CHAPTER 4

MATHEMATICAL MODEL FOR ARGUMENT REPRESENTATION

4.1 PHILOSOPHICAL PERSPECTIVES FOR ARGUMENTATION

4.1.1 Nyāya Tradition of Debates

The importance of Nyāya tradition in Indian philosophy has its significance particularly in rational debate and clear, logical argumentation. Nyāya means ‘that by which one is led to a conclusion’ or ‘correct reasoning’ and is often referred to as ‘the science of reasoning’ (Virupakshananda 1994). Analysis of inferential reasoning was central in establishing the proper rules for scholastic debate, a prominent practice of Indian philosophy. The roots of Nyāya lie in aspects concerning the nature of debate and its formal procedures. Logic in India developed out of vada tradition i.e. the tradition of debate which was concerned with dialectical tricks, eristic arguments and sophistry. According to Nyāya-Sutra, three kinds of vada (or debate) were identified (Chakraborti 2006):

- Discussion: Good debate in which the proof and refutation of thesis and antithesis are based on proper evidence and without contradicting any background or already established assumptions.
- Disputation: Devious or sly debate, in which the proof and refutation use unfair, measures such as hair-splitting empty pedantry, false rejoinders and defeat situations.

- Destructive criticism: Purely destructive or refutation-only debate, in which no positive counter-thesis is proved.

Nyāya postulates an additional category of vada, known as hypothetical reasoning or rational critique (tarka or tarka-vada), which is the exchange of arguments between the proponent and opponent with the objective of attaining valid knowledge. The purpose is to test the validity of inferential reasoning by demonstrating the absurd consequences that follow from an opponent’s position and therefore eliminate doubt in the mind of the enquirer.

In addition, Nyāya states various assumptions for conducting good debates. Two prominent assumptions are, both the active participants should possess identical fundamental world knowledge as common sense; and, both the participants have to be totally involved in the discussion without any deviation (Radhakrishnan and Moore 1989). Apart from these assumptions, there are certain criteria, which the participants involved in discussion, are expected to follow in principle during knowledge sharing (Vidyabhusana 1988). They are:

- Mutual Agreement: The participants (arguer and counter-arguer) must agree on the topic of discussion.

- Mutual Understanding: The arguer must agree to attempt to answer questions from the counter-arguer and vice versa.

- Mutual Reasoning: The participants must consider the reasoning process more important than facts.
• Mutual Criticism: The questions must expose errors in the opponent’s reasoning or beliefs.

• Mutual Consistency: The participants should possess similar abstract or common sense knowledge about the world at the foundational level, as per our first assumption; higher level domain specific knowledge differs between persons but without altering the consistency of the mutual fundamental abstract knowledge.

With the basic agreement of these policies, participating entities along with the above-mentioned assumptions ensure the active participation of members throughout the discussion. There are certain rules for conducting the scholastic debate as recommended by Nyāya. The following section discusses the scheme of argumentative discussion confining to the rules of debate.

4.1.2 **Rules of Scholastic Debate in Nyāya**

To illustrate the formal procedures of debate let us examine the following scheme of scholastic argumentation. A scholastic formal debate involves two interactive participants – the proponent (one who holds the initial position) and a respondent. The order and procedure of debate, according to King (1999) follows eight basic steps:

1. The initial proponent is asked to put forward his thesis (or ideas in the form of propositions)

2. If the thesis is thought to be erroneous the respondent may refute it immediately, but if the thesis is accepted, then the respondent asks the proponent to outline the reason for accepting the thesis
3. The proponent then offers a proof outlining the reasons why the thesis should be accepted

4. The respondent asks if the proof offered contains the logical relations required of a sound inference

5. The proponent replies by ‘removing the thorns’, that is, he negates the faulty relations and erroneous reasoning that may have occurred in the outlining of the proof of his thesis. This concludes the first part of the debate. If the respondent accepts both the thesis and the proof then the debate concludes. However, if the proof is deemed erroneous for some reason, then the second part of the debate is required.

6. The respondent offers a statement of refutation of the proponent’s thesis, which thereby constitutes the initial starting-point of his own exposition. The refutation that follows aims to demonstrate the errors and inconsistencies of the proponent’s position based upon the reasoning and evidence provided by the proponent. This stage, of course, may be entered much sooner (at stage 2) if the respondent does not accept the thesis of the proponent at the outset.

7. The proponent responds with a rejoinder if he thinks that his critic’s refutation is in some way erroneous. However, if the proponent accepts the soundness of the refutation, the respondent is asked to state a formal proof of the refutation in a positive form, that is, as an independent and formally stated inference.

8. Finally, the respondent offers a formal proof of the refutation in inferential form.
The inferential form is by means of five-membered syllogistic inference patterns which are aimed at convincing the opponent. This is actually referred as ‘making inference for the sake of others’. The other type of inference in Indian philosophy is ‘making inference for the sake of oneself’, one that is designed to alleviate doubt for one-self. This involves an inference drawn in one’s own mind as a result of repeated observation earlier.

