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Closed-form formulae have been analytically derived to estimate crosstalk due to stimulated Raman scattering (SRS) in a multipumped, broad, and flattened gain distributed Raman amplifier (DRA). The derived formulae are applied to study the crosstalk due to SRS in a multipumped DRA having different pumping configurations. System bounds for a typical wavelength division multiplexing system employing DRA have been evaluated theoretically. Performance of DRA for other parameters such as wavelength separation, input signal power, and bit rate of the system has also been investigated. © 2011 Optical Society of America

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1. Introduction

Optical fiber communication employs a wavelength division multiplexing (WDM) scheme to multiplex different information carrying optical signals in a single fiber [1–8], which is limited by attenuation, dispersion and nonlinear effects. To counter the effect of attenuation, signals are amplified in optical domain using erbium-doped fiber amplifiers (EDFA) and Raman amplifiers (RA). In RA, signals are amplified utilizing high power pump, which copropagates (forward pumping) or counter propagates (backward pumping) with the signals. The availability of high power diode pump laser has made Raman amplifiers commercially feasible both in lumped and distributed configurations [9, 10]. Distributed Raman amplifiers (DRA) allow long fiber spans, high bit rates and close wavelengths spacing because of improved noise figure and reduced nonlinear penalty of the system [9, 10]. In order to achieve broad and flat gain in DRAs, a “WDM pumping” technique [10] consisting of two or more pumps separated by few nanometers is used. Among all available pumping schemes with equal pump power and identical fiber length, backward pumping is preferred due to high gain and low noise figure [10]. However, in a wideband WDM system, because of overall high signal power confinement, nonlinear effects come into play that are greatly enhanced when high-gain DRA is included in the system [10]. One of the major nonlinear effects is stimulated Raman scattering (SRS), which adversely affects the performance of a WDM system due to transfer of power from lower wavelength channels to higher wavelength channels, resulting in stimulated Raman crosstalk.

Previous authors have studied the effect of SRS on optical communication systems with and without group velocity dispersion (GVD) and have evaluated the probability density function of SRS crosstalk for single and multiple fiber links [11–14]. Stimulated Raman crosstalk has been studied in single and multiple segments of fiber links [15, 16]. In [16], SRS crosstalk standard deviation has been evaluated in a periodically amplified and dispersion managed fiber link by applying statistical methods, where lumped amplifiers have been used for amplification of WDM signals. In the derivation of SRS crosstalk [16] mean and standard deviation gain is assumed to be constant. It has been reported that in the presence of lumped amplification and dispersion compensation, SRS crosstalk standard deviation is greatly enhanced [16]. The system performance
has been evaluated considering bit error rate (BER) at the receiver and system performance has been found to degrade significantly due to SRS [16]. This straightforward tool based on statistical methods [16] has been found to be extremely advantageous for evaluating SRS crosstalk in multiple segments of fiber link using lumped amplifiers [17].

In the present paper, statistical analysis of crosstalk due to SRS has been applied for the first time to WDM system employing DRA. In distributed Raman amplifiers, a high power pump co- and counter propagates with the WDM signals, providing a continuous gain to compensate for the attenuation of signals. Using statistical methods, closed form formulae have been derived to study SRS crosstalk in a single segment of fiber link employing DRA. Since the gain is continuously increasing along the transmission line, SRS crosstalk mean and standard deviation have been calculated for variable gain along the entire length of single mode fiber. Using these formulae, comparative study has been done between WDM systems using a lumped and distributed amplifier and between forward and backward pumped DRA. The analysis is further applied to WDM pumped Raman amplifiers in forward and backward configurations to obtain crosstalk for different DRA parameters such as input power, wavelength separation, and bit rate of the system. These results are then used to determine system bound for a typical WDM system using DRA as amplifier.

The paper has been divided into following sections. In Section 2, SRS crosstalk standard deviation in DRA has been derived. Using the formulae in Section 2, system performance has been evaluated in Section 3 for various parameters of DRA such as input power, interchannel separation, and bit rate of the system. Section 4 concludes the paper.

2. Crosstalk Variance in Distributed Raman Amplifier

In this section, crosstalk variance is derived in a distributed Raman amplifier for a forward and backward pumping scheme. A distributed Raman amplifier of length \( L \) with no repeater is considered. In \( N \)-channel WDM systems, SRS causes power change in the \( i \)th channel of the WDM system due to power amplification by shorter wavelength channels and depletion by longer wavelength channels.

Thus optical power of the \( i \)th channel after transmission through distance \( z \) is given as [15]

\[
P_i(z, t) = P_i(0, t) \exp \{-az - x_i(z, t)\},
\]

where

\[
x_i(z, t) = \sum_{j=1}^{N} x_{ij}(z, t) \quad (2)
\]

is the power change in the \( i \)th channel due to the remaining \((N - 1)\) channels due to SRS.

