CHAPTER V
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OPTIMUM PROCUREMENT QUANTITY FOR THE PERISHABLE COMMODITY:
A MATHEMATICAL PROGRAMMING APPROACH

5.1 INTRODUCTION:

It may be worth while to examine the functional behaviour of price affecting the procurement system pertaining to perishable commodity. In this chapter, the optimum value of procurement quantity is determined through the Amount Spent Per Unit Consumed (ASPU) approach for such procurement system. The model developed is illustrated numerically.

5.2 FUNCTIONAL BEHAVIOUR OF PRICE

For some of the cases, functional behaviour of price is observed. One of the demand function widely used to study the market behaviour is given by

\[ X = a e^{-bp} \]  \[5.2.1\]

Where \( X \) represents the quantity to be procured
\( p \) represents price per unit
\( a \) and \( b \) are constants.

The above functional relationship is also represented by

\[ p = \frac{1}{b} \log_e \left(\frac{a}{x}\right) \]  \[5.2.2\]
3 ASSUMPTIONS

The assumptions made for the development of this model are as under:

1. The demand is deterministic and uniform over the given cycle time. The demand rate is \(D\) units per cycle time.
2. Lead time \((t)\) is a random variable with doubly truncated normal distribution. (Truncated below at the point \(c\) and above at the point \(d\) respectively, Where \(c\) is the minimum lead time and \(d\) is the maximum possible lead time which are assumed to be known, \(0 \leq c < d \leq 1\)).
3. Inventory holding cost is \(C_1\) Rs. per unit per unit time.
4. Shortage cost is \(C_2\) Rs. per unit.
5. Procurement rate is finite.
6. Procurement cost is given by a functional form of the procurement quantity viz \((a + bQ)\), where \(a\) and \(b\) are known cost coefficients.
7. Order is placed as soon as the inventory level drops to zero.
8. At the most one order is outstanding at any time.
9. Commodity gets perished at a fixed perishability rate \(\theta_1\) during the transaction and thereafter at the rate \(\theta_2\) at the time when there is on hand inventory.
10. No replacement or repair of perished units is done during the entire cycle time.
5.4 NOTATIONS

The following notations are used in this chapter.

\[ N \quad \text{= Number of runs} \]
\[ D^* \quad \text{= Expected number of units that are to be procured during} \]
\[ \quad \text{the entire cycle time} \]
\[ P \quad \text{= Price per unit} \]
\[ t_i \quad \text{= Lead time for } i^{th} \text{ cycle} \]
\[ \mu \quad \text{= Average lead time} \]
\[ \sigma^2 \quad \text{= Variance of lead time} \]
\[ Q \quad \text{= Procurement quantity} \]
\[ Q^* \quad \text{= Optimum procurement quantity} \]
\[ TNU \quad \text{= Total number of units consumed} \]
\[ TNPU \quad \text{= Total number of perished units} \]
\[ ASPU(Q) \quad \text{= Amount spent per unit consumed} \]
\[ ASPU(Q^*) \quad \text{= Minimum value of amount spent per unit consumed} \]
\[ TC(Q) \quad \text{= Expected total cost} \]
\[ TC(Q^*) \quad \text{= Optimum value of expected total cost} \]
\[ I_1(Q) \quad \text{= Expected inventory holding cost} \]
\[ I_2(Q) \quad \text{= Expected shortage cost} \]
\[ I_3(Q) \quad \text{= Procurement cost} \]
\[ C(Q) \quad \text{= Purchase cost} \]
5.5 MATHEMATICAL FORMULATION

As procurement system considers variable lead time, there exists two possibilities for production house.

CASE A: When procurement system adopts backorder procurement policy approach and

CASE B: When procurement system adopts lost procurement policy approach.

For the given procurement system, let there be Q units to be procured per order. Thus during entire time span, N orders are required to meet the demand. Hence for case A, the expected number of units that are to be procured during the entire cycle time is given by the following relationship

\[ D^* = D + D^* \theta_1 + D^* (1 - \theta_1) \theta_2 - N \mu D \theta_2 \]

\[ [5.5.1] \]

Hence the value of N turns out as

\[ N = \frac{D}{(1 - \theta_1)(1 - \theta_2)Q + \mu D \theta_2} \]

\[ [5.5.2] \]
Thus for the case A, inventory level at the beginning of the $i^{th}$ cycle is given by

$$Q_i = (1 - \theta_1) Q - t_i D$$

[5.5.3]

Where $i = 1, 2, 3, \ldots, N$

and

$$T_i = (1 - \theta_2) Q_i / D$$

[5.5.4]

Where $i = 1, 2, 3, \ldots, N$

For the case A, Amount Spent Per Unit Consumed is given by

$$ASPU(Q) = TC(Q) / D$$

[5.5.5]

