Chapter 5

Summary

In this thesis, we have considered two classes of gluon fusion processes which may be important at a high energy hadron collider such as the LHC. We have reported on the production of a pair of electroweak vector bosons with a jet via gluon fusion within the Standard Model. We have taken a model of large extra dimensions, the ADD model, as an example of new physics and have also considered the associated production of an electroweak boson and KK-gravitons. These gluon fusion processes receive contributions from the quark loop diagrams at the leading order and they are finite. The amplitude calculation is based on the traditional Feynman diagram approach. We have developed general purpose codes to perform the reduction of one-loop tensor integrals. All the basic scalar integrals which may appear in a one-loop amplitude are derived analytically and have been implemented in a FORTRAN routine. A flexible Monte Carlo integration routine based on the VEGAS algorithm is used to obtain the total as well as differential cross sections in these processes. To reduce the run time in the calculations of the SM processes, we have run the code in a parallel environment using the AMCI package.

We have verified our one-loop calculations by performing numerous checks on the amplitudes. We have checked the ultraviolet and infrared finiteness of the amplitudes. We have checked the structure of amplitudes by making gauge invariance checks. We have also verified the expected behavior of these amplitudes in the heavy quark mass limit. The general result regarding the infrared finiteness of an individual fermion loop diagram is also confirmed in these processes. We have shown and verified that in a fermion loop amplitude, plagued from chiral anomaly, the anomaly is related to the rational part of the amplitude.
We have given a prescription for obtaining the correct rational part in a UV finite fermion loop amplitude utilizing the decoupling theorem.

In our study of the SM processes, we find that due to a large gluon flux available at the LHC, these processes are quite important. The typical hadronic cross section for the $gg \rightarrow \gamma\gamma g$ is about 1 pb and it is about 10% of the corresponding tree-level contribution. Like the $gg \rightarrow \gamma\gamma$ process, it is also important in the searches of a light Higgs boson. We find that the top quark loop contribution to the $gg \rightarrow \gamma\gamma g, \gamma Z g$ cross sections is negligible. For the processes $gg \rightarrow ZZ g$ and $gg \rightarrow W^+ W^- g$, we have kept only $\gamma Z g$-like contributions. Their cross sections are in the range of $4 - 15\%$ of the corresponding tree-level cross sections. We have observed a qualitative similarity of these processes with the corresponding di-vector boson production cases. At the 14 TeV centre-of-mass energy, the cross sections of $gg \rightarrow VV' g$ processes are 20-30% of those for the $gg \rightarrow VV'$ processes. In a detailed study of the $gg \rightarrow \gamma Z g$ process, we have compared this NNLO level contribution with the LO and the NLO predictions using the MCFM program. We note that the percentage contribution ($\sigma_{NNLO}/\sigma_{NLO}$) is about 2-3% which can be enhanced by choosing an appropriate set of kinematic cuts. The cross section of this process is dominated by the box contributions and therefore by the vector part of the amplitude. The axial-vector part of the amplitude contributes only about 10% towards the cross section. Being leading order process, the scale uncertainty in the cross section calculation is governed by the strong coupling parameter $\alpha_s$ and the parton distributions, and it is quite large ($\sim 25-40\%$). The observability of this process at the LHC is discussed considering the decay of the $Z$ boson into the charged leptons. We find that at 14 TeV and with 100 fb$^{-1}$ integrated luminosity, one can expect more than thousand events for $gg \rightarrow \gamma Z (\rightarrow l^+ l^-) g$ process at the LHC. The issue of numerical instability in our calculations is dealt by systematically ignoring the contributions from the exceptional phase space points. We have adopted three different strategies to ignore their contributions, all in agreement within the allowed range of uncertainty. We have seen that such phase space points are very few and the contributions from such phase space points do not dominate the cross section.

