CHAPTER VI
CHAPTER VI
OPTIMUM PROCUREMENT QUANTITIES FOR THE PROCUREMENT SYSTEM
OF PERISHABLE COMMODITY UNDER VARIABLE LEAD TIME :
AN APPROACH FOR THE SALES PROMOTIONAL SCHEME

6.1 INTRODUCTION

In the competitive market environment it may be necessary to attract the customers, to raise the market share or to clear the stock and hence most of the business firms have to adopt the sales promotional scheme either in the form of bulk quantity discount or in the form of additional units which are proportional to the order quantity.

To avail the maximum benefit from such kind of offer, one time decision is required to be taken for the purchase of the commodity in bulk quantum. In most of the cases, purchase of the perishable commodity in bulk quantum might adverse the situation due to perishable nature of the commodity. Thus need arises to take appropriate decision for the procurement of perishable commodity in adequate quantum to avail the maximum benefit from the sales promotional scheme.

In this chapter, an attempt is made to determine the optimum procurement quantities of the perishable commodity at two levels when temporary price discount is offered for the
procurement system under backorder procurement policy approach considering lead time as a random variable.

The determination of procurement quantities is done as under:

1. During the sales promotional scheme.
2. For the rest of time segment.

With the help of mathematical programming technique, the optimum procurement quantities are determined through Amount Spent Per Unit Consumed approach. For the above procurement system, the amount of benefit available through such kind of decision making approach is calculated. Comparison is also made for the case of the commodity which is not of perishable nature. This approach may also be useful in multiple price break situation by making minor modification in the formulated mathematical programming problem.

The developed procurement model is also applicable to the following restrictive situations.

**PERISHABLE COMMODITY HAVING FIXED LIFE TIME:**

The developed procurement system is also applicable to the perishable commodity having fixed life time cycle. This is done through imposing the restriction upon the inventory level such that it should not exceed the demand or otherwise consumption of commodity for predetermined time period.
SPECIAL INCENTIVE PRICE ONLY FOR HIGHER LEAD TIME:

The developed procurement system is also applicable to the sales promotional scheme in which the commodity is offered at a regular price along with the special incentive price. Lead time in the case of the special incentive price is invariably higher as compared to regular price. For such situation, the time point $T_s$ is calculated with the consideration of lead time of special incentive price offered, after this procurement quantity is determined through the above developed model. In this kind of decision making approach, the interest on amount given towards procurement quantity in the special incentive price scheme is also to be taken into account.
6.2 FUNCTIONAL BEHAVIOUR OF PERISHABILITY

Functional behaviour of perishability is observed in highly volatile commodity. It is observed that the number of units perished at the time of transaction of the commodity is proportional to the number of units transacted.

Hence $\theta_1 \propto Q$  

$\Rightarrow \quad \theta_1 = K_1 Q$ \hspace{1cm} [6.2.1]  

Where $K_1$ is the coefficient of perishability.

The number of units perished at the time when inventory is onhand depends upon the inventory level as well as inventory holding time. Thus it proportionately varies with the square of the inventory level.

Hence $\theta_2 \propto Q^2$  

$\Rightarrow \quad \theta_2 = K_2 Q^2$ \hspace{1cm} [6.2.2]  

Where $K_2$ is the coefficient of perishability.
6.3 ASSUMPTIONS

The assumptions made for the development of this model are as under:

1. The demand is deterministic and uniform over the given cycle time. The demand rate is $D_R$ units for the revised unit time.

2. Lead time $(t)$ is a random variable with doubly truncated normal distribution. (Truncated below at the point $c$ and above at the point $d$ respectively, where $c$ is the minimum lead time and $d$ is the maximum possible lead time and they are assumed to be known, $0 \leq c < d \leq 1$).

3. Shortages are allowed to occur with backorder procurement policy approach.

4. Inventory holding cost is $C_{1R}$ Rs. per unit for the revised unit time.

5. Shortage cost is $C_{2R}$ Rs. per unit.

6. Procurement rate is finite.

7. Procurement cost is given by a functional form of the procurement quantity viz $(a + bQ)$, where $a$ and $b$ are known cost coefficients.

8. Order is placed as soon as the inventory level drops to zero in the rest of the revised time segment.

9. At the most one order is outstanding at any time.

10. Perishability of commodity during the transaction and at the time when there is onhand inventory is given by functional form of procurement quantity and inventory level respectively.

