Chapter 4

Production of KK-gravitons with a Boson via Gluon Fusion

In this chapter, we will consider the associated production of an electroweak boson and the KK-graviton modes via gluon fusion in the ADD model. In particular, we will study leading order $gg \rightarrow HG_{KK}$, $gg \rightarrow \gamma G_{KK}$ and $gg \rightarrow ZG_{KK}$ processes. Like the SM processes discussed earlier, these processes also proceed via quark loop diagrams at the leading order and are finite. These processes contribute to the hadronic processes $pp \rightarrow BG_{KK}+X$, at the NLO in $\alpha_s$, where $B \in \{H, \gamma, Z\}$. We will calculate the inclusive cross sections of these gluon channel processes at the LHC and discuss some important kinematic distributions. In this chapter, by inclusive we mean the contribution from all the kinematically accessible KK-modes. We will also study the dependence of these results on the model parameters $\delta$ and $M_S$. This chapter is based on the work reported in [93–95]. A brief overview of the model is given in Sec. 1.2.

Like the other new physics scenarios, the searches for the model of large extra dimensions or the ADD model is also continuing at the LHC. With the 7 TeV LHC data, the bounds on the model parameters have also improved quite significantly [96–103]. In the context of the ADD model, studies of processes involving the exchange of KK-gravitons and those in which KK-gravitons are produced directly, have been reported for both the Tevatron and the LHC [104–112]. Due to a large gluon flux available at the LHC, the gluon initiated processes can be quite important. In this regard, we have investigated the KK-graviton production in association with a boson ($H/\gamma/Z$) via gluon fusion. Unlike
the cases of $\gamma/ZG_{\text{KK}}$ production, the $q\bar{q}$ initiated tree-level $HG_{\text{KK}}$ production process has very small cross section due to a vanishingly small coupling of the Higgs boson with light quarks. Even with a non-zero bottom quark mass, the tree-level cross section can be at best of the order $10^{-3}$ fb at the LHC. The contribution of the gluon-gluon channel is, therefore, expected to be relatively more important for the $HG_{\text{KK}}$ production at the LHC. The situation is analogous to the single Higgs boson production in the SM. There too the $gg \rightarrow H$ channel dominates the hadronic cross section. The rest of the chapter is organized as follows: In the next section, we give some details on the structure of the amplitudes. Various checks on our amplitudes and the method of computation are described in section 4.2 and 4.3. Numerical results are presented in section 4.4. An interesting issue related to the $ZG_{\text{KK}}$ amplitude calculation is added in the end.

4.1 The Structure of Amplitudes

At the leading order, the process $gg \rightarrow BG_{\text{KK}}$ proceeds via quark loop diagrams. The allowed vertices and their Feynman rules, in the ADD model, are listed in Ref. [48]. Depending upon the coupling of the KK-graviton with the standard model particles, there are three classes of diagrams: a triangle class of diagrams due to quark – boson – graviton coupling, another triangle class of diagrams due to boson – graviton coupling and the box class of diagrams due to quark – graviton coupling. The prototype diagram in each class is shown in Fig. 4.1. Other diagrams are obtained just by appropriate permutations of the external legs. There are total six box and twelve triangle diagrams for each quark flavor. However, due to the charge-conjugation property of the fermion loop diagrams, only half of the diagrams are independent. Since the coupling of the Higgs boson with quarks is proportional to the quark masses, in the $gg \rightarrow HG_{\text{KK}}$ case, we consider only bottom and top quark contributions. For a given massive quark in the loop, the $HG_{\text{KK}}$ amplitude has the following structure:

$$M_{q}^{ab}(gg \rightarrow HG_{\text{KK}}) = \frac{1}{8} y_{q} g_{s}^{2} \kappa \left(\delta_{ab}^{\text{tri}} \right) \mathcal{A}(m_{q}),$$

$$\mathcal{A}(m_{q}) = \left[2 \mathcal{A}_{\text{tri}}(m_{q}) - \mathcal{A}_{\text{box}}(m_{q})\right]. \quad (4.1)$$

Here the Yukawa coupling, $y_{q} = \frac{1}{2} g_{w} (m_{q}/M_{W})$ and $\kappa = \sqrt{2}/M_{P}$. Furthermore, $a$ and $b$ are color indices of the two gluons. $\mathcal{A}_{\text{tri}}$ is the net contribution from the two triangle
classes of diagrams, shown in Figs. 4.1 (a) and (b).

