CHAPTER VIII
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OPTIMISATION TECHNIQUE FOR PROCUREMENT PROBLEM OF AGROPROCESSING INDUSTRIES

8.1 INTRODUCTION

In a country like India, agriculture is the main economic activity. Agricultural sector has tremendous impact on national income generation which has its own impact upon the development of the country. In the era of globalisation, various agrobased industries are coming up which have substantial impact on country's economy. Thus necessity arises to study the procurement system of the agroprocessing industries for which agricultural crops are major inputs for the production.

Besides tremendous development achieved in the agricultural science, agroclimatic conditions and other related factors affect the quantity as well as quality of a crop substantially during non-season time, pre-season time, season time, peak season time and rest of season time. Due to such typical characteristics, the supply of agricultural crop is not uniform throughout the given cycle time (eg one year). Generally consumer behavior is to procure agricultural crop (product) during the season time and failing this, one has to make compromise with the quantity as well as the quality of yields.
In this chapter, an attempt is made to study the optimum procurement policy for an agricultural crop for the agroprocessing industries. The problem formulation is done with the help of linear programming technique for the determination of procurement quantity in each time segment which maximises the benefit of price variation that is existing among the different time segments. The developed model is also illustrated by means of an application for the processed food industry situated in Gujarat State.
.2 ASSUMPTIONS

The assumptions made for the development of this model are as under:

1. The demand is deterministic and uniform over given cycle time $T$.
2. Demand occurring during the non-season period $(T_0)$ is zero.
3. Shortages are not allowed to occur.
4. Procurement cost is expressed as a linear function of the procurement quantity $Q$ and it is given by $(a + b \cdot Q)$, where $a$, $b$ are known as cost coefficients.
5. Inventory holding cost per unit during $i^{th}$ time segment is denoted by $HCT_i$. $(i=1,2,3,4)$
6. Procurement rate is finite and it is denoted by $Q_i$ for the $i^{th}$ time segment. $(i=1,2,3,4)$
7. Units are procured at the beginning of each time segment itself.
8. The perishability rate during the transaction of the commodity is given by $\theta$ (in percentage).
9. The perishability rate at the time when there is onhand inventory is given by $\theta_i$ (in percentage) in the respective time segments. $(i=1,2,3,4)$
8.3 NOTATIONS:

The following are the notations used in this chapter.

\[
\begin{align*}
\mathbf{D} & = \text{Demand occurring during the cycle time } T \\
\mathbf{D^*} & = \text{Effective demand rate during the cycle period } T \\
\mathbf{D^{**}} & = \text{Number of units to be procured} \\
\mathbf{T_i} & = i^{th} \text{ time segment in the given cycle period } T. \\
\mathbf{Q_i} & = \text{Inventory level at the beginning of the } i^{th} \text{ time segment} \\
\mathbf{Q^i} & = \text{Units to be procured during the } i^{th} \text{ time segment} \\
\mathbf{P_i} & = \text{Price per unit during the } i^{th} \text{ time segment} \\
\mathbf{Q} & = \text{Procurement Quantities} = [0, Q_1, Q_2, Q_3, Q_4] \\
\mathbf{TC(Q)} & = \text{Total cost} \\
\mathbf{Q^*} & = \text{Optimum procurement quantities} \\
\mathbf{TC(Q^*)} & = \text{Optimum total cost} \\
\mathbf{I_1(Q)} & = \text{Inventory holding cost} \\
\mathbf{I_3(Q)} & = \text{Procurement cost} \\
\mathbf{C(Q)} & = \text{Purchase cost}
\end{align*}
\]
8.4 Problem Formulation

As the demand during non-season period is assumed to be zero, calculation of effective demand rate \( D^* \) is required to be incorporated for the problem formulation and it is given by

\[
D^* = D \sum_{i=1}^{4} T_i
\]  

[8.4.1]

(Note that \( T_0 D^* = 0 \))

As commodity is of perishable nature, the number of units perished during the transaction of the commodity and at the time when inventory is onhand are additionally required to meet the demand. Thus the number of units to be procured is given by the following relationship.

\[
D^{**} = D + D^{**} \theta + \sum_{i=1}^{4} Q_i \theta_i
\]  

[7.4.2]

It may be impossible to procure certain amount of quantity of agricultural crops at a time, as it depends upon the factors like nature of the cropping pattern, market conditions, consumer behaviour, transport facilities etc.. Thus without any loss of generality, it may be assumed that the required amount of quantity is procured at the beginning of the respective time segment itself at a single time point.
The mathematical formulation of the procurement system is self-explanatory from the above graphical presentation.
The recursive relationship between the inventory level and quantity to be procured in different time segments is given by

\[ Q_i = (1 - \theta) Q_{i+1} + (1 - \theta_{i-1}) Q_{i-1} - T_{i-1}D^\theta \]