In Eq. (2),

\[
x_{ij}(z, t) = \sum_{k=-\infty}^{\infty} b_k q_{ij} \left( t - \frac{z}{v_i} - kT \right)
\]

and

\[
q_{ij}(t) = K \int_{0}^{L} p(t - d_{ij}z') e^{-\alpha z'} g(z') dz',
\]

where

\[
K = \begin{cases} \frac{g_R f_i f_j}{v_i v_j f_i f_j} (|f_i - f_j| \leq 15 \text{THz}) \\ 0 (|f_i - f_j| > 15 \text{THz}) \end{cases}
\]

In Eq. (3), \( b_k \) is random transmission data that takes the binary value 1 or 0 and thus \( x_{ij} \) is considered to be a random variable. In Eq. (4), \( p(t) \) is pulse shaped, \( v_i \) is group velocity of \( i \)th channel, and \( d_{ij} (= 1/v_j - 1/v_i) \) is propagation time difference between the two different channels \((i, j)\) during a unit length transmission. In Eq. (5), the Raman gain function is approximated by a triangular function and \( g_R \) is the average slope of the triangular profile. The frequency of the \( i \)th \((j\)th\) channel is \( f_i (f_j) \), polarization constant \( \psi \) is equal to 2, \( T \) is bit period, and \( A_{\text{eff}} \) is effective area of cross section. The parameter \( K \) takes a positive (negative) value if \( f_i > f_j \) and that implies that optical power \( P_i \) is depleted (amplified) by the \( j \)th channel. If \( f_i \leq f_j \) then \( K \) becomes zero.

A typical DRA configuration is shown in Fig. 1. It shows number of signals at different wavelengths multiplexed in a single DRA. Each wavelength represents an independent communication channel being amplified by the DRA and ultimately demultiplexed at the output. When forward pumped DRA is used in optical transmission systems, the optical signal at the Stokes frequency exists from the beginning. So, pump depletion is considered in the analysis. Forward amplification gain [18] is
\[ g(z') = \exp \left( \gamma \frac{\sum \frac{1}{2} P_{so} + P_{po}}{A_{\text{eff}} \alpha} (1 - e^{-\alpha z'}) \right). \]  

(6)

where peak Raman gain \( \gamma = 6.5 \times 10^{-14} \), \( P_{po} \) and \( P_{so} \) are the initial pump and signal power, respectively, \( \lambda_p (\lambda_s) \) is signal (pump) wavelength.

When backward pumped DRA is used in optical transmission systems, the system is designed not to induce pump depletion because the signal level will fluctuate if pump depletion occurs. So, pump depletion is neglected. In this case, backward amplification gain [18] is

\[ g(z') = \exp \left( \gamma \frac{\sum \frac{1}{2} P_{so} + P_{PL}}{A_{\text{eff}} \alpha} (e^{\alpha z'} - e^{-\alpha L}) \right). \]

(7)

where \( P_{PL} \) is initial pump power, \( L \) is the length of DRA, and other parameters are the same as Eq. (6).

A. Mean and Variance of SRS Crosstalk for Forward Pumped DRA

Fourier transform of Eq. (4) for forward pumped DRA gives,

\[ Q_{ij}(\omega) = K \int_0^L P(\omega)e^{-jd dj_0 \omega} e^{-\alpha z'} \exp(K'(1-e^{-\alpha z'})) dz'. \]

(8)

where \( K' = \frac{P_{po}}{A_{\text{eff}} \alpha} \).

Substituting \( z' = \ln t \), \( dz' = \frac{1}{t} dt \), the equation becomes

\[ Q_{ij}(\omega) = K P(\omega) e^{K' \int_1^e \frac{1 - K't^{-\alpha}}{t^{\alpha + jd dj_0 \omega}} dt}. \]

(9)

Next, binomially expanding the exponent term \( (e^{-K't^{-\alpha}}) \) in the expression of \( Q_{ij}(\omega) \) and neglecting the higher order terms that tends to 0, gives

\[ Q_{ij}(\omega) = A P(\omega) \int_1^e \frac{1 - e^{-(\alpha + jd dj_0 \omega)L}}{\alpha + jd dj_0 \omega} dt,
\]

where \( A = Ke^{K'} \).

Integrating and substituting the limits we get

\[ Q_{ij}(\omega) = AP(\omega) \left( \frac{1 - e^{-(\alpha + jd dj_0 \omega)L}}{\alpha + jd dj_0 \omega} \right) \]

\[ -AK'P(\omega) \left( \frac{1 - e^{-2(\alpha + jd dj_0 \omega)L}}{2\alpha + jd dj_0 \omega} \right). \]

(11)

Considering rectangular nonreturn-to-zero pulse, Fourier transform of the pulse signal is

\[ P(\omega) = 2P_0 T \frac{\sin(\omega T)}{\omega T}. \]

(12)

The mean of SRS crosstalk is given as [16]

\[ \mu_x = \frac{Q(0)}{2T} = KP(0) e^{K'L_0(\alpha)} - KP(0) e^{K'L_0(2\alpha)}. \]

(13)

The variance of SRS crosstalk is given as [16]

\[ \sigma_x^2 = \frac{1}{8\pi T} \int_0^\infty |Q(\omega)|^2 d\omega. \]

(14)

The integral is performed using table of integral and series [19] and the result is given in Appendix A.

B. Mean and Variance of SRS Crosstalk for Backward Pumped DRA

Fourier transform of Eq. (4) for backward pumped DRA gives,

\[ Q_{ij}(\omega) = K \int_0^L P(\omega)e^{-jd dj_0 \omega} e^{-\alpha z'} \exp(K'(1-e^{-\alpha z'})) dz'. \]

(15)

Substituting \( z' = -L = -m, dz' = -dm \)

\[ Q_{ij}(\omega) = -KP(\omega) \int_0^L e^{-((\alpha + jd dj_0 \omega)L-\alpha \omega)} \exp(K'(1-e^{-\alpha z'})) dz'. \]

(16)

where \( K' = \frac{P_{po}}{A_{\text{eff}} \alpha} \).