Where $TC(Q) = C(Q) + I_1(Q) + I_2(Q) + I_3(Q)$

[5.5.6]

The purchase cost for the case A is given by $C(Q)$

$$C(Q) = D \mu P = N Q P$$

[5.5.7]

The procurement cost $I_3(Q)$ is given by

$$I_3(Q) = N (a+bQ)$$

[5.5.8]

The expected shortage cost $I_2(Q)$ is given by

$$I_2(Q) = E \left[ \sum_{i=1}^{N} t_i D \right] C_2 = N \mu D C_2$$

[5.5.9]

The expected inventory holding cost $I_1(Q)$ is given by

$$I_1(Q) = N (A_1 A_2 Q^2 - A_1 \mu Q C_1 + D (\mu^2 + \sigma^2) C_1 (1-\theta_2)/2)$$

Where $A_1 = (1-\theta_1)(1-\theta_2)$

$$A_2 = (1-\theta_1) C_1 / 2D$$

[5.5.10]

By substituting $Q, V_1$ and $V_2$ in place of $X, a$ and $b$ in equation [5.2.2], $p$ is represented by

$$p = (1/V_2) \log_e (V_1/Q)$$

[5.5.11]
Hence the expected total cost \( TC(Q) \) is given by

\[
TC(Q) = N \left( A_1 A_2 Q^2 + A_3 Q + A_5 \right)
\]

[5.5.12]

where \( A_3 = b + p - A_1 \mu C_1 \)

\[
A_4 = \mu D \theta_2
\]

\[
A_5 = D \left( \mu^2 + \sigma^2 \right) C_1 \left( 1 - \theta_2 \right)/2 + \mu D C_2 + a
\]

\[
p = \left( 1/V_2 \right) \log_e (V_1/Q)
\]

and \( A_1 \) and \( A_2 \) are as mentioned above.

This yields \( ASPU(Q) = TC(Q)/D \)

\[
= \left( A_1 A_2 Q^2 + A_3 Q + A_5 \right) / \left( A_1 Q + \mu D \theta_2 \right)
\]

[5.5.13]

The above cost function is a convex function of the procurement quantity \( Q \). Hence the problem is to determine the procurement quantity \( Q \) such that it yields the minimum value for the given \( ASPU(Q) \).

Thus the mathematical programming problem pertaining to case A turns out as under:

Minimise

\[
ASPU(Q) = \left( A_1 A_2 Q^2 + A_3 Q + A_5 \right) / \left( A_1 Q + \mu D \theta_2 \right)
\]

subject to

\[
Q > Q_0 \quad \text{(Procurement constraint)}
\]

\[
p = \left( 1/V_2 \right) \log_e (V_1/Q) \quad \text{(Price constraint)}
\]

\[
p > P_0 \quad \text{(Price constraint)}
\]

\[
A_1 = \left( 1 - \theta_1 \right) \left( 1 - \theta_2 \right)
\]

\[
A_2 = \left( 1 - \theta_1 \right) C_1 / 2D
\]

\[
A_3 = b + p - A_1 \mu C_1
\]

\[
A_4 = \mu D \theta_2
\]

\[
A_5 = D \left( \mu^2 + \sigma^2 \right) C_1 \left( 1 - \theta_2 \right)/2 + \mu D C_2 + a
\]

[5.5.14]
CASE B: When procurement system adopts lost procurement policy approach.

For the case B, let there be Q units to be procured per order. Thus during entire time span, N orders are required to meet the demand. Hence for case B, the expected number of units that are to be procured during entire cycle time is given by the following relationship

$$D^* = D + D^* \theta_1 + D^* (1 - \theta_1) \theta_2 - N \mu D$$  \[5.5.15\]

thus the value of N turns out as

$$N = \frac{D}{(1 - \theta_1)(1 - \theta_2)Q + \mu D}$$  \[5.5.16\]

For the case B, inventory level at the beginning of the $i$th cycle is given by

$$Q_i = (1 - \theta_1) Q$$ \[5.5.17\]

where $i = 1, 2, 3, ..., N$

and

$$T_i = (1 - \theta_2) Q_i / D$$ \[5.5.18\]

where $i = 1, 2, 3, ..., N$

For the case B, total number of units consumed is given by

$$TNU = N (1 - \theta_1)(1 - \theta_2) Q$$ \[5.5.19\]

Thus for the case B, Amount Spent Per Unit Consumed is given by

$$ASPU(Q) = TC(Q) / TNU$$ \[5.5.20\]

where $TC(Q) = C(Q) + I_1(Q) + I_2(Q) + I_3(Q)$ \[5.5.21\]
The purchase cost for the above procurement system is given by
\[ C(Q) = D^\# P = N Q \quad [5.5.22] \]

The procurement cost \( I_3(Q) \) is given by
\[ I_3(Q) = N (a+bQ) \quad [5.5.23] \]

