CHAPTER 4

4. COMPLEXITY REDUCTION USING LDPC-CODES IN PULSED – OFDM SYSTEM

4.1 INTRODUCTION

LDPC codes is used in pulsed-OFDM system to significantly improve the transmission performance. LDPC codes replaces interleaver and puncturer of the MB-pulsed OFDM system which reduces the number of blocks in the system leads to the advantage of reducing the complexity of the system. LDPC codes are linear block codes which are defined by a sparse parity check matrix [36]. The parity check matrix can be represented by a bipartite graph consisting of variable and check nodes which are characterized by their degree distribution. LDPC codes are decoded in an iterative process during which the variable and check nodes exchange soft information. The decoder is based on the sum-product algorithm. An appropriate and efficient construction method is provided by the Progressive Edge Growth (PEG) algorithm. The proposed LDPC codes in pulsed – OFDM codes will achieve a significant performance gain, by reducing the system complexity and a reduction of the transmission power by a factor of approximately three decibels.

4.2 FUNDAMENTALS OF LDPC CODES

LDPC codes are a class of linear block codes. The name comes from the characteristic of their parity-check matrix which contains only a few 1’s in comparison to the amount of 0’s. Their main advantage is that they provide a performance which is very close to the capacity for a lot of different channels and linear time complex algorithms for decoding which was first introduced by Gallager in his PhD thesis in 1960. Due to the computational effort in implementing coder and en-coder for such codes and the introduction of Reed-Solomon codes, mostly ignored until about ten years ago [33].
4.3 REPRESENTATIONS FOR LDPC CODES

There are two different possibilities to represent LDPC codes. Like all linear block codes they can be described via matrices. The second possibility is a graphical representation [35].

4.3.1 Matrix Representation

Let’s look at an example for a low-density parity-check matrix first. The matrix defined in equation (1) is a parity check matrix with dimension ‘n ×m’ for a (8, 4) code. The number of 1’s in each row is given by \( w_r \) and the number of 1’s in each column is given by \( w_c \). For a matrix to be called low-density the two conditions are \( w_c << n \) and \( w_r << m \).

\[
H = \begin{bmatrix}
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 & 1 & 0
\end{bmatrix}
\] (4.1)

4.3.2 Graphical Representation

Tanner introduced an effective graphical representation for LDPC codes. Tanner graphs are bipartite graphs. That means that the nodes of the graph are separated into two distinctive sets and edges are only connecting nodes of two different types. The two types of nodes in a Tanner graph are called variable nodes (v-nodes) and check nodes (c-nodes). Figure 4.1 is an example for a Tanner graph and represents the same code as the matrix in 4.1 [39].
Those should usually be avoided since they are bad for decoding performance. The creation of such a graph is rather straightforward. It consists of ‘m’ check nodes (the number of parity bits) and ‘n’ variable nodes (the number of bits in a codeword). Check node $f_i$ is connected to variable node $c_j$ if the element $h_{ij}$ of ‘$H$’ is 1.

### 4.4 REGULAR AND IRREGULAR LDPC CODES

A LDPC code is called regular if $w_c$ is constant for every column and $w_r = w_c \cdot (n/m)$ is also constant for every row. The example matrix from equation (4.1) is regular with $w_c = 2$ and $w_r = 4$. It is also possible to see the regularity of this code while looking at the graphical representation. The same number of incoming edges for every ‘v-node’ (variable nodes) and also for all the ‘c-nodes’ (check nodes). If ‘$H$’ is low density but the numbers of 1’s in each row or column are not constant the code is called an irregular LDPC code [40 - 42].
4.5 CONSTRUCTING LDPC CODES

Several different algorithms exist to construct suitable LDPC codes. Gallager introduced one and most common one is progressive edge growth algorithm.

4.6 DECODING LDPC CODES

The algorithm used to decode LDPC codes was discovered independently several times and as a matter of fact comes under different names. The most common ones are the belief propagation algorithm, the message passing algorithm and the sum-product algorithm [45].

4.7 PERFORMANCE & COMPLEXITY

The feature of LDPC codes to perform near the Shannon limit of a channel exists only for large block lengths. For example there have been simulations that perform within 0.04 dB of the Shannon limit at a bit error rate of $10^{-6}$ with a block length of $10^7$. The large block length results also in large parity-check and generator matrices. The complexity of multiplying a codeword with a matrix depends on the amount of 1’s in the matrix. If we put the sparse matrix ‘H’ in the form $[P^T \ I]$ via Gaussian elimination the generator matrix ‘G’ can be calculated as $G = [I \ P]$. The sub-matrix ‘P’ is generally not sparse so that the encoding complexity will be quite high. Since the complexity grows in $O(n^2)$ even sparse matrices don’t result in a good performance if the block length gets very high. So iterative decoding (and encoding) algorithms are used. Those algorithms perform local calculations and pass those local results via messages. This step is typically repeated several times. The term “local calculations” already indicates that a divide and conquer strategy, which separates a complex problem into manage- able sub-problems, is realized. A sparse parity check matrix now helps this algorithm in several ways. First it helps to keep both the local calculations simple and also reduces the complexity of combining the sub-problems by reducing the number of needed messages to exchange all the
information. Furthermore it was observed that iterative decoding algorithms of sparse codes perform very close to the optimal maximum likelihood decoder.