Substituting \( m = \ln t \) and \( dm = \frac{1}{t} dt \), the equation becomes

\[ Q_{ij}(\omega) = KP(\omega) e^{-K'e^{-\alpha L} \int_1^e \frac{1 - e^{-K't^{-\alpha}}}{t^{\alpha + jd dj_0 \omega}} dt}. \]

(17)

where \( K' = e^{-jd dj_0 \omega} L \).

Next, binomially expanding the exponent and neglecting the higher order terms as they tend to 0 yields

\[ Q_{ij}(\omega) = AP(\omega) e^{-jd dj_0 \omega L} \int_1^e t^{\alpha - 1} \frac{1 + K't^{-\alpha}}{t^{\alpha + jd dj_0 \omega}} dt,
\]

(18)

where \( A = Ke^{-K'e^{-\alpha L}} \).

Integrating and substituting the limits we get,

\[ Q_{ij}(\omega) = AP(\omega) e^{-jd dj_0 \omega L} \frac{e^{(\alpha + jd dj_0 \omega)L} - 1}{\alpha + jd dj_0 \omega} \]

\[ + AP(\omega) K' e^{-jd dj_0 \omega L} \frac{1 - e^{jd dj_0 \omega L}}{jd dj_0 \omega} \]

\[ = AP(\omega) \left( \frac{1 - e^{-(\alpha + jd dj_0 \omega)L}}{\alpha + jd dj_0 \omega} \right) \]

\[ + AP(\omega) K' e^{-\alpha L} \left( \frac{1 - e^{-(jd dj_0 \omega)L}}{jd dj_0 \omega} \right). \]

(19)
Same expression for $P(\omega)$ is assumed as in the former derivation.
The mean of SRS crosstalk is given as [16]

$$\mu_x = \frac{Q(0)}{2T} = KP(0)e^{-K\alpha L}L_c(\alpha).$$  \hspace{1cm} (20)

The variance of SRS crosstalk is given as [16]

$$\sigma_x^2 = \frac{1}{8\pi T} \int_{-\infty}^{\infty} |Q(\omega)|^2 d\omega.$$  \hspace{1cm} (21)

The integral is performed using table of integral and series [19] and the result is given in Appendix A.

3. Results and Discussion

In the derivation for the mean and variance following assumptions have been made:

- Pump power has been limited to 250 mW
- The value of exponential $e^{-K\alpha L}$ in Eq. (9) and $e^{K\alpha}$ in Eq. (17) is approximated to be the sum of two terms $(1-K\alpha L)$ in Eq. (10) and $(1+K\alpha)$ in Eq. (18), which introduces a maximum error of $6 \times 10^{-5}$.

The DRA configuration considered is shown in Fig. 1. Different interacting channels are assumed to transmit 1 mW power in wavelength range of 1515–1575 nm with interchannel separation of 1 nm. The multipumps used for obtaining wideband gain are in wavelength range of 1420–1460 nm, separated by 10 nm and each having power of 150 mW. Each pump provides gain to 10 signals around its peak Raman gain at a separation of approximately 100 nm, making the gain spectrum almost flat in signal wavelength range of 1515–1575 nm. The SRS crosstalk variations for forward and backward pumped DRAs are obtained solving Eq. (4) for one fiber span of 80 km. Other parameters used for calculation are same as in [16] and given in Table 1.

The validity of our approach is checked by taking gain $g$ as zero, which implies that the configuration shown in Fig. 1 reduces to a single span of single mode fiber. Pumps interact among themselves in the similar way as the signals propagating in single mode fiber. In Fig. 2, dashed curve shows the results obtained by integrating Eq. (4) for zero gain and coincides with that obtained by using the equation in [16] for a single span of single mode fiber. In Fig. 2, the solid curve shows the variation of SRS crosstalk standard deviation with channel number for distributed Raman amplifier. The minimum crosstalk due to SRS occurs in the central channels (around 1545 nm) for SMF as well as DRA. It is also observed that for DRA, SRS crosstalk increases in all channels by almost 20 times per span of DRA as compared to normal fibers.

Figure 3 shows variation of SRS crosstalk standard deviation with channel number for backward pumping configuration. It is observed that minimum crosstalk of about $1.986 \times 10^{-3}$ dB occurs around 1545 nm indicating that similar to forward pumped DRA the minima occurs in the central channel for backward pumped DRA. The reason for central channel suffering minimum crosstalk is that it gains power from lower wavelength channel and loses power to higher wavelength channel. Hence net power transfer due to SRS is smallest in central channel. It is observed that SRS crosstalk variance for backward and forward pumping are $2.685 \times 10^{-3}$ dB and 0.043 dB, respectively, for longest wavelength. Even in the low wavelength channels, backward pumped DRA shows SRS crosstalk variance of $2.89 \times 10^{-3}$ dB, which is comparatively less compared to that estimated in forward pumped DRA (0.046 dB). In the central wavelengths where the crosstalk is minimum at 0.031 dB for forward pumped DRA, the magnitude decreases to $1.986 \times 10^{-3}$ dB in case of backward pumped DRA.