11. No replacement or repair of perished units is done during the entire cycle.
4 NOTATIONS

The following notations are used in this chapter.

\[ D \] = Demand rate per unit time

\[ T_s \] = Time point of announcement made for temporary price discount

\[ T_R \] = Revised unit time

\[ P_R \] = Demand rate for the revised unit time

\[ P_{R}^* \] = Number of units that are to be procured during revised unit time

\[ t_i \] = Lead time for \( i \)th cycle

\[ t_{x_i} \] = Lead time for \( i \)th cycle in revised unit time

\[ \mu \] = Average lead time.

\[ \sigma^2 \] = Variance of lead time.

\[ C_R \] = Minimum lead time for revised unit time

\[ d_R \] = Maximum possible lead time for revised unit time

\[ h_R \] = Average lead time for revised unit time

\[ \sigma^2_R \] = Variance of lead time for revised unit time

\[ C_1 \] = Inventory holding cost per unit per unit time

\[ C_{1R} \] = Inventory holding cost per unit per revised unit time

\[ C_2 \] = Shortage cost per unit

\[ C_{2R} \] = Shortage cost per unit

\[ Q_{MR} \] = Minimum quantity requirement to avail price discount

\[ Q_M \] = Storage capacity

\[ P \] = Price per unit

\[ P_d \] = Price per unit during sales promotional scheme
\[ Q_1^* \] = Procurement quantity at the time when sales promotional scheme is effective
\[ Q_2 \] = Procurement quantity per order for the rest of revised time segment
\[ N \] = Number of times for which \( Q_2 \) units are to be procured
\[ K_1 \] = The coefficient of perishability during the transaction of the commodity. \( K_1 > 0 \)
\[ K_2 \] = The coefficient of perishability at the time when there is onhand inventory. \( K_2 > 0 \)

\[ \text{ASPU}(Q) \] = Amount spent per unit consumed
\[ \text{ASPU}(Q^*) \] = Optimum value of amount spent per unit consumed
\[ \text{TC}(Q) \] = Expected total cost
\[ \text{TC}(Q^*) \] = Optimum value of expected total cost
\[ C(Q) \] = Purchase cost
\[ I_1(Q) \] = Expected inventory holding cost
\[ I_2(Q) \] = Expected shortage cost
\[ I_3(Q) \] = Procurement cost

6.5 MATHEMATICAL FORMULATION

At initial stage, let the parametric space of procurement system be denoted by \( \Omega \) and it is given by
\[ \Omega = ( D, c, d, \mu, \sigma, a, b, C_1, C_2, K_1, K_2, P, D_M ) \] [6.5.1]
with their respective units of measurements.

For the above procurement system, let \( Q^* \) denote the optimum procurement quantity. After time \( T_S \) passes, suppose that vendor announces the temporary price discount on bulk quantity purchased.
Suppose that the sales promotional scheme is effective for time interval \([T_{S1}, T_{S2}]\) and during this time interval placement of only single order is feasible.

Let the time point \(t_a (\geq T_{S1})\) denote the time point at which inventory drops to zero in usual condition.

There are two possibilities for the time point \(t_a\).

\[
(1) \quad t_a \leq T_{S2} \quad \text{or} \quad (2) \quad t_a > T_{S2}
\]  

\[6.5.2\]

In the first case, \(T_S\) is substituted by \(t_a\) and in the second case, \(T_S\) is substituted by the minimum value of \(T_{S2}\) and \((t_a - \mu)\).

To avail maximum benefit from the sales promotional scheme, the calculation of revised unit time, demand rate and inventory holding cost per unit for the revised unit time are required to be incorporated in the prevailing procurement system.

Let \(Q_o\) denote the number of units on hand at time point \(T_S\). Thus the revised parametric space for the above procurement system is given by

\[
Q_r = \{ D_R, C_{1R}, C_{2R}, C_R, d_R, \mu_R, \sigma_R, a, b, K_1, K_2, P_d, P, \\
Q_0, Q_{MR} \}
\]  

\[6.5.3\]

with their respective units of measurements.
The values of $D_R$, $C_{1R}$, $C_{2R}$, $c_R$, $d_R$, $\mu_R$ and $\sigma_R$ are obtained through the following relations:

\[
\begin{align*}
T_R &= T - T_S = 1 - T_S \\
D_R &= D_T R \\
C_{1R} &= C_1 T_R \\
C_{2R} &= C_2 \\
C_R &= c / T_R \\
d_R &= d / T_R \\
\mu_R &= \mu / T_R \\
\sigma_R &= \sigma / T_R
\end{align*}
\]

[6.5.4]

For the revised procurement system, suppose that the decisions taken are as under:

1. Procure $Q_1'$ units during sales promotional scheme
2. Procure $Q_2'$ units in the rest of revised time segment per order.