Figure 4.1: Prototype Feynman diagrams for $gg \rightarrow H/\gamma/Z + G_{KK}$ in the ADD model.

For $gg \rightarrow \gamma G_{KK}$ case, we find that the amplitudes of the diagrams, related by charge-conjugation, are equal and opposite to each other. This implies that at the LO,

$$\mathcal{M}(gg \rightarrow \gamma G_{KK}) = 0. \quad (4.2)$$

By introducing charge-conjugation transformation of the KK-graviton field and using charge-conjugation properties of the gluon and photon fields, it can be shown that the $\gamma G_{KK}$ amplitude does vanish at the LO. This is just an implication of the extension of Furry’s theorem in the presence of gravitons, photons and gluons. The graviton field is considered even under charge-conjugation as it couples with the energy-momentum tensor only. This, we have explicitly verified using the ADD model Lagrangian [94]. This result would remain valid to all orders if only QED and/or QCD radiative corrections are included. In the presence of the weak interaction, this result may not hold to all orders.

The $gg \rightarrow Z G_{KK}$ amplitude has both the vector and axial-vector contributions coming from the $Z$ boson coupling to the quarks. The vector part of the amplitude is similar to the $\gamma G_{KK}$ amplitude and therefore at the LO, the process receives contribution only from the axial-vector part of the amplitude. For a given quark flavor in the loop, the amplitude has
the following structure:
\[
\mathcal{M}_{q}^{ab}(gg \rightarrow ZG_{\mathrm{KK}}) = g_{Z} g_{w}^{2} s \kappa \left(\frac{\delta^{ab}}{2}\right) c_{q}^{A} A(m_{q}),
\]
\[
A(m_{q}) = A_{\text{tri}}(m_{q}) - A_{\text{box}}(m_{q}).
\]

Here \( g_{Z} = g_{w}/\cos\theta_{w} \) and \( c_{q}^{A} = -T_{3}^{q}/2 \). \( A_{\text{tri}} \) includes the contributions from both types of triangle diagrams. We find that due to the nature of KK-graviton coupling with quarks, both the triangle and box diagrams are linearly divergent and therefore they will give rise to anomalous contributions to the amplitude. Of the six quark flavours, we treat the \( u, d, s \), and \( c \) quarks as massless. Since the amplitude of the process is proportional to \( T_{3}^{q} \) value, the first two generations do not contribute. Therefore, the full amplitude, including the contributions from all the six quarks, is
\[
\sum_{q} \mathcal{M}_{q}^{ab}(gg \rightarrow ZG_{\mathrm{KK}}) = -\frac{1}{4} g_{w} g_{w}^{2} s \kappa \left(\frac{\delta^{ab}}{2}\right) [A(m_{t}) - A(m_{b})].
\]

We note that the cross section is of \( \mathcal{O}(\alpha_{s}^{2}) \), and therefore this LO contribution can be included in \( \sigma_{\text{NLO}}(pp \rightarrow ZG_{\mathrm{KK}} + X) \) [110, 111].

The one-loop amplitudes for \( gg \rightarrow H/ZG_{\mathrm{KK}} \) are expected to be free of ultraviolet (UV) and infrared (IR) singularities for each quark flavour. The IR singularities (large logs in the mass regularization) are applicable to light quark cases only. We also expect gauge invariance with respect to the gluon and the KK-graviton currents. However, in the \( ZG_{\mathrm{KK}} \) case, due to the presence of anomalies, the amplitude for an individual quark flavour may not be gauge invariant with respect to the axial-vector current in the \( m_{q} \rightarrow 0 \) limit. Since the model is free from anomalies, we expect gauge invariance after summing over all the six quark flavours. The confirmation of the cancellation of UV and IR singularities and gauge invariance with respect to the vector and axial-vector currents are powerful checks on our calculation. We make all these checks as described in the next section.

### 4.2 Details of Calculation and Checks

Our one-loop calculation is based on the traditional Feynman diagram method. The amplitude of each diagram is written using the SM and ADD model Feynman rules. The ADD model Feynman rules we require are also listed in the appendix D.1. However, we need
not compute all the diagrams explicitly. We only need to compute the prototype diagrams. All other diagrams can be obtained by suitable permutations of the external momenta and polarizations. This works quite well in the $HG_{KK}$ case. However, due to the presence of $\gamma^5$ in the $ZG_{KK}$ amplitude, one needs to take extreme care in making these permutations in $n$ dimensions. The permutation should not be across the $\gamma^5$ vertex. Such permuted diagrams need to be computed explicitly. Due to the presence of a quark loop, the amplitude of each diagram is proportional to the trace of a string of gamma matrices. We compute these traces using FORM [64]. This is the most important part of the calculation. The presence of 4-dimensional $\gamma^5$ in the trace, leads to spurious anomalies in the amplitude. We, therefore, need an appropriate $n$-dimensional treatment of $\gamma^5$. We have used Larin’s prescription for $\gamma^5$ to calculate the trace in $n$ dimensions [113].