However, the ‘inference for the sake of others’ is not an informal matter. It requires demonstration of the inferential process as well as the evidence or ground for making the inference. The elaboration of the formal proof constitutes of proving the opponent what one has inferred and also showing how such inference had been arrived at.

In other words, the person involved in tarka, infers flaws in the opponent’s arguments. Nyāya Sastra refers to these flaws as ‘fallacies’. Fallacies are logical errors, which are to be avoided in any discussion. Fallacies can be present with the reason or with the manner in which the argument is proposed. (Note: The later is known as ‘argument fallacies’, which is out of scope of this work). These fallacies, otherwise called reason fallacies, are connected with inferential reasoning. In this work, we concentrate on ‘reason fallacies’ recommended by Nyāya Sastra.

The Nyāya-Sutra offered a five-step inference pattern for the conduct of fair and balanced debate. This five-step inference schema is known as the ‘classical five-membered inference pattern for proper argumentation’ (Chakraborti 2006). The concern clearly was to promote the notion and the practice of a good debate, and to differentiate it from the pointless, destructive debates. The following section describes clearly, the five-membered syllogisms stated by Nyāya Sastra (Sinha and Vidyabhusana 1930).
4.2 NYĀYA SYLLOGISM

4.2.1 Fundamentals of Nyāya Syllogism

Inferential arguments of Nyāya primarily have only three members: probandum or the object of inference; probans or the reason and the subject. These three members may be stated in any order. Apart from these members, there are two other members of syllogism, the analogous object and the dissimilar object (Oetke 2003). To arrive at a definite conclusion, the reason should ‘cover’ the subject; be present in the similar objects; and, be absent in dissimilar objects (Peckhaus 2001). According to Nyāya Sastra, an inferential proof is made up of the following members:

- The statement, premise or proposition – A proposition is a declaration of that which is to be established

- The cause or reason for the statement – The reason is the means for establishing that which is to be established through the homogenous nature of the example; reason is the supporting evidence which strengthens the proof

- The example – Example is a similar case that has occurred prior to the statement. It can be homogenous or heterogenous. Homogenous example is a well known instance which possesses the property sought to be established, and which points out that this property is necessarily contained in the reason set forth. Heterogenous example is a well known instance which is devoid of the property to be established, and which points out that the absence of this property is necessarily rejected in the reason set forth.
• The application of that example – Application is a summing up with reference to the example of what was sought to be established, as being ‘so’ or ‘not so’. The affirmative application is expressed by the word ‘so’; the negative application is expressed by the word ‘not so’.

• The conclusion – conclusion consists in re-stating the proposition after mentioning the reason.

Let us discuss the syllogistic inference scheme of Nyāya with two cases. In the cases discussed below, the ideas of argumentation have to be interpreted via the example scenarios and the words used are not to be interpreted per se.

**Case 1:** Consider an argumentation scenario, where two persons are arguing about the existence of ‘snake’ in a dark room. The sample arguments exchanged are given below:

<table>
<thead>
<tr>
<th>Arguer</th>
<th>Counter-arguer</th>
</tr>
</thead>
<tbody>
<tr>
<td>I guess this would be a snake</td>
<td>How? It appears to be a rope</td>
</tr>
<tr>
<td>It is long and is glossy</td>
<td>Even a rope can be long and glossy</td>
</tr>
<tr>
<td>Snakes generally lie in a curling circular pattern</td>
<td>Need not be! Somebody might have put a rope like that! (jumps over it) See it is not moving…</td>
</tr>
<tr>
<td>Yes, not moving. I have seen a snake in my childhood; snakes normally stay like this when they have eaten ‘rats’. This also looks like that…</td>
<td></td>
</tr>
<tr>
<td>Therefore this should be a snake</td>
<td></td>
</tr>
</tbody>
</table>
‘This is a snake’ (statement)
‘Because it is long and glossy, lies in curling circular pattern’ (reason)
‘Since whatever is long, glossy and curly, i.e. the snake in my childhood’ (example)
‘This is also like that’ (application)
‘Therefore, this is a snake’ (conclusion)

By following the above system of inference, argumentative reasoning can be seen as a methodology which introduces inferences through arguments for the sake of convincing the opponent.

**Case 2:** Here, we directly analyse the five-membered pattern of inference thus ignoring the order of arguments exchanged.

- ‘This hill has fire’ (statement)
- ‘Because it has smoke’ (reason)
- ‘Since whatever has smoke has fire e.g. an oven’ (example)
- ‘This hill has smoke, which is associated with fire’ (application)
- ‘Therefore, this hill has fire’ (conclusion)

After these five sentences have been employed there arises in the mind of the listener, consideration of the form, ‘this hill is full of smoke, which is in invariable concomitance with fire’, from which follows the conclusion, ‘this hill is full of fire’. Syllogism is therefore the name for the entire collection of these five sentences, each of which is called a part or member (Vidyabhusana 1988). On the employment of five sentences there
arises, at first, knowledge from each of them separately. Then arises collective knowledge from the five sentences combined together. This collective knowledge is based on each of the five sentences called a part or member, of the five membered syllogism. The following explanation of parts of Nyāya syllogism is taken from Vidyabhusana’s (1988) discussion of Nyāya Sutra.