6.5.1 DEVELOPMENT OF COST FUNCTION:

(1) CASE A:

In this case, there are no units left out.

viz. $Q_o = 0$ \quad \[6.5.5\]

For the case A, the number of units to be required during the revised unit time are given by

\[
D_R^* = D_R + K_1 D_R^* + K_2 (Q_1^2 + N Q_2^2)
\]

[6.5.6]

The inventory level $Q_1$ and $Q_2,i$ are given by

\[
\begin{align*}
Q_1 &= (1 - K_1) Q_1' - D_R t_{R,1} \\
Q_{2,i} &= (1 - K_1) Q_{2,i}' - D_R t_{R,i+1} (i=1,2,3,...,N)
\end{align*}
\]

[6.5.8]

Hence the cycle time $T_1$ and $T_{2,i}$ are given by

\[
\begin{align*}
T_1 &= (Q_1 - K_2 Q_1^2) / D_R \\
T_{2,i} &= (Q_2 - K_2 Q_2^2) / D_R (i=1,2,3,...,N)
\end{align*}
\]

[6.5.9]
The mathematical formulation of the procurement system is self-explanatory from the above graphical presentation.
Now as expected total cost comprises of the purchase cost, setup cost, expected inventory holding cost and expected shortage cost, for the revised procurement system expected total cost $TC(Q)$ is given by the following relationship.

$$TC(Q) = C(Q) + I_1(Q) + I_2(Q) + I_3(Q)$$  \[6.5.11\]

For this revised procurement system

$$C(Q) = Q^1 P_d + (D_R^* - Q^1) P$$

$$I_1(Q) = (Q^1 T_1 + N Q^2 T_2) C_{1R} / 2$$

$$I_2(Q) = (N+1) \mu_R D_R C_{2R}$$

$$I_3(Q) = (N+1) a + b D_R^*$$  \[6.5.12\]

Now ASPU$(Q)$ is given by the relationship

$$ASPU(Q) = TC(Q) / D_R$$  \[6.5.13\]

Thus mathematical programming problem turns out as under:

Minimise

$$ASPU(Q) = TC(Q)/D_R$$

Subject to

$$TC(Q) = Q^1 P_d + (D_R^* - Q^1) P + (Q^1 T_1 + N Q^2 T_2) C_{1R} / 2$$

$$+ (N+1) \mu_R D_R C_{2R} + (N+1) a + b D_R^*$$

$$D_R^* = D_R + K_1 D_R^* + K_2 (Q^2_1 + N Q^2_2)$$

$$D_R^* = Q^1 + N Q^2$$

$$Q^1 \leq Q_M$$

$$Q^2 \leq Q_M$$

$$Q^1 \geq Q_{MR}$$

$$Q^1 = (1 - K_1) Q^1 - \mu_R D_R$$

$$Q^2 = (1 - K_1) Q^2 - \mu_R D_R$$

$$T_1 = (Q^1 - K_2 Q^2) / D_R$$

$$T_2 = (Q^2 - K_2 Q^2) / D_R$$

All variables are non-negative.  \[6.5.14\]
FOR THE EXISTING PROCUREMENT POLICY:

As there is no effective sales promotional scheme, the quantity to be procured in the first level is equal to zero units. The mathematical programming problem pertaining to the existing procurement policy turns out as:

Minimise

$$\text{ASPU}(Q) = \frac{\text{TC}(Q)}{D_R}$$

Subject to

$$\text{TC}(Q) = D^*_R P + N Q_2 T_2 C_1 R / 2 + N \mu R D_R C_2 R + N a + b D^*_R$$

$$D^*_R = D_R + K_1 D^*_R + K_2 N Q^2_2$$

$$D^*_R = N Q^1_2$$

$$Q_2 \leq Q_M$$

$$Q_2 = (1 - K_1) Q^1_2 - \mu_R D_R$$

$$T_2 = (Q_2 - K_2 Q^2_2) / D_R$$

All variables are non-negative. [6.5.15]

(2) CASE B:

For this case, $T_S$ is given by the minimum value of $(T_{S2}, t_a - \mu)$. Let the number of units required to satisfy the demand during time period $\mu_R$ be denoted by $S$, then it is given by the following relationship

$$S - K_2 S^2 = \mu_R D_R$$ [6.5.16]