According to this prescription,

$$
\gamma_\mu \gamma^5 = -\frac{i}{6} \epsilon_{\mu\nu\rho\sigma} \gamma_\nu \gamma_\rho \gamma_\sigma.
$$

(4.6)

After calculating the trace, we express the amplitude in terms of appropriate tensor integrals. The box amplitude has rank-4 tensor integrals, while the triangle amplitude has rank-2 tensor integrals at the most. The tensor reduction into scalars is done in $n = (4 - 2\epsilon)$ dimensions using the methods of Odenborgh and Vermaseren [63], also described in the Sec. 2.1.3. All the required scalar integrals for the massless quark case are listed in the appendix B.2. We need only the UV and IR singular pieces of these scalars to verify the cancellation of UV and IR singularities. For both the cases of bottom and top quarks, we use FF library to calculate the required scalars [114]. Due to a very large and complicated expression of the amplitude, we compute the amplitude numerically before squaring it. This requires computation of the polarization vectors for the gauge bosons and for the KK-graviton. We have chosen the helicity basis for them. It also helps in making additional checks on our calculation by verifying relations among helicity amplitudes. The KK-graviton polarization tensor is constructed from the polarization vectors of two massive vector bosons as suggested in [48]. To obtain hadronic cross sections, we perform integrations over two body phase space and the gluon PDFs, using a Monte Carlo integration subroutine based on the VEGAS algorithm. Since the KK-gravitons are produced directly, we also require an additional integration over the KK-graviton mass parameter $M_{KK}$, to obtain an inclusive cross section.
As discussed in the previous section, our one-loop processes are expected to be finite. We verify that both the massive and massless contributions are UV finite. We observe that each triangle diagram is UV finite by itself, while the box amplitude is UV finite only after adding all the box contributions. As we discussed in the Sec. 2.3.1 fermion loop diagrams are known to be IR finite, in both the massive and massless fermion cases, for any kind and any number of external particles attached to the loop. In the massless quark case, we check that each diagram is IR finite and therefore IR finiteness holds for the full amplitude. Finally, we check the gauge invariance of the amplitude with respect to the two gluons by replacing their polarizations with their respective momenta. In the $HG_{KK}$ case, we observe that some of the triangle diagrams are separately gauge invariant with respect to both the gluons. To ensure the correctness of their contribution towards the full amplitude, we have also performed a gauge invariance check with respect to the KK-graviton current. In the $ZG_{KK}$ case, we find that (only after using $\gamma^5$ prescription in the trace) both the bottom and top quark contributions are separately gauge invariant with respect to the two gluons. In $ZG_{KK}$-triangle class of diagrams, involving graviton-gauge boson coupling, we have chosen the gauge-fixing parameter $\xi = 1$ (the Feynman gauge) for the gluon case and $\xi = \infty$ (the Unitary gauge) for the $Z$ boson case. As expected, the calculation does not depend on any specific choice of the gauge-fixing parameter. We also check gauge invariance of the amplitude with respect to the $Z$ boson. Because of the anomaly, the two contributions are not separately gauge invariant. However, the total amplitude is gauge invariant up to the top quark mass, as expected due to the explicit breaking of the chiral symmetry. All these checks on our amplitude have been made both numerically as well as analytically. The issue of anomaly in $ZG_{KK}$ amplitude is discussed in the end.

We have also cross-checked our calculations by taking the $m_t \to \infty$ limit. In the $ZG_{KK}$ case, for a given phase space point, we vary the top quark mass and observe that the amplitude-squared (which includes both the bottom and top quark contributions) approaches a constant value (the bottom quark contribution) as $m_t \to \infty$. This implies the complete decoupling of the top quark, i.e., the top quark contribution of the amplitude goes to zero in large $m_t$ limit. It is expected from the decoupling theorem [70]. This feature has been plotted in Fig. 4.2. The change in slope around $m_t = \sqrt{s}/2$ corresponds to a physical threshold after which the top quark propagators cannot go on-shell and the amplitude is
real. It is well known that the decoupling theorem does not hold for fermion loop amplitudes involving a Higgs boson. This is because, the Higgs boson coupling to fermions is proportional to the fermion masses. Like in the case of $gg \rightarrow H$ amplitude, we do observe non-decoupling of the heavy top quark in the $gg \rightarrow HG_{KK}$ amplitude. In Fig. 4.3, the rise in the curve, as $m_t$ increases in the beginning, is due to the explicit top quark mass dependence in the numerator ($m_t^2$ in the amplitude). As we approach larger values of $m_t$, the effective suppression ($\sim 1/m_t^2$ in the amplitude) due to the propagators dominates and the amplitude becomes independent of $m_t$.

