CHAPTER 7

MULTI-ITEM INVENTORY MODEL WITH DEMAND DEPENDENT UNIT COST AND VARYING LEAD TIME

7.1 INTRODUCTION

In real life, once production is discontinued i.e., once the labor force is dismantled, supply of raw materials is disrupted, machineries is kept in disorder, etc., it is obvious that to start the next production, some time is required to bring the above mentioned things in order. The cost related to above factors depends on the time gap between the decision to start the preparation and actual commence of production. If the gap is small, then everything will have to be arranged hurriedly and it costs more. Hence lead time crashing cost decreases with the increase of preparation time. The preparation time for the next production run is very important. This decision influences several costs like setup cost, production cost etc.

In this chapter, a multi-item inventory problem with floor space and finite production cost constraints were considered where the demand of the items are inversely related to the unit price. The varying purchase and leading time crashing costs were considered to be continuous functions of demand rate and leading time respectively. The model has been solved using Karush Kuhn-Tucker conditions approach with the optimal order quantity, demand rate and leading time as decision variables. An optimal total cost has been obtained under limited storage space and production cost restrictions.
The aim of this process is to derive an optimal solution of inventory model and minimize the annual total cost function based on the values of demand rate, order quantity and leading time using Karush Kuhn-Tucker conditions approach.

Numerical example has been solved for a single item to illustrate the model. The demand dependent unit cost is considered here in fuzzy environment.

7.2 ASSUMPTIONS

The following basic assumptions about the model are made.

(i) Time horizon is finite 
(ii) No shortages are allowed. 
(iii) Unit production cost is inversely related to the demand rate. 
\[ i.e \ p_i = AD_i^{\hat{\delta}} \]
where \( i = 1, 2, \ldots, n, \hat{\delta} \geq 1, A > 0 \), where \( \hat{\delta} \) is called the price elasticity. 
(iv) Lead time crashing cost is related to the lead time by a function of the form \( R(L_i) = aL_i^{-b}, i=1, 2, \ldots, n, a > 0, \ 0 < b \leq 0.5 \), where ‘a’ and ‘b’ are real constants selected to provide the best fit of the estimated cost function.

7.3 OBJECTIVE FUNCTION OF THE MODEL

The annual relevant total cost (sum of production, order, inventory carrying and lead time crashing costs) which according to the basic assumptions of the EOQ model is:
Annual Total cost = production cost+ordering cost+holding cost+lead time crashing cost

\[ TC(D_i, Q_i, L_i) = p_i D_i + S_i D_i \left( \frac{Q_i}{2} + k \sigma \sqrt{L_i} \right) H_i + \frac{D_i}{Q_i} R(L_i) \]  \hspace{1cm} (7.1) \]

Substituting \( p_i \) and \( R(L_i) \) in (7.1) yields the function

\[ TC(D_i, Q_i, L_i) = AD_i^{1-\beta} + \frac{S_i D_i}{Q_i} + \left( \frac{Q_i}{2} + k \sigma \sqrt{L_i} \right) H_i + \frac{D_i}{Q_i} aL_i^{-b} \]  \hspace{1cm} (7.2) \]

Hence the objective function of the model is to minimize the annual total cost given by

\[ Min TC(D_i, Q_i, L_i) = \sum_{i=1}^{n} \left[ AD_i^{1-\beta} + \frac{S_i D_i}{Q_i} + \left( \frac{Q_i}{2} + k \sigma \sqrt{L_i} \right) H_i + \frac{D_i}{Q_i} aL_i^{-b} \right] \]  \hspace{1cm} (7.3) \]

7.4 CONSTRAINTS OF THE MODEL

There are some restrictions on available resources in inventory problems that cannot be ignored to derive the optimal total cost.

(i) There is a limitation on the available warehouse floor space where the items are to be stored.

\[ (i.e) \sum_{i=1}^{n} w_i Q_i \leq W \]  \hspace{1cm} (7.4) \]

(ii) Investment amount on total production cost cannot be infinite. It may have an upper limit on the maximum investment.
(i.e) \( \sum_{i=1}^{n} AD_i^{-\delta} Q_i \leq B \) \hspace{1cm} (7.5)

Thus the model can be stated as

\[
\text{Min} \text{TC}(D_i, Q_i, L_i) = \sum_{i=1}^{n} \left[ AD_i^{1-\delta} + \frac{SD_i}{Q_i} + \left( \frac{Q_i}{2} + k\sigma_i \sqrt{L_i} \right) H_i + \frac{D_i}{Q_i} aL_i^{-b} \right]
\]

subject to the inequality constraints

\[
\sum_{i=1}^{n} w_i Q_i \leq W \quad \text{and}
\]

\[
\sum_{i=1}^{n} AD_i^{-\delta} Q_i \leq B
\]