It has been reported in literature [16] that SRS crosstalk variance depends on wavelength separation, input power, and bit rate of the signals for WDM systems employing lumped amplifiers. The effects of variation in the above mentioned three factors were investigated in DRA. Figure 4 shows the variation of SRS crosstalk standard deviation for forward

<table>
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<th>Table 1. Parameters for Evaluating Crosstalk Variance</th>
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<tr>
<td>Parameters of SMF [16]</td>
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<tr>
<td>$g_R$ (Slope of Raman gain curve) $0.57 \times 10^{-16}$ W$^{-1}$</td>
</tr>
<tr>
<td>$A_{eff}$ (Effective Area) $80 \mu m^2$</td>
</tr>
<tr>
<td>Zero Dispersion Wavelength $1265.5$ nm</td>
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<tr>
<td>Dispersion Slope $0.058 \times 10^7$ ps/nm/nm/km</td>
</tr>
<tr>
<td>$\alpha$ (attenuation coefficient) $0.2$ dB/km</td>
</tr>
<tr>
<td>$L$ (length) $80$ km</td>
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</tbody>
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pumped DRA for different values of wavelength separation. For the three values of wavelength separation, i.e., 1 nm, 2.5 nm, and 5 nm, keeping the wavelength range (1515 nm–1575 nm) and other parameters the same as previous analysis, it can be seen that crosstalk decreases. This is because the number of channels in the system decreases from 60 channels for 1 nm spacing to 24 channels for 2.5 nm spacing and 12 channels for 5 nm spacing. Figure 5 shows the variation of SRS crosstalk standard deviation for backward pumped DRA for different values of wavelength separation. The results are similar to the analysis of forward pumped DRA, i.e., with the increase in wavelength separation, keeping wavelength range constant, crosstalk decreases.

In forward pumped DRA, minimum crosstalk occurs in the central wavelength for all the three values of wavelength separation and has a magnitude of 0.031 dB, 0.020 dB, and 0.015 dB for 1 nm, 2.5 nm, and 5 nm wavelength separations, respectively. In backward pumped DRA, minimum crosstalk of $1.986 \times 10^{-5}$ dB for 1 nm, $1.308 \times 10^{-5}$ dB for 2.5 nm, and $0.959 \times 10^{-5}$ dB for 5 nm wavelength separation occurs at around 1545 nm.

In forward pumped DRA, it is observed that the amount of increase in crosstalk is equal for all wavelengths when wavelength separation is increased by a given amount. When the change in wavelength separation is from 1 nm to 2.5 nm, crosstalk decreases by almost 0.015 dB for all wavelengths. Similarly, when wavelength separation is increased from 2.5 nm to 5 nm, crosstalk decreases by almost 0.007 dB. Similarly, in the analysis of backward pumped DRA when wavelength separation was increased from 1 nm to 2.5 nm crosstalk decreased by 0.001 dB for all wavelengths. Similarly, when wavelength separation was increased from 2.5 nm to 5 nm crosstalk decreased by 0.0005 dB for all wavelengths. Thus, all wavelengths suffered crosstalk change by almost similar amount.

Figures 6 and 7 show variation of SRS crosstalk standard deviation with input power of the signals for forward and backward pumped DRA, respectively. The values of input power considered are 0 dBm, 5 dBm, and 10 dBm. With the increase in signal strength, crosstalk increases for both forward and backward pumped DRA. In case of forward pumped DRA minima occurs in the central wavelength for all three cases of power input. But unlike the case of increase in wavelength separation, the amount of increase is not equal for all signals when power is increased by equal amount. Longer and shorter wavelengths suffer greater increase in crosstalk than central wavelengths. In the analysis of

![Figure 4: Variation of SRS crosstalk standard deviation (dB) with signal wavelength for Δλ = 1, 2.5, and 5 nm wavelength separation for forward pumped DRAs.](image)

![Figure 5: Variation of SRS crosstalk standard deviation (dB) with signal wavelength for Δλ = 1, 2.5 and 5 nm wavelength separation for backward pumped DRAs.](image)

![Figure 6: Variation of SRS crosstalk standard deviation (dB) with signal wavelength for power input = 0, 5, and 10 dBm for forward pumped DRAs.](image)

![Figure 7: Variation of SRS crosstalk standard deviation (dB) with signal wavelength for power input = 0, 5, and 10 dBm for backward pumped DRAs.](image)
backward pumped DRA, crosstalk was found to increase by a factor of 10 for all signal wavelengths when input power of the signal increases from 0 to 10 dBm.

It is observed from Figs. 8 and 9 that with the increase in bit rate of the system SRS crosstalk variance decreases for forward and backward pumped DRA. The SRS crosstalk suffered by each channel almost halves for increase in bit rate of system from 2.5 Gbps to 10 Gbps and from 10 Gbps to 40 Gbps for forward pumped DRA and backward pumped DRA.

Using these results, system bound for a typical DRA configuration given in Fig. 1 is evaluated theoretically and is given in Figs. 10 and 11. It has been given in literature \[15,16\] that for lognormal distribution of SRS crosstalk and Gaussian distribution of noise, SRS crosstalk standard deviation should be less than 0.4 dB for power penalty to be less than 1 dB. Under the condition that \(\sigma_x < 0.4\) dB, we evaluate the average input power to the WDM system for different bit rates of the system. It can be seen from Fig. 10 that for forward pumping configuration, the limits of average input power increases with the increase in bit rate of the system. Similarly as seen from Fig. 11 for backward pumping configuration the limits of average input power increases with the increase in bit rate of the system.