4.2.2 Parts of Nyāya Syllogism

**Proposition:** The proposition is a sentence which gives rise to an inquiry necessitating the mention of the reason, e.g. ‘This hill has fire’. (Why so? Because it is smoky).

**Reason or Probans:** The reason or probans is one which produces knowledge, whose object is not the probandum, but which contributes to, the production of the entire knowledge that gives rise to consideration, e.g. because it is full of smoke. The loci of the reason can be in any of the three atomic states of existence: presence, which produces valid conclusion; absence, which produces invalid conclusion; presence in some part and absence in another part, which produces doubtful conclusion (almost equal to invalid conclusion) (Randle 2001; Vijay 1990).

The reason is said to possess either one or more relations with the object of inference, analogous object and the dissimilar object. The presence and absence of these relations and nature of these relations define the strength and weakness of any argument. Technically speaking, the notion of tarka in the context of knowledge sharing is the means of assessing the validity of the relation of pervasion or universal concomitance between the thesis requiring proof and the reason offered.
**Example:** An example is one, which, while producing knowledge of connection of the form that the locus of probans is constantly occupied by the probandum, causes another knowledge, which, proceeds from the sentence expressive of consideration.

All that has smoke has fire, as a kitchen.

(So this hill has smoke)

**Application:** The application is a member that produces consideration, e.g. All that has smoke has fire, this hill too has smoke.

**Conclusion:** Conclusion is a sentence which produces knowledge of the probandum as indicated by that of the probans through its invariable concomitance with the probandum and its nature of abiding in the subject, e.g. In this hill there is smoke, which is in invariable concomitance with fire. Therefore, in this hill there is fire, or therefore this hill is full of fire.

Therefore, according to ancient Nyāya logic, the proper formulation of inference utilises the above mentioned five-membered syllogism. To test the validity of inferential reasoning, Nyāya incorporates the unique property of invariable concomitance, the relation of simultaneous existence that binds any two concepts. The following section discusses briefly the importance of invariable concomitance in tarka.

**4.2.3 Invariable Concomitance and Tarka**

To discuss the nature and usage of invariable concomitance, let us elaborate the notion of ‘tarka’ in knowledge sharing. Classically ‘tarka’ means rational exchange of arguments with the objective of attaining valid knowledge. If the arguments require proof for syllogistic explanations to
demonstrate one’s stand, supporting arguments are framed (Matilal 1990). These supporting arguments actually refer to inferential evidences deposited in commonsense knowledge, like, invariable concomitance relation between ‘smoke’ and ‘fire’. Supporting arguments are also exchanged in a debate fashion.

In the previous example, the object to be inferred is ‘fire’ and the subject is, the ‘mountain’. A similar instance is kitchen, for it is well known that there is ‘fire’ in a ‘kitchen’. A dissimilar instance can be a ‘lake’, for it is well known that there is no ‘fire’ in a ‘lake’. The application informs us that the subject, i.e., the ‘mountain’, possesses the reason, i.e., ‘smoke’. More precisely, it informs us that the ‘mountain’ possesses ‘smoke’, which is ‘invariably’ connected with ‘fire’. The conclusion states that the hypothesis is deduced from the application.

The order of a five-membered syllogism is not in accordance with the process of getting inferential cognition. An inferer must first comprehend the relation between smoke (the reason) and fire (the object of inference), i.e., the invariable concomitance between smoke and fire, before starting to infer about fire. When the inferer sees smoke on the mountain (the subject), he remembers the invariable concomitance between smoke and fire. Then he gets the cognition that ‘the mountain has smoke’, which invariably coexists with fire. Finally he obtains the inferential cognition that the mountain has fire.

As seen from the earlier explanation, the instrument for generating this cognition is the knowledge of the invariable concomitance between smoke and fire. This knowledge is assumed to be available as part of the common-sense knowledge. This forms the intermediate operation for producing the inferential cognition i.e. the mountain has smoke which invariably coexists with fire, which is technically called confirmatory
cognition (Wada 1990). To put it more precisely, the inference relates to the relation of smoke to fire, and not the relation of fire to smoke. This is because smoke invariably coexists with fire and the reverse is not true.

Thus, invariable concomitance is the relation by which the probandum and probans are related to one another in any proposition. Whether the enquirer infers this relation by the co-presence or co-absence of both probans and probandum lies in the formal definition of invariable concomitance. The definition of invariable concomitance relation has too many variations: the five provisional definitions stated in Vyaptipancakam, objectionable definitions stated in Purvapaksah, conclusive definition of invariable concomitance stated in Siddhantalaksanam, invariable concomitance of special forms listed in Visesa-Vyapti (Vidyabhusana, 1988).