Let $LU$ denote the units left out from $Q_0$ units after the time period $\mu_R$ has elapsed. Then $LU$ is given by the following relationship

$$LU = Q_0 - S - 2 K_2 (LU)^2$$ and $LU \geq 0$ [6.5.17]
The mathematical formulation of the procurement system is self-explanatory from the above graphical presentation.
Thus the number of units to be required during the revised unit time is given by following relationship

\[
\dot{D}_R = D_R + K_1 D_R + K_2 (Q_1^2 + NO_2) - Q_0 + S - \mu_R D_R \\
+ 2 K_2 (LU)^2
\]

[6.5.18]

The inventory level \( Q_1 \) and \( Q_{2,i} \) are given by

\[
Q_1 = (1-K_1) Q_1^i + LU \\
Q_{2,i} = (1-K_2) Q_2^i - t_{R,i+1} D_R \quad (i=1,2,3,...,N) \quad [6.5.19]
\]

Hence the cycle time \( T_1 \) and \( T_{2,i} \) are given by

\[
T_1 = (Q_1 - K_2 Q_1^2) / D_R \\
T_{2,i} = (Q_2 - K_2 Q_{2,i}^2) / D_R \quad (i=1,2,3,...,N) \quad [6.5.20]
\]

As expected total cost comprises of the purchase cost, setup cost, expected inventory holding cost and expected shortage cost, expected total cost for the revised procurement system is given by

\[
TC(Q) = C(Q) + I_1(Q) + I_2(Q) + I_3(Q) \quad [6.5.21]
\]

Where

\[
C(Q) = Q_1^i P_d + (D_R^* - Q_1^i) P \\
I_1(Q) = (LU \mu_R + S \mu_R / 2) C_{1R} + (Q_1 T_1 + N Q_2 T_2) C_{1R} / 2 \\
I_2(Q) = N \mu_R D_R C_{2R} \\
I_3(Q) = (N+1) a + b D_R^*
\]

Thus mathematical programming problem pertaining to the revised procurement system now turns out as under:
Minimise

$$\text{ASPU}(Q) = \frac{TC(Q)}{D_R}$$

Subject to

$$TC(Q) = Q_1^1 P_d + (D_R^1 - Q_1^1) P + (LU \mu_R + \mu_R s / 2) C_{1R}$$
$$+ (Q_1 T_1 + N Q_2 T_2) C_{1R} / 2 + N \mu_R D_R C_{2R}$$
$$+ (N+1) a + b D_R$$

$$D_R^1 = D_R + K_1 D_R^1 + K_2 (Q_1^2 + N Q_2^2) - Q_o + s - \mu_R D_R +$$
$$2 K_2 (LU)^2$$

$$D_R^2 = Q_1^1 + N Q_2^1$$

$$Q_1 \leq Q_M$$

$$Q_2 \leq Q_M$$

$$Q_1^1 \geq Q_{MR}$$

$$Q_1 = (1 - K_1) Q_1^1 + LU$$

$$Q_2 = (1 - K_1) Q_2^1 - \mu_R D_R$$

$$T_1 = (Q_1 - K_2 Q_1^2) / D_R$$

$$T_2 = (Q_2 - K_2 Q_2^2) / D_R$$

$$S = \mu_R s^2 + \mu_R D_R$$

$$LU = Q_o - s - 2 K_2 (LU)^2$$

All variables are non-negative. [6.5.22]

**FOR THE EXISTING PROCUREMENT POLICY:**

As there is no effective sales promotional scheme, the quantity to be procured for the first level is equal to zero units. The mathematical programming problem pertaining to the existing procurement policy turns out as:
Minimise

\[ \text{ASPU}(Q) = \frac{TC(Q)}{D_R} \]

Subject to

\[ TC(Q) = D_R^{*} P + (Q_0 T_1 + N Q_2 T_2) C_{1R} / 2 + N \mu_R D_R C_{2R} \]

\[ + N a + b D_R^{*} \]

\[ D_R^{*} = D_R + K_1 D_R^{*} + K_2 (Q_0^2 + N Q_2^2) - Q_0 \]

\[ D_R^{*} = N Q_2^{*} \]

\[ Q_2 \leq Q_M \]

\[ Q_2 = (1 - K_1) Q_2^{*} - \mu_R D_R \]

\[ T_1 = (Q_0 - K_2 Q_0^2) / D_R \]

\[ T_2 = (Q_2 - K_2 Q_2^2) / D_R \]