7.5 **FUZZIFICATION OF COST PARAMETER**

In this chapter, the unit production cost \( p_i \) has been defined under fuzzy environment. The membership function for the fuzzy variable \( p_i \) is defined as follows.

\[
\mu_{p_i}(X)=\begin{cases} 
1, & p_i \leq L_{p_i} \\
\frac{U_{p_i} - p_i}{U_{p_i} - L_{p_i}}, & L_{p_i} \leq p_i \leq U_{p_i} \\
0, & p_i \geq U_{p_i}
\end{cases}
\]

Here \( U_{p_i} \) and \( L_{p_i} \) are upper limit and lower limit of \( p_i \) respectively.

7.6 **KARUSH KUHN-TUCKER CONDITIONS**

When there is only one item the objective function reduces to the form
\[ TC(D, Q, L) = AD^{1-\delta} + \frac{SD}{Q} + \left( \frac{Q}{2} + k\sigma \sqrt{L} \right) H + \frac{D}{Q} aL^{-b} \]  

subject to the constraints

\[ wQ \leq W \]

and \( AD^{-\delta} Q \leq B \)

According to Kuhn-Tucker conditions the objective function and the constraints together can be written as follows by introducing the slack variables \( s_1 \) and \( s_2 \).

\[
Z = \left[ AD^{1-\delta} + \frac{SD}{Q} + \left( \frac{Q}{2} + k\sigma \sqrt{L} \right) H + \frac{D}{Q} aL^{-b} \right] - \lambda_1 [W - wQ - s_1^2] - \lambda_2 [B - AD^{-\delta} Q - s_2^2]
\]

Differentiating the Lagrangian function (7.7) with respect to the decision variables \( D, Q \) and \( L \) gives

\[
\frac{\partial Z}{\partial D} = (1-\delta)AD^{-\delta} + SQ^{-1} + aQ^{-1}L^{-b} - AQ\lambda_2 D^{-\delta-1}
\]

\[
\frac{\partial Z}{\partial Q} = -SDQ^{-2} + 0.5H - aDQ^{-2}L^{-b} + w\lambda_1 + A\lambda_2 D^{-\delta}
\]

\[
\frac{\partial Z}{\partial L} = \frac{k\sigma H}{2} L^{-1/2} - abDQ^{-1}L^{-b-1}
\]

Kuhn-Tucker conditions yields the following equations:

\[
\frac{\partial Z}{\partial D} = 0 \Rightarrow \frac{\partial Z}{\partial D} = (1-\delta)AD^{-\delta} + SQ^{-1} + aQ^{-1}L^{-b} - AQ\lambda_2 D^{-\delta-1} = 0 \quad (7.8)
\]

\[
\frac{\partial Z}{\partial Q} = 0 \Rightarrow \frac{\partial Z}{\partial Q} = -SDQ^{-2} + 0.5H - aDQ^{-2}L^{-b} + w\lambda_1 + A\lambda_2 D^{-\delta} = 0 \quad (7.9)
\]
\[
\frac{\partial Z}{\partial L} = 0 \Rightarrow \frac{\partial Z}{\partial L} = \frac{k \sigma H}{2} L^{1/2} - ab D Q^{-1} L^{b-1} = 0
\]  
(7.10)

Equation (7.10) gives \[\frac{k \sigma H}{2} L^{1/2} = ab D Q^{-1} L^{b-1}\]

i.e. \[L^{b+0.5} = \frac{ab D Q^{-1}}{Hk\sigma}\]  
(7.11)

The above set of equations can be solved to obtain the optimal values of the decision variables and hence the corresponding unit price and the annual total cost can be evaluated.

7.7 NUMERICAL EXAMPLE

The decision variables (optimal order quantity \(Q_1\), optimal demand rate \(D_1\) and optimal lead time \(L_1\)) whose values are to be determined to minimize the annual relevant total cost were obtained by solving the Equations (7.8), (7.9) and (7.11) with the input parameters given in Table 7.1.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Number of items</td>
<td>(n)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Constant</td>
<td>(A)</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>Setup cost of the item 1</td>
<td>(S_1)</td>
<td>$200</td>
</tr>
<tr>
<td>4</td>
<td>Holding cost of the item 1</td>
<td>(H_1)</td>
<td>$0.8</td>
</tr>
<tr>
<td>5</td>
<td>Storage space for the item 1</td>
<td>(w_1)</td>
<td>2 sq.ft.</td>
</tr>
<tr>
<td>6</td>
<td>Storage space available</td>
<td>(W)</td>
<td>100 sq.ft.</td>
</tr>
<tr>
<td>7</td>
<td>Total investment cost</td>
<td>(B)</td>
<td>$200</td>
</tr>
<tr>
<td>8</td>
<td>Standard deviation</td>
<td>(\sigma)</td>
<td>6 unit/year</td>
</tr>
<tr>
<td>9</td>
<td>Constant</td>
<td>(k)</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>Lower limit of the unit cost of the item 1</td>
<td>(L_{L_1})</td>
<td>$6</td>
</tr>
<tr>
<td>10</td>
<td>Upper limit of the unit cost of the item 1</td>
<td>(U_{L_1})</td>
<td>$9</td>
</tr>
</tbody>
</table>
The values of the decision variables were calculated for some different values of the index parameter \( \delta \).