In our work, we go on to explain and follow only the plain definition of invariable concomitance as stated in Nyāya Sastra (Sinha and Vidyabhusana 1930), thus ignoring the internal variations. The process of determining the nature of existence of invariable concomitance in the proponent’s argument starts by exploration of argument in light of various fallacies over the elements of argument. This exploration further results in identifying the possible defects in the proposition, for which the nature and definition of invariable concomitance is utilised.

Syllogistic interactions are a measure to resolve knowledge inconsistencies through argument analysis by which defects are thoroughly explored. It is this perspective which motivated us to apply this to knowledge sharing. In other words, defect identification and elimination in a repeated manner keeps the discussion going, thereby, sharing the valid knowledge between the participants involved in discussion. Identification of defects becomes easier when they are neatly represented to facilitate their exploration.
Therefore, not only the style and nature of argument defects, but also, the representation of defects packed in the syllogistic elements of arguments and the inference gained during the process of hole-finding leads to refinement in knowledge-sharing through procedural arguments.

4.3 MOTIVATION AND NEED FOR INDIAN LOGIC BASED ARGUMENT REPRESENTATION

The family of DL systems are mere knowledge representation systems and there was no focus on argumentation and knowledge sharing. This motivated us to design a special knowledge representation formalism which completely inherits all the classification recommendations of Nyāya Sastra and provide a detailed perspective for the conduct of ‘tarka’ based argumentation by applying five membered syllogisms. Therefore, from the viewpoint of argumentative procedures, concepts, and relations between concepts, described by the ontology should be enriched to perform reasoning and inferencing from arguments. Hence, there arose a need to propose the relation enhancement of extended DL systems, expressive enough to enable the mimicking of human inferencing.

In addition, during argumentative discussions, generation of defects could be appropriate if and only if the submitted argument is interpreted correctly. To analyse the input argument properly, the elements of arguments should have a provision for correct mapping in the knowledgebase. Quick as well as complete knowledge representation formalisms are required which play a good role in finding the defects of the submitted arguments.

In a nutshell, procedural argumentation in its very own Nyāya tradition demanded an improved ontology to handle the non-monotonic nature of argumentation. In addition, the analysis of strength of any argument in terms of holes (defects) associated with the elements of argument and the
invariable concomitance (tautologies as specified by the fundamental Nyāya ontology) relation which is essential in determining the validity of inference obtained, were taken into consideration while constructing the Indian logic-based argument representation formalism. The incorporation of invariable concomitance into the relation elements of arguments re-defines the argument representation formalism from Nyāya perspective.

One of the most difficult problems in argumentation systems involves representing knowledge and belief of arguing agents in dynamic environments. New perceptions modify the current knowledge about the world and consequently the beliefs. Such revision and updating process should be performed efficiently by the arguing agent, particularly in the context of real time constraints. Therefore, we have proposed three different layers for handling argumentation (and for knowledge updation) in a systematic fashion. The following section explains our contribution of argumentation framework based on Nyāya Sastra.

4.4 LAYERED PERSPECTIVE OF ARGUMENTATION FRAMEWORK BASED ON NYĀYA SAstra

The Indian-logic based mathematical framework for argumentation, proposed in this work (Figure 4.1), is viewed from three perspectives.

- The bottom layer is the knowledge layer where world knowledge is classified in terms of concepts and relations into Indian logic ontology. A Nyāya Ontology Reference Model (NORM) for cognitive knowledge representation is proposed which involves Nyāya Description language (NDL) for defining the knowledge base. In addition, Gautama, an Indian–logic based ontology editor is also proposed to build the knowledge layer.
The middle layer is the logical layer where the main concern is about the use of appropriate logic to represent arguments. The definition of argument as proposed by Nyāya logics is defined in the logical layer.

The upper most layer is the argument gaming layer where the relationships between arguments are analysed, so that which portions of arguments attack or support which arguments can be identified, and the arguments can be evaluated which results in rewards. PONAG (Partially observable nondeterministic argument gaming) model for evaluating the arguments is also proposed.

<table>
<thead>
<tr>
<th>Argument Gaming Layer (PONAG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logical Layer (Nyāya Logics)</td>
</tr>
<tr>
<td>Knowledge Layer (Nyāya Ontology Reference Model)</td>
</tr>
</tbody>
</table>

Figure 4.1 Indian Logic based Argumentation Framework

4.4.1 Knowledge Layer - NORM for Cognitive Knowledge Representation

The Nyāya-Vaiśesika is a self-contained system of philosophy. It proposes a unique categorisation of world knowledge elements (Vidyabhusana 1988). Through the epistemological definitions of Nyāya-Vaiśesika system, the treatment of world knowledge elements was very special which contributed to the uniqueness of ontological categorization. The methodology of categorization was inaugurated by Gautama-Akshapada which consists in enumeration and classification of world knowledge entities
into specific categories which were recommended, argued and analysed by the followers of Gautama (Vidyabhusana 1988).