(i=1,2,3,4, \( Q_0 = 0 \), \( \theta_0 = 0 \) and \( T_0 D^\theta = 0 \)) \[ (8.4.3) \]

As \( \theta_i \) percentage units are perished at the time when inventory is on hand in respective time segments, the inventory level at the beginning of the cycle must satisfy the following inequalities.

\[ Q_i \geq D^\theta T_i / (1 - \theta_i) \]

\( 0 < \theta_i < 1 \) and \( i=1,2,3,4 \) \[ (8.4.4) \]

As total quantity procured is equal to the number of units to be procured during the cycle time \( T \)

\[ \sum_{i=1}^{4} Q_i = D^{\theta^{**}} \]

\[ (8.4.5) \]

From [8.4.3], [8.4.4] and [8.4.5], we get

\[ Q_4 = T_4 D^\theta / (1 - \theta_4) \]

\[ (8.4.6) \]

For the above procurement system under consideration, total cost \( TC(Q) \) is given by

\[ TC(Q) = \text{Purchase cost} + \text{Inventory holding cost} + \text{Procurement cost} \]

\( i.e \ TC(Q) = C(Q) + I_1(Q) + I_3(Q) \)

\[ (8.4.7) \]

\[ (8.4.8) \]
Where \( C(Q) = \sum_{i=1}^{4} P_i Q^i_1 \) \[8.4.9\]

\[ I_3(Q) = \sum_{i=1}^{4} (a + b Q^i_1) \] \[8.4.10\]

and

\[ I_1(Q) = \sum_{i=1}^{4} \left[ (1 - \theta_i) Q_i - T_i D^* + \left( T_i D^* + \theta_i Q_i \right) / 2 \right] HCT_i \] \[8.4.11\]

Thus the above procurement problem for the perishable commodity will be formulated as under:

Determine the procurement quantities \( Q_i (i=1,2,3,4) \) such that the total cost

\[ TC(Q) = \sum_{i=1}^{4} \left[ P_i Q^i_1 + a + b Q^i_1 + \left\{ (1 - \theta_i) Q_i - T_i D^* + \left( T_i D^* + \theta_i Q_i \right) / 2 \right\} HCT_i \right] \] \[8.4.12\]

is minimum subject to the conditions given by \[8.4.13\], \[8.4.14\], and \[8.4.15\].

Where

\[ Q_1 = (1 - \theta) Q^i_1 \]
\[ Q_2 = (1 - \theta) Q^2_2 + (1 - \theta) Q_1 - T_1 D^* \]
\[ Q_3 = (1 - \theta) Q^3_3 + (1 - \theta) Q_2 - T_2 D^* \]
\[ Q_4 = (1 - \theta) Q^4_4 + (1 - \theta) Q_3 - T_3 D^* \] \[8.4.13\]

The above set of equations are the recursive relation constraints.
\[ Q_1 \geq T_1 b^* / (1 - \theta_1) \]
\[ Q_2 \geq T_2 b^* / (1 - \theta_2) \]
\[ Q_3 \geq T_3 b^* / (1 - \theta_3) \]
\[ Q_4 = T_4 b^* / (1 - \theta_4) \] 

These inequalities are known as procurement constraints.

\[ Q^i_1 \geq 0 \quad (i=1,2,3,4) \] 

This set of inequalities is known as the set of the non-negativity constraints.

The above linear programming problem has 8 unknown variables (i.e. 4 independent variables which are procurement variables and 4 dependent variables which are the variables for inventory level) with 12 constraints. Above problem can be solved by the standard simplex algorithm and optimum values of the procurement quantities can be obtained accordingly.

8.4.1 ALTERNATIVE STRATEGY

As a result of better business negotiation, the cost of perished units during the transaction of the commodity is reimbursed through the supplier and thus the linear programming problem becomes
Minimise the cost function

\[
TC(Q) = \sum_{i=1}^{4} \left[ (1 - \theta_i) P_i Q_i^t + a + b Q_i^t + \left\{ (1 - \theta_i) Q_i - T_i D^t + (T_i D^t + \theta_i Q_i) / 2 \right\} HCT_i \right]
\]  

[8.4.16]

subject to the conditions given by [8.4.13], [8.4.14] and [8.4.15].

8.4.2 PARTICULAR APPLICATION

If commodity has very negligible rate of perishability, the rate of perishability can be taken as zero and in this case the linear programming problem turns out as:

Minimise

\[
TC(Q) = \sum_{i=1}^{4} \left[ P_i Q_i^t + a + b Q_i^t + \left\{ Q_i - T_i D^t / 2 \right\} HCT_i \right]
\]  

[8.4.17]

subject to the conditions given by [8.4.18], [8.4.19] and [8.4.20].