The expected shortage cost \( I_2(Q) \) is given by
\[ I_2(Q) = E \left( \sum_{i=1}^{N} t_i D \right) C_2 = N \mu D C_2 \quad [5.5.24] \]

The expected inventory holding cost \( I_1(Q) \) is given by
\[ I_1(Q) = E \left( \sum_{i=1}^{N} Q_i \frac{t_i}{Z} \right) C_1 = N A_1 A_2 Q^2 \]

Where \( A_1 = (1-\theta_1)(1-\theta_2) \)
\[ A_2 = (1-\theta_1) \frac{C_1}{2D} \quad [5.5.25] \]

By substituting \( Q, V_1 \) and \( V_2 \) in place of \( X, a \) and \( b \) in equation [5.2.2], \( p \) is represented by
\[ p = \frac{1}{V_2} \log_e(V_1/Q) \quad [5.5.26] \]

Hence the expected total cost \( TC(Q) \) is given by
\[ TC(Q) = N (A_1 A_2 Q^2 + A_3 Q + A_5) \quad [5.5.27] \]

Where \( A_3 = b + p \)
\[ A_6 = \mu D \]
\[ A_5 = D C_2 + a \]
\[ p = \frac{1}{V_2} \log_e(V_1/Q) \]

and \( A_1 \) and \( A_2 \) are as mentioned above.
This yields \( \text{ASPU}(Q) = \frac{\text{TC}(Q)}{\text{TNU}} \)

\[
= \frac{(A_1A_2 Q^2 + A_3 Q + A_5)}{A_1 Q}
\]

[5.5.28]

The above cost function is a convex function of the procurement quantity \( Q \). Hence the problem is to determine the procurement quantity \( Q \) such that it yields the minimum value for the given \( \text{ASPU}(Q) \).

Thus the mathematical programming problem pertaining to case B turns out as under:

Minimise

\[
\text{ASPU}(Q) = \frac{(A_1A_2 Q^2 + A_3 Q + A_5)}{A_1 Q}
\]

subject to

\[
\begin{align*}
Q & > Q_0 \quad \text{(Procurement constraint)} \\
P & = \frac{1}{V_2} \log_e \left( \frac{V_1}{Q} \right) \quad \text{(Price constraint)} \\
P & > P_0 \quad \text{(Price constraint)} \\
A_1 & = (1-\theta_1)(1-\theta_2) \\
A_2 & = \frac{(1-\theta_1) C_1}{2D} \\
A_3 & = b + p \\
A_4 & = \mu D \\
A_5 & = \mu D C_2 + a
\end{align*}
\]

[5.5.29]
5.6 NUMERICAL SOLUTION

The parametric space is denoted by

\[ \Omega = ( D, \mu, \sigma, c, d, C_1, C_2, a, b, V_1, V_2, \theta_1, \theta_2, Q_0, P_0) \]

with their respective units of measurements.

The above developed procurement model is applied to the production process of one industry situated in Gujarat State. For the procurement system, parametric space is given by

\[ \Omega = (15000, .010, 0.02, 0.005, .04, 8, 10, 1000, 0.25, 5000, 0.0075, .015, .025, 1500, 125) \]

The solution obtained for both the cases are given in the following table.
### TABLE 5.6.1
OPTIMUM VALUES ASSOCIATED WITH THE PROCUREMENT SYSTEM

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>UNIT</th>
<th>OPTIMUM VALUE</th>
<th>CASE A</th>
<th>CASE B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^*$</td>
<td>Units</td>
<td>1958.00</td>
<td>1958.00</td>
<td></td>
</tr>
<tr>
<td>$D^*$</td>
<td>Units</td>
<td>15587.00</td>
<td>14465.00</td>
<td></td>
</tr>
<tr>
<td>TNU</td>
<td>Units</td>
<td>15000.00</td>
<td>13892.00</td>
<td></td>
</tr>
<tr>
<td>TPU</td>
<td>Units</td>
<td>587.00</td>
<td>573.00</td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>Rs.</td>
<td>125.00</td>
<td>125.00</td>
<td></td>
</tr>
<tr>
<td>$TC(Q^*)$</td>
<td>Rs.</td>
<td>1978950.00</td>
<td>1837355.90</td>
<td></td>
</tr>
<tr>
<td>ASPU($Q^*$)</td>
<td>Rs.</td>
<td>131.93</td>
<td>132.26</td>
<td></td>
</tr>
</tbody>
</table>
The above table reveals that for both the cases, procurement quantity turns out to be equal and amount spent per unit consumed turns out as Rs. 131.93 and Rs. 132.26 respectively. This indicates that, in all Rs. 6.93 and Rs. 7.26 per unit will be spent towards maintaining inventories and total number of perished units turn out as 587 units and 573 units respectively. This indicates that the backorder procurement policy does not have much impact on determination of procurement quantity.