NORM is the Nyāya based Ontology Reference Model, which defines the standards for constructing ontology, based on the recommendations of the epistemology definitions of Nyāya-Vaiśesika school of Indian philosophy. NORM is organized as two-layer ontology, where the upper layer represents the abstract fundamental knowledge and the lower layer represents the domain knowledge. According to NORM, a node in the ontology is composed of an enriched concept which is related implicitly to its member qualities and explicitly to other peer concepts, by means of relations (Wada 1990).

4.4.1.1 Structure of Concept and Relation in NORM

A node of Nyāya-Vaiśesika based ontology has the following structure (Figure 4.2). Every concept of the world knowledge shall be thoroughly classified as per NORM structure. The abstract and domain concepts form a strict classification hierarchy. The traits of the top-level concepts are applicable down the hierarchy.

Every concept in the NORM model has links to other concepts by external relations (Figure 4.2a). A concept is made of qualities. In addition, the qualities are bounded to the concept by internal relations. The qualities may also be related to each other, which are depicted as dotted edges (Figure 4.2b). Every quality has a set of values, and the values are bounded to the qualities by grouping relations (Figure 4.2c). This model (Figure 4.2) is inspired by the various recommendations of classifications of world knowledge according to Nyāya-Vaiśesika. The following section discusses the system of classification of Nyāya Sutra.
Figure 4.2 NORM Model for Cognitive Knowledge Representation

(a) ontology with concepts as nodes and external relations as edges, (b) a concept with qualities as nodes, internal relations as thin edges, tangential relations as dotted edges, (c) a quality with values as nodes, grouping relations as edges

4.4.1.2 Nyāya System of Classification

According to Nyāya Sastra (Sinha and Vidyabhusana 1930), every concept is classified into seven categories: substance, quality, action, generality, particularity, inherence and negation. Among these, the substance is of nine kinds: earth, water, light, air, ether, time, space, soul and mind. Every substance is threefold: body, organ and object. The object of light is fourfold: earthly, heavenly, gastric and mineral. Every substance is said to possess some quality. The quality is of twenty-four varieties which in turn possess values (Figure 4.3).

The permissible action associated with the substance is of five types: upward motion, downward motion, contraction, expansion, and motion. Generality is either more comprehensive or less comprehensive. Negation is of four varieties: antecedent negation (or prior negation), destructive negation (or posterior negation), absolute negation and mutual negation. Out of the nine substances, odour persists only in earth and is inherent. Earth exists in all the seven colors. Air has no color; water is pale-white in color and light is bright-white in color. Air has touch. Water has cold-touch and light has hot-touch. Dimension (or magnitude), distinctness, conjunction and disjunction are present in all the nine substances. Remoteness and Proximity is found in
earth, water, light, air and mind. Heaviness or Weight is only in earth and water. Viscidity is present only in the substance, Water.

- **Color**: white, blue, yellow, red, green, brown, varied
- **Taste**: sweet, acid, saline, pungent, astringent, bitter
- **Odour**: fragrant, foul
- **Touch**: cool, hot, lukewarm
- **Number**
- **Magnitude**: atomic, large, long, short
- **Separateness**
- **Conjunction**
- **Disjunction**
- **Remoteness**: spatial, temporal
- **Proximity**: spatial, temporal
- **Weight**
- **Fluidity**: natural, artificial
- **Viscosity**
- **Sound**: articulate, inarticulate
- **Intelllect**
- **Pleasure**: Remembrance, apprehension. Erroneous apprehension, valid apprehension.
  - Valid apprehension: perception, inference, analytical knowledge and verbal testimony

**Figure 4.3 Ontological Classification of Nyāya-Vaiśesika Qualities**

The detailed structure of a node in Nyāya-Vaiśesika ontology is shown in Figure 4.4. The structure incorporates almost all the recommendations of Nyāya-Vaiśesika school along with the detailed definitions of relations at every level, between concepts, between concept and member qualities, between qualities, and between quality and member values. Appendix 1 describes the ontology editor, Gautama for editing the world knowledge in the form of Indian logic ontologies.
Figure 4.4 The Node Ontology Architecture of NORM

Having analysed an acceptable structure of knowledge representation to suit procedural argument gaming, the following section defines the mathematical definitions for logical layer and knowledge layer (Figure 4.1), in our procedural argument gaming system which embeds world knowledge in the form of NORM.