4. Conclusion
In this paper a closed formed formula has been derived to estimate crosstalk due to SRS in a WDM system employing DRA. The derived formulae have then been applied to different pumping configurations of DRA and system bounds for a typical WDM system have been found. It is observed that backward pumped DRA suffers from less SRS crosstalk compared to forward pump DRA. It is found that with the increase in bit rate of the system, limits of average input power increases for backward pumping configuration and decreases for forward pumping configurations. The effect of wavelength separation between channels, input power, and bit rate of system on SRS crosstalk is also investigated. The results show that for both pumping scheme crosstalk decreases with increase in wavelength separation and increases with increase in power input. Moreover with the increase in bit rate of system crosstalk decreases for backward and forward pumped DRA.

One of the authors (Anamika) would like to acknowledge ISM, Dhanbad for encouragement and financial support.

Appendix A

\[ |Q(\omega)|^2 = |M + N|^2 = (M + N) \times (M + N)^* = |M|^2 + |N|^2 + MN^* + M^*N = |M|^2 + |N|^2 + 2 \times \text{Re}(MN^*). \]

Substituting the values of \(M\) and \(N\) from equation of \(Q(\omega)\) and simplifying we get the following terms.
Forward pumped DRA:

\[ |M|^2 = \frac{A^2}{a^2 + (d_{ij} \omega )^2} \left[ (1 - e^{-aL})^2 + 4e^{-aL} \sin^2 \left( \frac{d_{ij} \omega L}{2} \right) \right], \]  
\[ (A1) \]

\[ |N|^2 = \frac{A^2K_y^2}{4a^2 + (d_{ij} \alpha )^2} \left[ (1 - e^{-2aL})^2 + 4e^{-2aL} \sin^2 \left( \frac{d_{ij} \alpha L}{2} \right) \right]. \]  
\[ (A2) \]

\[ 2\text{Re}(MN^*) = \frac{2A^2K_y^2(2a^2 + d_{ij} \omega^2)}{(a^2 + d_{ij} \omega^2)^2} \left[ 1 + e^{-3aL} - (e^{-2aL} + e^{-aL}) \left( 1 - 2\sin^2 \left( \frac{d_{ij} \omega L}{2} \right) \right) \right. \]
\[ + \frac{ad_{ij} \omega \sin d_{ij} \omega L}{(a^2 + d_{ij} \omega^2)^2} \left( e^{-aL} - e^{-2aL} \right). \]  
\[ (A3) \]

Breaking the terms of Eq. \( (A3) \) into partial fractions

\[ \frac{2a^2 + d_{ij} \omega^2}{(a^2 + d_{ij} \omega^2)^2} = \frac{1}{3} \frac{a^2 + d_{ij} \omega^2}{a^2 + d_{ij} \omega^2} \]
\[ + \frac{2}{3} \frac{d_{ij} \omega^2}{a^2 + d_{ij} \omega^2}. \]

For integration, the following trigonometric formulae and integrals are used \[19\]

\[ \sin^2 \frac{aT}{2} \omega \times \sin^2 \frac{d_{ij} \omega L}{2} \]
\[ = \frac{1}{4} \left[ 2\sin^2 \frac{aT}{2} \omega + 2\sin^2 \left( \frac{d_{ij} \omega L}{2} \right) \right. \]
\[ - \sin^2 \left( T + d_{ij} L \right) \omega - \sin^2 \left( T - d_{ij} L \right) \omega \]
\[ = \frac{2}{4} \left( e^{-aL} + aLw - 1 \right). \]

Thus,

\[ \frac{4P_0^2T^2}{8\pi a} \int_{-\infty}^{\infty} \frac{\sin^2 \frac{\omega T}{2}}{\frac{\omega^2}{4}} |M|^2 \]
\[ = \frac{A^2P_0^2}{e^2L_w} \left[ (1 - e^{-aL})^2(e^{-aL} + aLw - 1) \right. \]
\[ + e^{-aL} \left( 2(e^{-aL} + e^{-2aL} - 1) - e^{-aL+L_w} \right) \]
\[ + e^{-aL-L_w} + \alpha([L + L_w] - |L - L_w|) \right]. \]  
\[ (A4) \]

\[ \frac{4P_0^2T^2}{8\pi \alpha} \int_{-\infty}^{\infty} \frac{\sin^2 \frac{\omega T}{2}}{\frac{\omega^2}{4}} |N|^2 \]
\[ = \frac{A^2K_y^2P_0^2}{8\pi e\alpha L_w} \left[ (1 - e^{-2aL})^2(e^{-2aL} + 2aLw - 1) \right. \]
\[ + e^{-2aL} \left( 2(e^{-2aL} + e^{-2aL} - 1) \right. \]
\[ - (e^{-2aL+L_w} + e^{-2aL-L_w}) \]
\[ + 2\alpha([L + L_w] - |L - L_w|) \right]. \]  
\[ (A5) \]

