For the given procurement system, the expected number of units that are to be procured in given cycle time is given by the following relationship

\[ D^* = D + D^* \theta_1 + D^* (1 - \theta_1) \theta_2 - N \mu D \]  \hspace{1cm} [3.4.1]

thus value of \( N \) turns out as

\[ N = \frac{D}{(1 - \theta_1)(1 - \theta_2)Q + \mu D} \]  \hspace{1cm} [3.4.2]

Inventory level at the beginning of the \( i \)th cycle is given by

\[ Q_i = (1 - \theta_1) Q \]  \hspace{1cm} [3.4.3]

where \( i = 1, 2, 3, ..., N \)

and

\[ T_i = (1 - \theta_2) Q_i / D \]  \hspace{1cm} [3.4.4]

where \( i = 1, 2, 3, ..., N \)

For the procurement system under consideration, the expected total cost \( TC(Q) \) is given by

\[ TC(Q) = T_1(Q) + T_2(Q) + T_3(Q) + T_4(Q) \]  \hspace{1cm} [3.4.5]

The loss occurred due to perished unit is given by

\[ T_4(Q) = N \sum_{i=1}^{N} (\theta_1 + \theta_2 - \theta_1 \theta_2) Q \]  \hspace{1cm} [3.4.6]

The procurement cost \( T_3(Q) \) is given by

\[ T_3(Q) = N (a + bQ) \]  \hspace{1cm} [3.4.7]
The mathematical formulation of the procurement system is self explanatory from the above graphical presentation.
The expected shortage cost \( I_2(Q) \) is given by

\[
I_2(Q) = E \left[ \sum_{i=1}^{N} t_i D \right] C_2
\]

\[
= N \mu D C_2
\]

[3.4.8]

The holding cost \( I_1(Q) \) is given by

\[
I_1(Q) = E \left[ \sum_{i=1}^{N} Q_i T_i \right] C_1 / 2
\]

\[
= N A_1 A_2 Q^2
\]

[3.4.9]

Where

\[
A_1 = (1-\theta_1)(1-\theta_2)
\]

\[
A_2 = (1-\theta_1) C_1 / 2D
\]

Hence the expected total cost \( TC(Q) \) is given by

\[
TC(Q) = N \left[ A_1 A_2 Q^2 + A_3 Q + A_5 \right]
\]

[3.4.10]

Where

\[
A_3 = P (\theta_1 + \theta_2 - \theta_1 \theta_2) + b
\]

\[
A_4 = \mu D
\]

\[
A_5 = A_4 C_2 + a
\]

and \( A_1 \) and \( A_2 \) are as mentioned above.

The above cost function is a convex function of the procurement quantity \( Q \) as a decision variable \( Q \). The problem is to determine the optimum value of \( Q \) such that \( TC(Q) \) is minimum. The necessary and sufficient conditions are given by

\[
\frac{\partial TC(Q)}{\partial Q} = 0, \quad \text{and} \quad \frac{\partial^2 TC(Q)}{\partial Q^2} > 0 \text{ at } Q = Q^*.
\]

[3.4.11]

From the necessary and sufficient conditions, the optimum value of procurement quantity \( Q^* \) is obtained as under.

\[
Q = - \frac{A_4}{A_1} + \sqrt{\frac{A_6}{A_2}}
\]

[3.4.12]
The above value of procurement quantity Q is optimum provided that

\[ A_2A_4^2 - A_3A_4 + A_1A_5 > 0 \]  \[ \text{[3.4.13]} \]

3.4.1 PARTICULAR CASES:

Different particular cases obtained from the above developed model are as under:

Case A: Alternative strategy
Case B: Non-perishable commodity
Case C: Zero lead time with single rate of perishability
and
Case D: Non-perishable commodity with zero lead time

CASE A: ALTERNATIVE STRATEGY

As a result of better business negotiation, the cost of units perished during the transactions of the commodity is reimbursed through the supplier, then TC(Q) and the optimum value of procurement quantity turn out as under:

\[ TC(Q) = N \left( A_1A_2 Q^2 + A_7 Q + A_5 \right) \]  \[ \text{[3.4.14]} \]

\[ Q = - \frac{A_4}{A_1} + \frac{\sqrt{A_8}}{A_1} \]  \[ \text{[3.4.15]} \]

Where \( A_7 = P \left( \theta_2 - \theta_1 \theta_2 \right) + b \)
\( A_8 = A_4^2 + (A_1A_5 - A_7A_4) / A_2 \)

and \( N, A_1, A_2, A_4 \) and \( A_5 \) are as mentioned above.

The above value of procurement quantity Q is optimum provided that

\[ A_2A_4^2 - A_7A_4 + A_1A_5 > 0 \]  \[ \text{[3.4.16]} \]
CASE B: NON-PERISHABLE COMMODITY

The commodity having negligible rate of perishability can be considered as NON-PERISHABLE COMMODITY. Considering this aspect, the above formulae reduce to

\[ TC(Q) = \left[ \frac{D}{Q + \mu D} \right] \left[ \frac{C_1 Q^2}{2D} + bQ + \mu D C_2 + a \right] \]  \[ 3.4.17 \]

and

\[ Q = -\mu D + \left[ (\mu D)^2 + (\mu D C_2 + a - \mu Db) 2D/C_1 \right]^{1/2} \]  \[ 3.4.18 \]

The above value of procurement quantity Q is optimum provided that

\[ C_1(\mu D)^2 + (\mu D C_2 + a - \mu Db) 2D > 0 \]  \[ 3.4.19 \]

CASE C: ZERO LEAD TIME WITH SINGLE RATE OF PERISHABILITY

It is a general practice to calculate the single rate of perishability rather than the split one. In this case, perishability rate \( \theta \) is given by the following approximation.

\[ \theta \approx \theta_1 + \theta_2 \]  \[ 3.4.20 \]

Taking zero lead time situation, optimum procurement quantity is determined by the standard EOQ formula.