In the ADD model processes, we have argued that the $gg \rightarrow \gamma G_{KK}$ amplitude vanishes at the leading order due to an extension of Furry’s theorem which includes the graviton. We find that, as expected, the gluon-gluon contribution to $pp \rightarrow H G_{KK} + X$ dominates its cross section. For the model parameter values, $\delta = 2$ and $M_S = 2$ TeV, the hadronic cross
section is only about 0.6 fb at 14 TeV LHC. We have cross-checked our full one-loop calculation by working in an effective theory of gluon-Higgs coupling. In the effective theory, the process becomes a tree-level process. We find that for the physical top quark mass there is no agreement between the two calculations. This difference can be attributed to the fact that we have a large scale, the mass of the KK-gravitons, present in the theory. However, in the limit of a very heavy top quark the two results are in complete agreement as desired. In the case of $gg \rightarrow ZG_{KK}$ process, we find that the amplitude gets contribution solely from the axial-vector part of the $Z$ boson coupling to a quark and the quarks of a massless/mass-degenerate generation do not contribute. Due to the nature of graviton-quark coupling, the box diagrams are also anomalous along with the triangle diagrams. We also learned that the anomaly should be regulated using a suitable $\gamma^5$ prescription in $n$ dimensions to ensure various symmetries of the diagrams and the amplitude. As expected, we find that the axial-vector current conservation in the amplitude holds only after including both the bottom and top quark contributions. The typical cross section at 14 TeV is about 2 fb and it is about 10% of the NLO cross section for $\delta = 2$ and $M_S = 2$ TeV. In both the $HG_{KK}$ and $ZG_{KK}$ cases, we find that there is a significant cancellation at the amplitude level between the triangle and the box contributions. The smallness of the cross sections for these processes, particularly in the $HG_{KK}$ case, may be due to this cancellation. The cross sections of these processes go down with increasing $\delta$ and/or $M_S$. We see that the massive KK-graviton modes contribute significantly towards the inclusive cross section. Like in the case of the SM processes, we do observe a large uncertainty ($\sim 5-20\%$) due to the scale variation. To check the sensitivity of our results on the scale of Gravity $M_S$, we have calculated the cross section in both the truncated and untruncated schemes and the results differ by about 20%.

We have seen that at higher energies, the contributions of all these gluon fusion processes increase. However, their cross sections suffer from large scale uncertainties as mentioned above. The scale uncertainties can be reduced by calculating radiative corrections to these processes. The radiative corrections, which will also involve calculation of two-loop diagrams, are particularly important to our gluon fusion SM processes. Unlike the one-loop calculation, the calculation of two-loop amplitudes is not very common and its techniques are not yet standardized [119–122]. Very few two-loop calculations of phenomenological importance, are available even for three and four-point functions. We have mentioned that
beyond four-point function, the one-loop calculations are subject to numerical instabilities near exceptional phase space points. In the traditional approach of tensor reduction, this issue can be resolved to a certain degree by employing special expressions for the reduction of higher point tensor integrals near such points [91]. One may also use modern techniques (on-shell methods), based on the generalized unitarity cut, of calculating one-loop amplitudes [66, 123–125]. These techniques are results of many important ideas which have been developed over the years and due to them the automation of one-loop calculations, like that of tree-level calculations, seems feasible [126–130]. Like the case of \( gg \rightarrow VV' \), the compact analytic expressions for \( gg \rightarrow VV'g \) amplitudes can be calculated using these techniques [79, 131, 132]. These analytic expressions will certainly reduce the computation time and can also be used for precision calculation at the LHC. The SM gluon fusion processes \( gg \rightarrow VV'g \) with soft jet form the real radiation part of the radiative correction to \( gg \rightarrow VV' \) processes. Therefore, a full radiative correction of the LO \( gg \rightarrow VV' \) processes can be a fruitful exercise [133]. I would like to conclude by quoting L. D. Landau,

“A method is more important than a discovery,
since the right method will lead to new
and even more important discoveries.”