Where

\[
\begin{align*}
Q_1 &= Q_t \\
Q_2 &= Q_2^i + Q_1 - T_1 D^t \\
Q_3 &= Q_3^i + Q_2 - T_2 D^t \\
Q_4 &= Q_4^i + Q_3 - T_3 D^t
\end{align*}
\]  

[8.4.18]

The above set of equations are the recursive relation constraints.
This set of inequalities is the set of procurement constraints.

\[ q_i \geq t_i d^* \]

[8.4.19]

This set of inequalities is known as the set of non-negativity constraints.

8.5 APPLICATION

The above developed model is applied to one processed food industry situated in Gujarat State. Procurement problem pertaining to the above industry is narrated as below.

\[ D = 50,000 \text{ Units per time cycle} \]

Time segment \( T \) is split up as

\[
\begin{align*}
T_0 &= 0.1666 \text{ year} \quad (2 \text{ Months}) \quad \text{Non-Season Time} \\
T_1 &= 0.1666 \text{ year} \quad (2 \text{ Months}) \quad \text{Pre-Season Time} \\
T_2 &= 0.25 \text{ year} \quad (3 \text{ Months}) \quad \text{Season Time} \\
T_3 &= 0.1668 \text{ year} \quad (2 \text{ Months}) \quad \text{Peak of the Season Time} \\
T_4 &= 0.25 \text{ year} \quad (3 \text{ Months}) \quad \text{Rest of the season time}
\end{align*}
\]
The holding costs for respective time segments are given as

\[ HCT_1 = 20 \text{ Rs/Unit} \]
\[ HCT_2 = 28 \text{ Rs/Unit} \]
\[ HCT_3 = 35 \text{ Rs/Unit} \]
\[ HCT_4 = 25 \text{ Rs/Unit} \]

Price for respective time segments are given as

\[ P_1 = 300 \text{ Rs/Unit} \]
\[ P_2 = 275 \text{ Rs/Unit} \]
\[ P_3 = 225 \text{ Rs/Unit} \]
\[ P_4 = 260 \text{ Rs/Unit} \]

The procurement cost is given by \( a + b Q \) where

\[ a = 1000 \text{ Rs} \]
\[ b = 0.25 \text{ Rs per Unit} \]

The perishability rates are given as under

\[ \theta = .02 \]
\[ \theta_1 = .03 \]
\[ \theta_2 = .035 \]
\[ \theta_3 = .05 \]
\[ \theta_4 = .04 \]

The optimum solution corresponding to the above agroprocessing industries is obtained as under.
<table>
<thead>
<tr>
<th>Associated Values</th>
<th>Original</th>
<th>Alternative Strategy</th>
<th>No Perishability Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procurement Quantity (in Unit)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_1 )</td>
<td>10520</td>
<td>10520</td>
<td>10000</td>
</tr>
<tr>
<td>( Q_2 )</td>
<td>15862</td>
<td>15862</td>
<td>15000</td>
</tr>
<tr>
<td>( Q_3 )</td>
<td>10741</td>
<td>10741</td>
<td>25000</td>
</tr>
<tr>
<td>( Q_4 )</td>
<td>15943</td>
<td>15943</td>
<td>00000</td>
</tr>
<tr>
<td>Cost (in Rs)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C(Q) )</td>
<td>1,40,79,955=00</td>
<td>1,37,98,465=00</td>
<td>1,27,50,000=00</td>
</tr>
<tr>
<td>( I_1(Q) )</td>
<td>7,00,235=00</td>
<td>7,00,235=00</td>
<td>11,97,500=00</td>
</tr>
<tr>
<td>( I_3(Q) )</td>
<td>17,266=50</td>
<td>17,266=50</td>
<td>15,500=00</td>
</tr>
<tr>
<td>( TC(Q^*) )</td>
<td>1,47,97,457=50</td>
<td>1,45,15,966=50</td>
<td>1,39,63,000=00</td>
</tr>
<tr>
<td>Number of Perished Units</td>
<td>3066</td>
<td>3066</td>
<td>00</td>
</tr>
<tr>
<td>Difference Amount (in Rs.)</td>
<td>8,34,457=50</td>
<td>5,52,966=50</td>
<td>00</td>
</tr>
</tbody>
</table>

N.B.: Difference amount is calculated by taking difference between the \( TC(Q^*) \) and the value of \( TC(Q^*) \) obtained in the case of No Perishability.
From the Table 8.5.1, it reveals the fact that the rate of perishability plays vital role in determination of procurement quantities. In the alternative strategy, as procurement quantities remain unchanged, the difference in total cost amount is only due to reimbursement of perished units during the transaction time received through the supplier.