The optimum procurement quantity and associated cost function are given by

\[ Q^* = \left[ \frac{2D^* a}{C_1} \right]^{1/2} \]  \[ 3.4.21 \]

and

\[ TC(Q^*) = \left[ 2D^* C_1 a \right]^{1/2} + bD^* + D^* \theta P \]  \[ 3.4.22 \]

respectively.

Where \( D^* = D/(1-\theta) \)
CASE D: NON-PERISHABLE COMMODITY WITH ZERO LEAD TIME

If one considers non-perishable commodity with zero lead time, then the optimum value of procurement quantity is reduced to

\[ \sqrt{\frac{2D}{a}} \]

and hence

\[ TC(Q^*) = \left( \frac{2DC_1}{a} \right) + bD \]

which is the standard formula for EOQ model.

3.5 NUMERICAL SOLUTION

The parametric space is denoted by

\[ \Omega = (D, \mu, \sigma, c, d, C_1, C_2, a, b, p, \theta_1, \theta_2) \]

with their respective units of measurements.

The above developed model is applied to procurement system of one agroprocessing unit situated in Gujarat State. For this agroprocessing unit, the parametric space is given by

\[ \Omega = (40000, .001, 0.01, 0.005, .01, 25, 40, 500, 0.25, 150, .01, .025) \]

with their respective units of measurements.

The optimum values associated with the procurement system along with the particular cases are given in Table 3.5.1.
### TABLE 3.5.1

**OPTIMUM VALUES ASSOCIATED WITH THE PROCUREMENT SYSTEM**

<table>
<thead>
<tr>
<th>OPTION</th>
<th>( Q^* )</th>
<th>( TC(Q^*) ) (in Rs.)</th>
<th>NUMBER OF PERISHED UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>2464</td>
<td>287343.90</td>
<td>1416</td>
</tr>
<tr>
<td>A</td>
<td>2505</td>
<td>226203.62</td>
<td>1417</td>
</tr>
<tr>
<td>B</td>
<td>2546</td>
<td>73660.65</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>1288</td>
<td>260171.73</td>
<td>1451</td>
</tr>
<tr>
<td>D</td>
<td>1265</td>
<td>41623.00</td>
<td>0</td>
</tr>
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TABLE 3.5.2
SENSITIVITY ANALYSIS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>10%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q^*</td>
<td>TC(Q^*)</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INC</td>
<td>8.64</td>
<td>9.71</td>
</tr>
<tr>
<td>DEC</td>
<td>-8.65</td>
<td>-9.71</td>
</tr>
<tr>
<td>μ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INC</td>
<td>3.50</td>
<td>0.74</td>
</tr>
<tr>
<td>DEC</td>
<td>-3.63</td>
<td>-0.77</td>
</tr>
<tr>
<td>C_1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INC</td>
<td>-4.73</td>
<td>1.02</td>
</tr>
<tr>
<td>DEC</td>
<td>5.50</td>
<td>1.07</td>
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<tr>
<td>C_2</td>
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<td></td>
</tr>
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<td>INC</td>
<td>4.25</td>
<td>0.90</td>
</tr>
<tr>
<td>DEC</td>
<td>-4.44</td>
<td>-0.94</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INC</td>
<td>1.35</td>
<td>0.29</td>
</tr>
<tr>
<td>DEC</td>
<td>-1.37</td>
<td>-0.29</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INC</td>
<td>0.028</td>
<td>0.35</td>
</tr>
<tr>
<td>DEC</td>
<td>0.028</td>
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<tr>
<td>P</td>
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<td>θ_1</td>
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<td>DEC</td>
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</table>
From Table 3.5.1, it may be seen that for the original case and the cases A and C, the values of the optimum procurement quantity and the associated cost function differ due to different situations prevailing with the procurement system, however in case A, almost equal number of units get perished as compared to original case.

From the optimum procurement quantity and the associated cost function, for standard EOQ model and case B, it reveals that the random nature of lead time plays a very vital role in decision making approach. The associated cost with the system is higher than the later case. The above result reveals that one cannot ignore the fact that lead time and the perishability of the commodity are of specific nature.

From table 3.5.2, it can be seen that the parameters D, P and $\theta_2$ are the most sensitive parameters, while $\mu$, a, b, $C_1$ and $C_2$ are the least sensitive parameters. The parameter $\theta_1$ is reasonably effective with respect to expected total cost. The parameters D is the most sensitive parameter, while $\theta_1$, $\theta_2$, b and p are the least sensitive parameters. The parameter $a$, $\mu$, $C_1$ and $C_2$ are reasonably effective with respect to the procurement quantity, while the parameters c, d and $\phi$ have almost no effect at all with respect to the procurement quantity as well as the expected total cost.

It may be noted that the optimum procurement quantity is inversely proportional to the parameters b, p, $C_1$, $\theta_1$ and $\theta_2$ while optimum value of expected total cost observes the proportional effect in the same direction.