![Decoupling of the top quark](image1)

![Non-decoupling of the top quark](image2)

**Figure 4.2**: Decoupling of the top quark as $m_t \rightarrow \infty$, in $gg \rightarrow ZG_{KK}$.

**Figure 4.3**: Non-decoupling of the top quark as $m_t \rightarrow \infty$, in $gg \rightarrow HG_{KK}$.

### 4.3 $gg \rightarrow HG_{KK}$ Calculation in the Effective theory of Gluon-Higgs coupling

In the heavy top quark limit ($m_t >> M_H/2$), the interaction of gluons with the Higgs boson can be described by an effective Lagrangian [115],

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} g_{\text{eff}} G_{\mu\nu}^a G^{a,\mu\nu} H,$$

where the effective coupling is

$$g_{\text{eff}} = \frac{\alpha_s}{3\pi v} \left[ 1 + \mathcal{O}(M_H^2/4m_t^2) \right].$$
It is known that the full calculation of the Higgs production via gluon fusion (including its radiative corrections), matches quite well with the calculation performed using this effective Lagrangian, even for physical top quark mass $m_t = 175$ GeV and $M_H = 120$ GeV [116–118]. We can also use this Lagrangian, in the ADD model to calculate $gg \rightarrow HG_{KK}$ process. Furthermore, we can compare this effective theory calculation with the full calculation. In the heavy top quark limit, both calculations should be in complete agreement. In this case, the diagrams contributing to the process are displayed in the Fig. 4.4. The

![Feynman diagrams](image)

**Figure 4.4:** Feynman diagrams for $gg \rightarrow HG_{KK}$ in the heavy top quark limit.

Feynman rule for the last diagram is derived following Ref. [48] and it is added in the appendix [D.1] In this effective theory, the amplitude-squared of $gg \rightarrow HG_{KK}$ process is

$$
\sum_{\text{pol.}} |\mathcal{M}(gg \rightarrow HG_{KK})|^2 = \frac{1}{6(M_H^2 - s)^2 t^2 u^2} \left[ 6M_H^2 (t + u)^2 + 7s^2 t^2 u^2 (t + u)^2 - 12M_H^{10} 
\right.
\left.
((t + u)^3 + 2s(2 + tu + u^2)) + 6M_H^6 ((t + u)^2 (t^2 + 4tu + u^2) + 2s^2 (3t^2 + tu + 3u^2) +
2s(t + u)(3t^2 + 4tu + 3u^2)) - 4M_H^2 stu(3tu(t + u)^2 + 3s^2 (t^2 + tu + u^2) + s(t + u)
(3t^2 + 8tu + 3u^2)) - 12M_H^6 (tu(t + u)^3 + 2s^3 (t^2 + u^2) + 3s^2 (t + u)(t^2 + tu + u^2) +
s(t^4 + 7t^3u + 10t^2u^2 + 7tu^3 + u^4)) + 2M_H^4 (3t^2u^2 (t + u)^2 + 12stu(t + u)^3 + 3s^4
(t^2 + u^2) + 6s^3 (t + u)(t^2 + tu + u^2) + s^2 (3t^4 + 30t^3u + 44t^2u^2 + 30tu^3 + 3u^4)) \right],
$$

(4.9)
where a summation over external polarizations is included. Here $s$, $t$ and $u$ are Mandelstam variables and they satisfy, $s + t + u = M_H^2 + M_{KK}^2$.