4.4.2 Formal Definitions of Knowledge Representation in Indian Logic Arguments

A knowledge base basically consists of statements.

\[ \varepsilon : \text{Knowledge base} \]  \hspace{1cm} (4.1)

A statement is basically classified as two disjoint sets: propositions and assumptions.

\[ P \subseteq \{ p_1, p_2, \ldots, p_n \} \text{elements are propositions} \]  \hspace{1cm} (4.2)
Here propositions are statements of facts that take part in an argumentative procedure. Propositions can be of two types: facts or proven facts. Factual propositional statements are extracted from current input statements; proven propositional statements exist in the knowledge-base. A degree of belief or strength of proof factor is attached to each of the proven propositional statements. Propositions can be plain propositions discussing information regarding the current subject, or else, can be reasons that support or oppose the belief of already existing propositions. Assumptions are special type of propositions defined as initial statements assumed to be true until proved otherwise. Assumptions do not possess strength of proof factor and are completely open for attack.

A knowledge base is the context in which the process of argumentation takes place and consists of assumptions $\alpha$, propositions $\Pi$ and a language $L$ which is a set of combinations of assumptions and propositions. $\xi$ is the statement whose truth is to be determined in the context of the knowledge base $\varepsilon$

$$\Lambda_{\alpha\Phi}$$: propositional language,
$$\xi$$: arbitrary statement in $L_{\alpha\Phi}$, \hspace{1cm} (4.6)

then, triple ($\xi, \Pi, \alpha$) forms the propositional argumentation system. $\xi$ can be any element of $\alpha \cup P$; i.e. $\xi$ can either be a proposition or an assumption
\[ \mathcal{E} = \sum_{i=0}^{r} \xi_i \]

where \( \xi_i \in \Lambda_{\text{conf}}, \rho = \text{maximum number of arbitrary statements} \) \( \xi \) in the knowledge base \( \mathcal{E} \).

An ontological classification \( O_T \) is defined as the collection of all concepts under the given domain boundary

\[ O_T = \sum_{i=1}^{n} c_i \]

An ontological commitment \( O_D \) is defined as the collection of all constraints defined by operator set \( \text{Op} \) over concepts \( C \) of ontological classification \( O_T \) under the given domain boundary

\[ O_D = O_D(\text{Op}, O_T) – \text{collection of constraints} \]

**Definition 1 (Abstract concept)** A concept is an abstract entity which embeds several qualifying attributes of its own. The attributes are bound to the concept by relation existence. An attribute is a sub-property of a concept / relation which is used to define the concept set. Attributes are optionally supported by values.

An abstract concept \( c \) is a 7-tuple

\[ c \equiv (O, Q, V, R^e, R', R', R^e) \]

where,

\[ c = \{ c_1, c_2, \ldots, c_n \} \]

\( O \) is a set of object of the concept, \( O = \{ o_1, o_2, o_3, \ldots, o_n \} \)
Q is a nonempty set of attributes, \( Q = \{q_1, q_2, q_3, \ldots, q_m\} \) \hfill (4.13)

\( V \) is a set of values, \( V = \{v_1, v_2, v_3, \ldots, v_p\} \) \hfill (4.14)

\( R^e \subseteq O \times O \) is a set of external relations; i.e. relations between a pair of concepts \hfill (4.15)

\( R^i \subseteq O \times Q \) is a set of internal relations; i.e. relations between a concept and its member attributes \hfill (4.16)

\( R^t \subseteq Q \times Q \) is a set of tangential relations; i.e. relations between the member attributes of a given concept \hfill (4.17)

\( R^g \subseteq Q \times V \) is a set of grouping relations between the member attribute and the values owned by it \hfill (4.18)

The presence of \( R^i \) with a particular concept may also contribute to and qualify the exclusiveness of a given concept. The presence of \( R^g \) with a particular concept may also contribute to and qualify the exclusiveness / exceptions of a given concept.

**Definition 2: (Abstract Quality)** An abstract quality \( Q \) associated with an abstract concept \( C \) under an ontological classification \( O_T \) is a 4-tuple which is defined as:

\[
Q \equiv (Q_{con}, V, R^e, R^i)
\]

where,

\( Q_{con} \) is a set of constraints , \( Q_{con} \subseteq \{Q_m, Q_o, Q_e, Q_x\} \) \hfill (4.20)

i.e. mandatory, optional, exceptional and exclusive respectively.
\( Q \) is a nonempty set of attributes, \( Q = \{q_1, q_2, q_3, \ldots, q_m\} \)

\( V \) is a set of values, \( V = \{v_1, v_2, v_3, \ldots, v_p\} \)

\( R^e \equiv R' \subseteq Q \times Q \) is a set of external relations; i.e. relations between qualities

\( R^i \equiv R^e \subseteq Q \times V \) is a set of internal relations; i.e. relations between a quality and the values owned by it.

An attribute \( Q \) of a concept is called mandatory, if and only if the presence of the attribute exclusively qualifies the nature of the particular concept. An attribute \( Q \) of a concept is called optional, if the presence or absence of the attribute does not exclusively qualify the nature of the particular concept.

An attribute \( Q \) of a concept / relation is called exclusive, if the added presence of the attribute exclusively qualifies the nature of the particular concept / relation. An attribute \( Q \) of a concept / relation is called exceptional, if the attribute is present in the particular concept / usage of the relation and absent in every other concept / usage of that particular or other relations. Quality is the property associated with every concept and is said to describe the concept more expressively.