\[ \frac{4P_0^2T^2}{8\pi \alpha} \int_{-\infty}^{\infty} \frac{\sin^2 \frac{\omega T}{2}}{\frac{\omega^2}{4}} 2\text{Re}(MN^*) \]
\[ = \frac{-2A^2K_y^2P_0^2}{3\alpha eL_w} \left[ (1 + e^{-3aL} - e^{-2aL} - e^{-aL}) \right. \]
\[ \times (e^{-aLw} + aLw - 1) \]
\[ + \frac{1}{2} (e^{-aL} + e^{-2aL}) \left( 2(e^{-aLw} + e^{-aL} - 1) \right. \]
\[ - (e^{-aL+L_w} + e^{-aL-L_w}) + \alpha([L + L_w] - |L - L_w|) \right] \]
\[ - \frac{A^2K_y^2P_0^2}{6\alpha eL_w} \left[ (1 + e^{-3aL} - e^{-2aL} - e^{-aL}) \right. \]
\[ \times (e^{-2aLw} + 2aLw - 1) \]
\[ + \frac{1}{2} (e^{-aL} + e^{-2aL}) \left( 2(e^{-2aLw} + e^{-2aL} - 1) \right. \]
\[ - (e^{-2aL+L_w} + e^{-2aL-L_w}) \]
\[ + 2\alpha([L + L_w] - |L - L_w|) \right]. \]  
\[ (A6) \]
The SRS crosstalk variance of DRA using a forward pumping technique is given by adding the three Eqs. (A4)–(A6):

\[
\sigma^2 = K_1 \{ (1 - e^{-\alpha L})^2 P_1(\alpha) + e^{-\alpha L} P_2(\alpha) \} + K' \{ (1 - e^{-2\alpha L})^2 P_1(2\alpha) + e^{-2\alpha L} P_2(2\alpha) \} - \frac{2K'}{3} \{ M_\sigma P_1(\alpha) + N_\sigma P_2(\alpha) \} - \frac{K'}{6} \{ M_\sigma P_1(2\alpha) + N_\sigma P_2(2\alpha) \} - \frac{K'}{3} \{ \text{sign}(d_{ij} L)(2 - 2e^{-\alpha L}) + \text{sign}(\gamma)(1 - e^{-|\gamma|/2}) + \text{sign}(\beta)(1 - e^{-|\beta|/2}) \} + \frac{K'}{12} \{ \text{sign}(d_{ij} L)(2 - 2e^{-2\alpha L}) + \text{sign}(\gamma)(1 - e^{-|\gamma|/2}) + \text{sign}(\beta)(1 - e^{-|\beta|/2}) \}.
\]

(A7)

Different terms used in the above equation are \( K_1 = \frac{A^2 P_2}{a L_{\text{nw}}} \); \( P_1(\alpha) = (e^{-\alpha L_w} + \alpha L_w - 1) \); \( P_2(\alpha) = 2(e^{-\alpha L_w} + e^{-\alpha L_w} - 1) - (e^{-\alpha L_w} + e^{-\alpha L_w} + \alpha(L + L_w) - |L - L_w|) \); \( M_\sigma = (1 + e^{-3\alpha L} - e^{-2\alpha L} - e^{-\alpha L}) \); \( N_\sigma = \frac{1}{2} (e^{-\alpha L} + e^{-2\alpha L}) \); \( \gamma = T - d_{ij} L \); \( \beta = T + d_{ij} L \); \( L_w = T / |d_{ij}| \); \( A = Ke^{K'} \); \( K' = \frac{A e^{K}}{a \tau} \).

Backward pumped DRA:

\[
|M|^2 = \frac{A^2}{\sigma^2 + (d_{ij} \omega)^2} \left( (1 - e^{-\alpha L})^2 + 4e^{-\alpha L} \sin^2 \left( \frac{d_{ij} \omega L}{2} \right) \right),
\]

(A8)

\[
\frac{4P_0^2 T^2}{8 \pi T} \int_{-\infty}^{\infty} \frac{\sin^2 \omega T}{\omega^2} |M|^2 = \frac{A^2 P_2}{a \tau L_{\text{nw}}} \left( 1 - e^{-\alpha L} \right)^2 \times (e^{-\alpha L_w} + e^{-\alpha L_w} - 1) + e^{-\alpha L} (2(e^{-\alpha L_w} + e^{-\alpha L_w} - 1) - (e^{-\alpha L_w} + e^{-\alpha L_w} + \alpha(L + L_w) - |L - L_w|) + \alpha(L + L_w - |L - L_w|) \right).
\]

\[
|N|^2 = \frac{A^2 K^2 e^{-2\alpha L}}{(d_{ij} \omega)^2} \left[ 4 \sin^2 \left( \frac{d_{ij} \omega L}{2} \right) \right],
\]

(A9)

\[
2 \text{Re}(MN^*) = -\frac{2A^2 K'}{(\sigma^2 + (d_{ij} \omega)^2)} \left( (1 - e^{-\alpha L})^2 \left( \cos d_{ij} \omega L - 1 \right) \right) \times \left\{ -ad_{ij} \omega \sin d_{ij} \omega L (1 - e^{-\alpha L}) \right\}.
\]

(A10)

For integration the following trigonometric formulae and integrals are used [19]:

\[
\int_0^\infty \frac{\sin^2 ax \sin bx}{x^2 + c^2} \, dx = \frac{\pi}{8c^2} \left[ \text{sign}(b) \{ 2 - 2e^{-|b|c} \} + \text{sign}(2a - b) \{ 1 - e^{-2|a-b|c} \} - \text{sign}(2a + b) \{ 1 - e^{-2|a+b|c} \} \right].
\]

\[
\int_0^\infty \frac{\sin^2 ax \cos bx}{x^2 + c^2} \, dx = \frac{\pi}{8c^2} \left[ -2|b||c| + |2a - b||c| + |2a + b||c| - e^{-2|a-b|c} \right].
\]