### 4.4 Numerical Results

In this section, we present results for $gg \rightarrow H/ZG_{KK}$ processes at the LHC. The results depend on the two parameters of the ADD model - (i) the number of extra-space dimensions $\delta$ and (ii) the fundamental scale of Gravity $M_S$. We will study this dependence and other features of our processes in the following. In Fig. 4.5, we have plotted the hadronic cross sections as a function of the collider centre-of-mass energy for both the processes. Due to a large gluon flux at higher energies, the cross sections also increase. Here we have chosen $\delta = 2$ and $M_S = 2$ TeV. This combination of the model parameters has already been ruled out. Nevertheless, for a study of the qualitative features of many of the results presented here, this combination is as good as any other combination. In addition, we have applied following kinematic cuts:

$$p_T^H > 20 \text{ GeV}, \quad p_T^Z > 30 \text{ GeV}, \quad |\eta^{H/Z}| < 2.5, \quad \sqrt{s} < M_S. \quad (4.10)$$

The cut on the partonic centre-of-mass energy or equivalently on the $HG_{KK}/ZG_{KK}$ invariant

![Figure 4.5](image.png)

Figure 4.5: Collider energy dependence of the hadronic cross sections for $gg \rightarrow H/ZG_{KK}$ at the LHC.

ant mass is known as a *truncated scheme* in the literature. This cut is related to the fact that theoretical predictions within the ADD model, which is an effective field theory, can
be valid only below the fundamental scale $M_S$. It will be interesting to probe the sensitivity of our predictions on this kind of a constraint. We will further comment on the issue at the end of this section. Since our gluon fusion processes are finite, the partonic cross sections do not depend on the factorization scale $\mu_f$. Also, their dependence on the renormalization scale $\mu_r$ is only through the strong coupling parameter $\alpha_s$. We have chosen the transverse energy ($E_T = \sqrt{M^2 + (p_T)^2}$) of the weak bosons $H/Z$, as the common scale for the $\mu_f$ and $\mu_r$. In principle, we can work with both the LO and NLO PDFs. We have used the LO CTEQ6L1 PDF, in the $HG_{KK}$ case and the NLO CTEQ6M PDF, in $ZG_{KK}$ case [88]. We note that at the 14 TeV LHC energy, the cross sections are 0.75 fb and 2.13 fb for the $HG_{KK}$ and $ZG_{KK}$ cases, respectively. The $HG_{KK}$ cross section is much smaller than expected. The smallness of the cross section is due to two-orders of magnitude cancellation in the amplitude between the box and the triangle contributions. This destructive interference occurs due to the relative minus sign between the two contributions. However, the triangle and the box amplitudes are not separately gauge invariant. A similar cancellation is also seen in the $ZG_{KK}$ amplitude. In the $HG_{KK}$ case, we find that the bottom quark loop contribution to the cross section is less than a percent. In the $ZG_{KK}$ case, we have also calculated the hadronic cross section for a $p_T^Z > 400$ GeV to avoid the SM background as suggested in [47]. We find that the cross section is about 0.2 fb and it is almost 10% of the NLO QCD correction calculated in [110, 111].

Figure 4.6: Dependence of the cross sections on the number of extra dimensions $\delta$, for the scale $M_S = 2$ TeV.

Figure 4.7: Dependence of the cross sections on the scale $M_S$, for the number of extra dimension $\delta = 2$. 
We now illustrate various kinematic aspects of these processes at the 14 TeV centre-of-mass energy. In Figs. 4.6 and 4.7, we show the dependence of inclusive cross sections of the two processes on the ADD model parameters $\delta$ and $M_S$. As $\delta$ or $M_S$ is increased, the density of states for KK-graviton modes falls (see Eq. 1.33), and therefore the cross sections also go down. The transverse momentum distributions of the Higgs and the $Z$ boson are plotted in Figs. 4.8 and 4.9, respectively. The $p_T$ distributions are peaked about the masses of the weak bosons as one would expect. In the direct production processes of KK-graviton, all the kinematically allowed modes are produced. Figs. 4.10 and 4.11 show the KK-graviton mass distributions. Since the density of KK-graviton modes increases with the KK-graviton mass $M_{KK}$, the differential cross sections also increase before the phase space suppression takes over. As the Eq. 1.33 suggests, the peak in the distributions will depend on the ADD model parameters $\delta$ and $M_S$. Next, we study the scale dependence of the cross sections. We vary the common scale of the factorization and renormalization around its central value, $\mu_0 = E_T^{H/Z}$. The cross sections change by about $25 - 30\%$ by changing $\mu$ in the range between $\mu_0/2$ and $2\mu_0$. We find that the uncertainty in our calculations, due to the choice of different PDF sets, is in the range of $5 - 20\%$ for the two processes.