The qualifying attributes of a concept are not present as only the feature set of that concept; instead, every qualifying attribute is related to or bound to the concept by relations.

The ‘mandatory’ attribute of a concept shall be made ‘exclusive’ or ‘exceptional’ but not ‘optional’.
The ‘optional’ attribute of a concept shall be made ‘mandatory’ which contributes more to inferential learning; but the vice versa is not allowed and is considered as violation of ontology definitions.

**Definition 3: (Mathematical Framework for Argumentation)**

The procedural argumentation framework is a pair \( AF = <X, A_T> \), where \( X \) is a set of arguments and \( A_T \subseteq X \times X \) is the attack relationship for \( AF \). \( A_T \) comprises a set of ordered pairs of distinct arguments in \( X \). A pair \(<x, y>\) is referred to as ‘\( x \) attacks \( y \)’ (Capon 2002). Attack is defined as a binary relation between already existing arguments within the argumentation framework. An argument \( x \) attacks argument \( y \) (\( y \leftarrow x \)), if the conclusion of \( x \) contradicts the conclusion of sub-argument \( y’ \) of \( y \) and \( y’ \) is not stronger than \( x \).

\[
y \leftarrow x ; \text{ if } \text{conc}(y'/y) \neq \text{conc}(x) \& \& A_{str}(y’) < A_{str}(x) \quad (4.21)
\]

The strength of the argument is denoted as \( A_{str} \). The arguments and counter-arguments constructed as a result of \( A_T \) accumulate to form an argument hierarchy.

**Definition 4: (Argument A)** An argument is a set of propositions related to each other in such a way that all but at least one of them (the premise) are supported to provide support for the remaining (the conclusion). An argument \( A \) over argumentation framework \( AF \) is defined as a tuple

\[
A = <A_{id}, f(c, r), A_{state}, A_{status}, A_{str}>
\]

(4.22)

where \( f(c, r) = c_{cat} \times r_{cat} \) is a function of argument concepts and argument relations

\( A_{id} \) is the argument index;
A_{state}, the state of argument; A_{state} \subseteq \{premise, inference, conclusion\};

A_{status}, the defeat status of arguments; A_{status} \subseteq \{defeated, undefeated, ambiguous, undetermined and

A_{str}, the strength or conclusive force of the argument.

**Definition 5: (Concept in Argument)** A concept in the argumentation framework is defined as a combination of abstract concept with other categorical properties of concept existence in argument gaming.

\[ C_{AG} \equiv (c, C_{con}, C_{Cat}, C_{cf}) \] (4.23)

where \( c \) is the abstract concept in def. 1,

\( C_{con} \) is the constraint set under which concept \( C \) is said to exist;

\( C_{con} \subseteq O_D \)

\( C_{Cat} \) is the category of concept in the procedural argumentation scenario; the category can be of three types; \( C_{Cat} \subseteq \{ C_S, C_{ol}, C_R \} \)

\( C_{cf} \) is the confidence factor (a numeric value) associated with every abstract concept in the knowledgebase.

**Definition 6: (Relation in Argument)** A relation in the argumentation framework is defined as a combination of abstract relation with other categorical properties of relation existence in argument gaming.

\[ R_{AG} \equiv (r, r_{q}, r_{con}, r_{cat}, r_{cf}) \] (4.24)

where \( r \) is the abstract relation (refer def. 1),
\( r \subseteq \{R^e, R^i, R^r, R^g\} \)

\( r_g \) is the set of attributes of the abstract relation, \( r_g \subseteq \{IC, D, X, Xp\} \)

\( IC \) is the set of invariable concomitance relations, where

\( IC \subseteq \{\text{symmetric}, +IC, -IC, \text{neutral}\} \)

\( D \) is the set of direct relations, where \( D \subseteq \{\text{is-a, has-a, part-of}\} \)

(Note: For convenience, direct relations are notated by \( r \) throughout the rest of our work)

\( X \) is the set of exclusive relations where

\( X \subseteq X \land (X \subseteq r) \land (X \neq \phi) \)

\( Xp \) is the set of exceptional relations where

\( Xp \subseteq Xp \land ((Xp \subseteq r) \land (Xp \neq \phi); (xp[V] = \text{true}) \text{ for some element} \)

\( c_k \subseteq c \); and false for other elements of \( c \)

\( r_{con} \) is the constraint set under which relation \( r \) is said to exist;

\( r_{con} \subseteq \{\text{reflexive, symmetric, anti-symmetric, asymmetric, transitive}\} \)

\( r_{cf} \subseteq \{R_{S-0l}, R_{S-R}, R_{R-0l}\} \)

\( r_{cf} \) is the confidence factor (a numeric value) associated with every abstract relation in the knowledgebase