\[
\int_0^\infty \frac{\sin^2 ax \sin^2 bx}{x^4} \, dx = \frac{\pi}{6} \min(a^2, b^2) \left[ 3 \max(a, b) - \min(a, b) \right].
\]
\[
\frac{4P_0^2T^2}{8\pi T} \int_0^\infty \frac{\sin^2 \frac{\omega T}{2}}{\omega^2} \frac{d^2 \Psi}{\omega^2} \left( \frac{\partial^2}{\partial x^2} + K_{\text{L}} - \frac{\pi}{V} \right)
\]

\[
= -A^2 K' e^{-al} P_0^2 \left[ (1 + e^{-al}) (-2al + |L_w - L||\alpha|)
\right.
\]

\[
+ (1 + e^{-al})(e^{-2al}L_w - 2e^{-2al}L_w + e^{-2al}L_w - L_w - L||\alpha|)
\]

\[
+ \frac{2A^2 K' e^{-al} P_0^2}{\alpha^3 L_w} \left[ (1 + e^{-al})(e^{-2al}L_w + 2alw - 1)
\right.
\]

\[
+ \frac{A^2 K' e^{-al} P_0^2}{\alpha^3 L_w} \left[ (1 + e^{-al})F(T, d_j L)ight]
\]

\[
- \frac{A^2 K' e^{-al} P_0^2}{\alpha^3 L_w} \left[ K_4 \right.
\]

\[
= \frac{A^2 P_0^2}{\alpha^3 L_w} K_2 + \frac{8A^2 K' e^{-al} P_0^2}{3d_j^2 T} - K_5
\]

\[
- \frac{A^2 K' e^{-al} P_0^2}{\alpha^3 L_w} (K_4 + K_5) + \frac{2A^2 K' e^{-al} P_0^2}{\alpha^3 L_w} K_6.
\]

The SRS crosstalk variance of DRA using backward pumping technique is obtained by adding the Eqs. (A11)-(A13):

\[
\sigma_x^2 = \frac{A^2 P_0^2}{\alpha^3 L_w} K_2 + \frac{8A^2 K' e^{-al} P_0^2}{3d_j^2 T} - K_5
\]

\[
- \frac{A^2 K' e^{-al} P_0^2}{\alpha^3 L_w} (K_4 + K_5) + \frac{2A^2 K' e^{-al} P_0^2}{\alpha^3 L_w} K_6.
\]

Different terms used in the above equation are

\[
K_2 = [(1 - e^{-al})^2(e^{-al} + 2alw - 1)
\]

\[
+ e^{-al}(2(e^{-al} + 2alw - 1) - (e^{-al} + 2alw - 1)
\]

\[
+ \alpha(L + L_w - |L - L_w|)]
\]

\[
K_3 = \min \left( \frac{T^2}{4}, \frac{d_j^2 L_w^2}{4} \right)
\]

\[
\times \left\{ 3 \max \left( \frac{T}{2}, \frac{d_j L_w}{2} \right) - \min \left( \frac{T}{2}, \frac{d_j L_w}{2} \right) \right\}
\]

\[
K_4 = (1 + e^{-al}) \left\{ -2aL + |L_w - L||\alpha| + |L_w + L||\alpha| - 2e^{-2al} + e^{-|L_w - L||\alpha|} + e^{-|L_w + L||\alpha|} \right\}
\]

\[
K_5 = (1 - e^{-al}) \left\{ \text{sign}(d_j L)(2 - 2e^{-al}) + \text{sign}(T - d_j L)(e^{-|L_w - L||\alpha|}) + \text{sign}(T + d_j L)(e^{-|L_w + L||\alpha|}) \right\}
\]

\[
K_6 = \left\{ (1 + e^{-al})(e^{-2al}L_w + 2alw - 1) \right\}
\]

\[
A = Ke^{-K' e^{-al} P_0^2} \frac{d_j}{\alpha^3 T}
\]

References

Stimulated Raman crosstalk in bi-directional pumped distributed Raman amplifier for DPSK and OOK modulation format

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ABSTRACT

Novel closed form formulae are derived to study crosstalk degradation due to stimulated Raman scattering (SRS) in WDM systems employing bi-directional pumped distributed Raman amplifier (DRA). They are applied to evaluate the crosstalk performance of different modulation formats, especially differential phase-shift keying (DPSK) and ON–OFF keying (OOK), which are widely used in optical communication. Further, SRS crosstalk is evaluated for different data rates and pulse shapes prevalent in optical data transmission. Next, crosstalk is calculated for different pumping schemes as special cases of bi-directional pumped DRA. The study shows that minimum SRS crosstalk can be achieved for 40 Gb/s RZ-DPSK signal with 33.3% duty cycle in WDM system employing backward pumped DRA.

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1. Introduction

Wavelength division multiplexing (WDM) scheme is used to multiplex different information carrying wavelength channels on a single fiber [1]. The WDM scheme has many advantages in terms of effective cost and efficient performance. However, it is limited by various factors such as attenuation, dispersion and nonlinear effects. Stimulated Raman scattering is one such nonlinear effect which greatly limits the system performance [2–11]. Stimulated Raman scattering (SRS) is an inelastic effect that causes transitions in energy states due to interactions of photons with molecules. In a WDM system, SRS causes power transfer from a lower wavelength to a higher wavelength resulting in crosstalk.

The limitation of attenuation is overcome in N-channel WDM system by using lumped and distributed amplifiers such as erbium-doped fiber amplifier (EDFA) and Raman amplifier (RA). In distributed Raman amplifier (DRA), signals are amplified using high power pumps which co-propagate (forward pumping) or counter propagate (backward pumping) with the signals. As the gain spectrum of Raman amplifier depends on pump wavelength, it gives flexibility to achieve gain at any desired signal wavelength. In order to achieve broadband signal gain, “WDM pumping” [12,13] is used which incorporates multiple high power pumps separated in wavelength by a few nanometers.