We discussed in the Sec. 4.3 that the process $gg \to HG_{KK}$ can also be calculated in an effective theory of $ggH$-coupling. The cross section calculations in the full theory and in the effective theory do not agree for $M_H = 120$ GeV and $m_t = 175$ GeV. However, the two calculations agree very well for a very large top quark mass value ($m_t \geq 1.2$ TeV) as
Figure 4.10: KK-graviton mass distribution in $HG_{KK}$ case, for $M_S = 2$ TeV and $\delta = 2$.

Figure 4.11: KK-graviton mass distribution in $ZG_{KK}$ case, for $M_S = 2$ TeV and $\delta = 2$.

required. This is shown in Fig. 4.12. Note that unlike the SM $gg \to H$ case, in our case, there is one extra scale present – namely, the mass of the KK-graviton $M_{KK}$. We have seen that the $M_{KK}$ value is significantly larger than the mass of the top quark most of the time, see Fig. 4.10. Because of this, $\sqrt{s}$, which is larger than $M_H + M_{KK}$, can go much beyond $2m_t$. Therefore, one cannot expect the effective theory calculation to agree with the full calculation for $M_H = 120$ GeV and the physical top quark mass $m_t = 175$ GeV. Finally,

Figure 4.12: A comparison of $gg \to HG_{KK}$ cross sections calculated in full theory and in the effective theory.

we comment on the UV sensitivity of our theoretical results. As mentioned before, we
have presented all the results for which the partonic energy $\sqrt{s}$ is below the fundamental scale $M_S$, i.e., the truncated scheme. It has been argued in Ref. [47] that, if relaxing this constraint does not change the results significantly, then results of the effective field theory can be trusted. We find that the cross sections in the truncated and untruncated schemes (that is, with and without cut on $\sqrt{s}$) differ by 20% for $\delta = 2$ and $M_S = 2$ TeV. This difference, as expected, increases for larger values of $\delta$ while it decreases with increasing $M_S$ [93].

4.5 A Discussion on $ZG_{KK}$ Calculation

The coupling of the KK-graviton with the quarks is such that the triangle and the box diagrams, in the $ZG_{KK}$ case, are linearly divergent in loop momentum and they have $VVA$ and $VVVA$ coupling structures respectively. The issue of chiral anomaly is well known in a linearly divergent fermion loop triangle diagram having $VVA$-structure. Because of the linear divergence, the $VVVA$-box diagrams also give anomalous contributions to the amplitude. This can be confirmed by checking the relations among charge-conjugated box diagrams, in the absence of a suitable $n$-dimensional $\gamma^5$ prescription. The presence of anomaly affects the gauge invariance of the amplitude and for reliable predictions, our amplitude should be gauge invariant with respect to all the currents. We have already mentioned that if we do not regulate anomaly by using a suitable prescription for $\gamma^5$, the amplitude for the bottom/top quark in the loop, is not gauge invariant with respect to the gluons due to spurious anomalies. Even the relations among charge-conjugated diagrams and the Bose symmetry of the amplitude do not hold. It turns out that the $n$-dimensional $\gamma^5$ prescription, given in Eq. 4.6 respects various symmetries of diagrams and the amplitude. We have mentioned that, with this $\gamma^5$ prescription, both the bottom and top quark contributions are separately gauge invariant with respect to the gluons and only the axial-vector current, corresponding to the $Z$ boson, is anomalous. The anomaly in axial-vector current, being independent of the quark mass, also goes away in the full amplitude. See Eq. 4.5. We have learned from our general discussion on the chiral anomaly that the anomalous contributions to the amplitude, including the spurious ones in 4 dimensions, affect only the fermion mass independent rational part $\mathcal{R}$, see Sec. 2.3.2. The quark mass independence of the rational part, for an individual quark, is checked explicitly. In the full amplitude, shown in Eq. 4.5 the
rational terms will cancel between the bottom and the top quark contributions. Therefore, even if we work with 4-dimensional $\gamma^5$, the full amplitude is going to be gauge invariant with respect to all the currents. Although, the amplitude for an individual quark flavor in the loop will not be gauge invariant anymore, their difference in Eq. 4.5 is always gauge invariant. We have verified this in a separate calculation. All the numerical results, presented above, agree with this way of doing calculation. It is definitely more economical because with 4-dimensional $\gamma^5$ we require only one prototype box amplitude and two prototype triangle amplitudes (one for each class) to generate the full amplitude. On the other hand, if we use $\gamma^5$ prescription given in Eq. 4.6, we need two prototype box amplitudes and four prototype triangle amplitudes (two for each case) to generate the full amplitude. Also, with this $\gamma^5$ prescription, the trace calculation gives rise to a bigger expression for the amplitude of each diagram.