The optimal values of the production batch quantity \( Q_1 \), demand rate \( D_1 \), lead time \( L_1 \) and minimum annual total cost were given in the Table 7.2

**Table 7.2 Optimal solution table**

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( D_1 )</th>
<th>( Q_1 )</th>
<th>( L_1 )</th>
<th>Min TC</th>
<th>( p_1 )</th>
<th>( \mu_{p_1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.08</td>
<td>23.21</td>
<td>9.6x10^{-6}</td>
<td>28.03</td>
<td>8.57</td>
<td>0.1433</td>
</tr>
<tr>
<td>5</td>
<td>1.08</td>
<td>29.39</td>
<td>6.6x10^{-6}</td>
<td>26.63</td>
<td>6.81</td>
<td>0.7300</td>
</tr>
<tr>
<td>8</td>
<td>1.05</td>
<td>29.55</td>
<td>6.2x10^{-6}</td>
<td>26.18</td>
<td>6.77</td>
<td>0.7433</td>
</tr>
<tr>
<td>10</td>
<td>1.04</td>
<td>29.60</td>
<td>6.1x10^{-6}</td>
<td>26.04</td>
<td>6.76</td>
<td>0.7467</td>
</tr>
</tbody>
</table>

From the Table 7.2 it follows that minimum TC = $26.04 that corresponds to the maximum membership function 0.7467. Hence the required optimal solution is

\[ p_1 = 6.76 \quad D_1 = 1.04 \quad Q_1 = 29.60 \quad L_1 = 6.1 \times 10^{-6} \quad \text{Min TC} = 26.04 \]

It has been seen that as the constant \( \delta \) increases the demand, the lead time, the unit price decreases and hence the annual total cost also decreases. But the lot size value increases with the increase in the parameter \( \delta \).

### 7.8 SENSITIVITY ANALYSIS

This chapter has been devoted to study multi-item inventory model that consider the order quantity, the demand rate and the leading time as three
decision variables with demand dependent unit price. These decision variables were evaluated and the minimum annual total cost has been deduced for a single item. The smallest value of the minimum total cost is found at the largest value of $\delta$.

A sensitivity analysis of inventory model has been performed with respect to the change of index parameter $\delta$ which is presented in Table 7.2. In the formulation of the model demand has been expressed as some quantities power of the unit price. Similarly leading time crashing cost is some quantities power to the lead time. Hence the said indexes are very effective in determining optimal unit price, lot size, lead time and minimum annual total cost. It is observed that the decision variables demand, lead time and the objective value decreases when the index parameter increases whereas the lot size value increases with increase in the parametric value $\delta$.

![Graph of demand versus $\delta$ value](image)

**Figure 7.1 Graph of demand versus $\delta$ value**

The Figure 7.1 shows that as the value of the parameter $\delta$ increases the demand value decreases from 1.08 to 1.04.
Figure 7.2 Graph of lot size versus $\delta$ value

The Figure 7.2 shows that as the value of the parameter $\delta$ increases the lot size value increases from 23.21 to 29.60.

Figure 7.3 Graph of Annual total cost versus $\delta$ value
The Figure 7.3 shows that as the value of the parameter $\delta$ increases the annual total cost decreases from $28.03$ to $26.04$.

![Graph of unit price versus $\delta$ value](image)

**Figure 7.4 Graph of unit price versus $\delta$ value**

The Figure 7.4 shows that as the value of the parameter $\delta$ increases the unit price decreases from $8.57$ to $6.76$.

### 7.9 SUMMARY

Though simple EOQ model have been considered here, the technique illustrated in this chapter can be easily applied to other inventory problems with deterioration, shortages, discount, fixed time horizon etc. This technique is an appropriate tool to tackle the real-life inventory problems in realistic environments. The model can be evaluated for different values of the other parameters namely ‘a’ and ‘b’. Also it can be solved with other constraints like limited budget, setup cost etc.