All variables are non-negative. \[ 6.5.23 \]
6.6 NUMERICAL SOLUTION

The above developed model is supported with an application for agroprocessing unit. The parametric space for this production process unit is as shown below:

\[ D = 25000 \text{ units per unit time} \]

\[ c = 0.0075 \text{ unit time} \]

\[ d = 0.015 \text{ unit time} \]

\[ \mu = 0.01 \text{ unit time} \]

\[ \delta = 0.025 \text{ unit time} \]

\[ C_1 = 5 \text{ Rs. per unit per unit time} \]

\[ C_2 = 8 \text{ Rs. per unit} \]

\[ a = 1000 \text{ Rs. per order} \]

\[ b = 0.50 \text{ Rs. per unit} \]

\[ K_1 = 0.02 \]

\[ K_2 = 0.00005 \]

\[ P = 75 \text{ Rs. per unit} \]

\[ Q_M = 5000 \text{ units} \]

For the procurement system having above parametric set, the optimum procurement quantity turns out as 898 units.

Suppose that at time \( T_s = 0.06816 \) time, vendor announces the temporary price discount on purchase of bulk quantity to promote his sales. To avail the maximum benefit from the announcement made for the temporary price discount by the vendor, the parametric space of the existing procurement system is required to be revised.
Let the revised parametric space for the procurement system incorporated with temporary price discount be as under.

\[ T_R = 1 - 0.06816 = 0.93184 \text{ unit time} \]
\[ D_R = 23296 \text{ units per revised unit time} \]
\[ C_{1R} = 4.66 \text{ Rs. per unit per revised unit time} \]
\[ C_{2R} = 8.00 \text{ Rs. per unit} \]
\[ a = 1000 \text{ Rs. per order} \]
\[ b = 0.50 \text{ Rs. per unit} \]
\[ C_r = 0.008049 \text{ unit time} \]
\[ d_R = 0.016097 \text{ unit time} \]
\[ \mu_R = 0.0107314 \text{ unit time} \]
\[ \sigma_R = 0.0268286 \text{ unit time} \]
\[ K_1 = 0.02 \]
\[ K_2 = 0.00005 \]
\[ P = 75 \text{ Rs. per unit} \]
\[ P_d = 70 \text{ Rs. per unit} \]
\[ Q_{MR} = 1500 \text{ units} \]
\[ Q_M = 5000 \text{ units} \]

For this revised procurement system, the optimum procurement quantities are calculated by mathematical programming technique as explained above. The solutions obtained through the mathematical programming problem are given in the following table.
**TABLE 6.6.1**

OPTIMUM VALUES ASSOCIATED WITH THE REVISED PROCUREMENT SYSTEM.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Optimum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Perishable commodity</td>
</tr>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>$Q_0$ (Units)</td>
<td>0</td>
</tr>
<tr>
<td>$G_1^*$ (Units)</td>
<td>0</td>
</tr>
<tr>
<td>$Q_2^*$ (Units)</td>
<td>898</td>
</tr>
<tr>
<td>Number of Perished units</td>
<td>1024</td>
</tr>
<tr>
<td>TC($Q^*$) (Rs)</td>
<td>1918441.86</td>
</tr>
<tr>
<td>ASPU($Q^*$) (Rs.)</td>
<td>82.35</td>
</tr>
</tbody>
</table>

NB.:  
A: Existing procurement policy  
B: Procurement policy when sales promotional scheme is effective.
The above decision making approach suggests that, the
decision makers associated with the procurement system should
take decision for the procurement of the quantity as under:

Procure 1500 units during sales promotional scheme.
Procure 889 units during rest of the revised time
segment.

This decision making approach yields amount spent per unit
consumed as Rs. 82.10 under the sales promotional scheme
while the existing procurement policy (no sales promotional
scheme) yields amount spent per unit consumed as Rs. 82.35.
Thus sales promotional scheme reduces the amount spent per
unit consumed, while in the case of non-perishable commodity,
amount spent per unit consumed turns out as Rs. 68.85 and for
the existing procurement policy (no sales promotional scheme)
amount spent per unit consumed turns out as Rs. 69.51.

This decision making approach yields better results for both
the type of the commodity. This approach also reveals that
for perishable and non-perishable nature of commodity, the
procurement quantity remains almost the same for the rest of
time segment as compared to the existing procurement policy.