**Definition 7: (Invariableness \( \vdash \))** A relation \( r \) is called invariable iff for any two concepts \( c_1 \) and \( c_2 \), \( c_1 \vdash c_2 \) or \( c_1 \) entails \( c_2 \)

**Definition 8: (Affirmative Invariableness \( \vdash_T \))** A relation \( r \) is called affirmatively invariable iff for any two concepts \( c_1, c_2 \), \( c_1 \vdash \top c_2 \) and if \( c_1 \) is true and \( c_1 \vdash c_2 \) then \( c_2 \) is true. This property of invariable relation is also known as ‘agreement in presence’, which is represented as \( c_1 \vdash_T c_2 \).
**Definition 9: (Negative Invariableness)** A relation $r$ is called negatively invariable iff for any two concepts $c_1, c_2, c_1 \not r c_2$ and if $c_2$ is false and $c_1 \not= c_2$ then $c_1$ is false. This property of invariable relation is also known as ‘agreement in absence’, which is represented as $c_1 \not F c_2$.

**Definition 10: (Affirmative-Negative Invariableness)** A relation $r$ is called affirmative-negatively invariable iff for any two concepts $c_1$ and $c_2$, $c_1$ is affirmatively invariable with $c_2$, and $c_2$ is negatively invariable with $c_1$. This is represented as $c_1 \not T/F c_2$, i.e. Wherever $c_1$ is true, $c_2$ is true; Wherever $c_1$ is false, $c_2$ is false.

**Definition 11: (Exclusive Relation)** A relation $r$ is called exclusive iff for any two concepts $c_1$ and $c_2$, $(c_1 \not< c_2) \triangleq (c_1 \not r c_2) \land (\text{Sup}(c_1) \not r \text{Sup}(c_2))$ in the name of $r$. The superset of concepts are denoted as $\text{Sup}(c_1), \text{Sup}(c_2)$. (Note: $\triangleq$ has to be interpreted as ‘is defined as’).

**Definition 12: (Exceptional Relation)** A relation $r$ is called exceptional iff for any two concepts $c_1$ and $c_2$, $c_1 \not< c_2 \triangleq ((c_1 \not r c_2)$ when $r$ is true; $\text{Sup}(c_1) r \text{Sup}(c_2)$ when $r$ is false) in the name of $r$.

### 4.5 CASE STUDY

The analysis of invariable concomitance relation (if present) helps in proving the proposed argument during argumentative discussions. The application presented in this section presents two knowledge sharing scenarios, the first with no invariable relation between smoke and fire.
(Figure 4.5a), and the second, with the presence of invariable relation between the two relating concepts, smoke and fire (Figure 4.5b).

Two knowledge sharing agents discuss among themselves regarding the presence of fire in a distant mountain. The argument is initiated due to the perception of smoke from the mountain by agent 2. The knowledge base of the discussion agents is different in both the scenarios. In both the scenarios, agent 2 starts the discussion. In the first scenario (Figure 4.5a), agent 2 has a direct relation between smoke and fire; agent 1 has a direct relation between smoke and falls. Therefore, when agent 2 argues about the relation between smoke and fire which is its’ own valid belief, it is not universally valid because the relation is not ‘invariably concomitant’.

![Figure 4.5 Sample argumentative discussion](image)

(a) with no invariable concomitance relation – Agent 1 concludes
(b) with invariable concomitance relation – Agent 2 concludes

In addition, agent 1 has a direct relation between smoke and falls; falls is a water-body which is ‘absence of fire’, according to the ontological classification. Therefore, agent 1 does not accept the claim of agent 2 and ensures this again by the argument ‘does smoke exists with fire?’.
counter-argument of agent 2 is not convincing. Therefore, when agent 1 counter-attacks the argument by stating the relation between smoke and falls, which is a water-body, agent 2 has no point in arguing back and gives up its proposal. Therefore, the claim of agent 2 gets defeated and this is visible from the last argument which concludes, ‘there is no fire in the mountain’.

In the second scenario (Figure 4.5b), agent 2 has an invariable relation existing between smoke and fire. Here too, agent 1 has no relation between smoke and fire. Therefore, when agent 2 argues about the relation between smoke and fire which is its’ own and universally valid belief, agent 1 has no point in arguing back again. Therefore, the claim of agent 2 is proved to be valid and this is visible from the last argument which concludes as ‘there is fire in the mountain’.

The procedural approach of argumentation according to Indian philosophy is made possible by the definition of fundamental concept and relation elements of argument procedures represented based on Indian logics. The mathematical model for argument representation provides detailed definitions of fundamental elements of arguments, and, their associations with a conceptual ontology has been modified so as to enable alternate cycling of attack and defeat of arguments. This kind of alternation of arguments and counter-arguments shall be modeled as a sequence of moves and counter-moves. In a nutshell, counter-moves are finalised after inferencing and reasoning over the previous moves. This game sequence of moves and counter-moves shall be modeled as argument gaming. The following chapter discusses the fundamentals of argument gaming.