4.8 LDPC CODER

LDPC coding and decoding algorithm with example is given below

**Encoding Algorithm**

i. Convert the sequential bit stream into frames of bits.
ii. Select the polynomial
iii. Compute the parity check matrix $H$ for the required polynomial
iv. Compute the $G$ matrix from the $H$ matrix
v. Multiply the message frame with $G$ matrix to get the required code word.
vi. Save the code word
vii. Repeat the steps 5 and 6

**Decoding Algorithm**

The standard decoding algorithm has the following steps:

i. (Initialization) Iteration 0
ii. Update the Check node
iii. Update Bit node
iv. Decision

Example:

Let the polynomial be $(6, 3)$
Rate be $1/2$

The bipartite graph for the selected polynomial is
Let the first message, m frame be \([1 \ 0 \ 1]\)

Code word = \([m] \ [G]\)

\[
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0
\end{bmatrix}
\]

Generated code word = \([1 \ 0 \ 1 \ 0 \ 1 \ 1]\), similarly the code word is generated for each frame. So for a 3 bit frame 6 bit code word is generated

To check the code word generated is correct or not, the following steps is to be followed.

Let the parity check matrix be

\[
H^T = \begin{bmatrix}
1 & 0 & 1 \\
1 & 1 & 1 \\
1 & 1 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Code word generated be 101011
\[ Z = dH^T \rightarrow [10101] \]
\[
\begin{bmatrix}
1 & 0 & 1 \\
1 & 1 & 1 \\
1 & 1 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix} = (0 \ 0 \ 0)
\]

So the code word is correct

4.9 PROPOSED LDPC-CODED PULSED-OFDM SYSTEM

In order to reduce the complexity of Pulsed OFDM system and to achieve higher code rates without puncturing, the LDPC coding is proposed for the Pulsed-OFDM system. The performance of LDPC codes in Pulsed-OFDM UWB system will be studied.

(a) Pulsed-OFDM transmitter using low duty-cycle pulse generator

(b) Proposed LDPC coded-Pulsed-OFDM transmitter
LDPC coder replaces the convolutional code, puncturer and interleaver of the Pulsed-OFDM System is shown in Figure 4.2. The proposed system reduces the number of blocks (puncturer and interleaver) which provides less complexity compared to Pulsed-OFDM System.

4.10 SYSTEM PERFORMANCE

To compare the performance of the LDPC-Pulsed-OFDM and Pulsed-OFDM systems, two channel models (namely CM-3 and CM-4) are proposed. The simulations are carried out for both channels at extreme fading conditions using parameters shown in table 4.1. Figure 4.3 & 4.4 shows the results over the CM3 and CM4 channel under Log normal fading conditions for various code rates.

Table 4.1 Simulation parameters for LDPC coded pulsed OFDM system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of OFDM subcarriers (NFFT)</td>
<td>32</td>
</tr>
<tr>
<td>Number of data subcarriers (ND)</td>
<td>25</td>
</tr>
<tr>
<td>Number of defined pilot subcarriers</td>
<td>07</td>
</tr>
<tr>
<td>$\Delta f$: Subcarrier frequency spacing</td>
<td>4.125 MHz</td>
</tr>
<tr>
<td>Modulation</td>
<td>QPSK</td>
</tr>
<tr>
<td>Coding</td>
<td>LDPC</td>
</tr>
<tr>
<td>Coding rate</td>
<td>1/2,2/5</td>
</tr>
</tbody>
</table>
From Figure 4.3 it is inferred that for the code rate 1/2 in CM-3 to achieve a BER of $10^{-6}$ a SNR of 10.5dB is required for LDPC-Pulsed- MB-OFDM but for CM-4 to achieve a BER of $10^{-6}$ a SNR of 11.5dB is required.
From Figure 4.4 it is inferred that for the code rate 2/5 in CM-3 to achieve a BER of $10^{-6}$ a SNR of 12 dB is required for LDPC-Pulsed- MB-OFDM but for CM-4 to achieve a BER of $10^{-6}$ a SNR of 13.5dB is required. It is observed that for higher code rates power consumption is increased and CM-3 performs well compared to CM-4.

Table 4.2 Simulation parameters to compare LDPC and Convolutional codes

<table>
<thead>
<tr>
<th>Parameter</th>
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<tr>
<td>Number of OFDM subcarriers (NFFT)</td>
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</tr>
<tr>
<td>Coding</td>
<td>LDPC/Convolution</td>
</tr>
<tr>
<td>Coding rate</td>
<td>1/2, 2/3, 2/5</td>
</tr>
</tbody>
</table>

Simulations are carried out using CM-3 parameters for convolutional code and LDPC code with code rate of 1/2, 2/3 & 2/5 is shown in Figure 4.5.

Figure 4.5 Comparison of LDPC code and Convolutional code with different coding rate in channel model 3
Simulated results of Figure 4.5 shows that LDPC codes with a code rate of \( \frac{1}{2} \) requires less SNR of 1dB compared to convolutional codes, LDPC codes with a code rate of \( \frac{2}{3} \) requires less SNR of 1.5dB compared to convolutional codes and LDPC codes with a code rate of \( \frac{2}{5} \) requires less SNR of 2dB compared to convolutional codes.

Simulations are carried out using CM-4 parameters for convolutional code and LDPC code with code rate of 1/2, 2/3 & 2/5 is shown in Figure 4.6.

![Figure 4.6 Comparison of LDPC code and Convolutional code with different coding rate in channel model 4.](image)

From Figure 4.6 it is inferred that the LDPC codes with a code rate of 1/2 requires less SNR of 1dB compared to convolutional codes, LDPC codes with a code rate of \( \frac{2}{3} \) requires less SNR of 1.5dB compared to convolutional codes and LDPC codes with a code rate of \( \frac{2}{5} \) requires less SNR of 2dB compared to...
convolutional codes. It is observed that for various code rates the proposed LDPC coded pulsed-OFDM system performs well compared to convolutional coded UWB systems.

4.11 CONCLUSION

Proposed LDPC codes in Pulsed-MBOFDM system provides

- Computation complexity is reduced by replacing puncturer and interleaver with LDPC encoder
- Reduction in signal power by 2dB