Design of an efficient WDM optical communication system depends on the judicious choice of modulation format and pulse shape. The commonly used modulation formats in optical communication are ON–OFF keying (OOK) and differential phase-shift keying (DPSK). In the present work, OOK and DPSK are considered with both modulation formats having non-return-to-zero (NRZ) and return-to-zero (RZ) pulse shapes with different duty cycles. There has been considerable research work in the field of optical communication which focuses on comparison of OOK and DPSK modulation format and pulse shape for WDM system employing EDFA [14–23].

Raman crosstalk in WDM system with lumped amplifier has been exhaustively studied both theoretically and experimentally [24–26]. In one of such studies, soliton propagation in optical fiber communication systems was considered taking into account the effects of delayed Raman response and the random character of pulse sequences [27]. It was concluded that in such systems the Raman-induced energy exchange due to collisions leads to lognormal statistics for the pulse amplitudes. The PDF of SRS crosstalk has been approximated by lognormal distribution [24,28–30]. In Ref [30], it is shown that in order to use PDF for system performance evaluation, methods to evaluate standard deviation is essential. In the current work, standard deviation of SRS crosstalk in DRA is calculated which will be supportive for characterizing the PDF of SRS crosstalk.

Raman crosstalk dynamics in OOK soliton WDM systems demonstrated the combined effect of delayed Raman response and bit pattern randomness on pulse propagation in optical fiber communication systems [27–29,31]. The propagation was described...
by a perturbed stochastic nonlinear Schrödinger equation and extensive numerical simulations were performed with the model to analyze the dynamics of the frequency moments, the bit-error rate, and the mutual distribution of amplitude and position. In another study DPSK modulation was considered and stability of WDM DPSK transmission against Raman crosstalk effects under the influence of deterministic effects of inter-pulse Raman crosstalk in amplified WDM optical fiber transmission systems [32] was quantitatively explained. The above studies along with other reported literatures [14–23], show the applicability of OOK and DPSK modulation format for distributed Raman amplified WDM system.

In the recently published work [33] SRS crosstalk is evaluated in forward and backward pumped DRA. In the paper, performance of backward and forward pumped DRA had been investigated and system bounds were found for a typical WDM system employing DRA. In the present work, the statistical method has been used to develop closed form formulae to study the performance of different modulation formats prevalent in optical communication system. The generalized formulae are derived for WDM system employing bi-directional pumped DRA. The forward and backward pumpings are considered to be special cases of bi-directional pumping. To the best of authors’ knowledge no work has been reported to evaluate the effect of modulation format and pulse shape on crosstalk behavior of WDM system with DRA.

In this paper, using statistical methods, close form formulae have been derived to evaluate SRS crosstalk in WDM system employing bi-directional pumped DRA. In DRA, high power pumps provide a continuous gain to compensate for the attenuation of signals. Since the gain is continuously increasing along the transmission line, power penalty due to SRS crosstalk has been calculated for variable gain along the entire length of fiber. A detailed study using the closed formed formulae has been done on different modulation formats, pulse shapes and pumping scheme. On the basis of the study we have shown that 40 Gb/s RZ-DPSK signal in WDM system employing backward pumped DRA is most tolerant to SRS crosstalk.

In literature, results are available that show that error generation can be due to interplay between SRS induced cross frequency shift (XFS) and SRS crosstalk rather than SRS crosstalk alone [27,31]. Moreover perturbations like cross phase modulation (XPM), Raman self frequency shift (SFS) and Raman cross frequency shift (XFS) also contribute to signal degradation and set a bound on the minimum frequency spacing for stable transmission due to Raman induced interplay between amplitude and frequency dynamics [32]. The above effects in DRA are not considered in the present work but will be considered in future work.

The present work based on statistical method [30,33] is evolved to study the signal crosstalk due to SRS in DRA. The amplified noise is not considered in present analysis as it is presumed that transfer of noise from one channel to other due to SRS will be low and will affect the overall noise in the channel instead of affecting the signal degradation due to crosstalk. Hence the equations developed by us do not incorporate the effect of noise [34] but are valid for evaluation of SRS crosstalk in DRA. The present statistical approach to analyze SRS ignores the effect of group velocity dispersion (GVD) on interchannel power transfer [30].

The paper is organized as follows. In Section 2 closed formed formulae have been developed to study crosstalk in WDM system employing bi-directional pumped DRA. Section 3 uses the closed formed formulae to evaluate crosstalk in a typical configuration of WDM system and analyze the performance of different modulation formats and pulse shapes. Section 4 concludes the paper.

2. Theory

In this section, crosstalk variance is derived in a distributed Raman amplifier for bi-directional pumping scheme. A single span of DRA of length $l$ with no repeater is considered. In $N$-channel WDM systems, SRS causes power change in $i$th channel of the WDM system due to power amplification by shorter wavelength channels and depletion by longer wavelength channels. Thus optical power of the $i$th channel after transmission through distance $z$ is given as [30]

$$P_i(z,t) = P_i(0,t) \exp(-az - x_i(z,t)) \tag{1}$$

$P_i(0,t)$ is the input at $z=0$ and $t = t - z/v_i$ with $v_i$ being the group velocity of $i$th channel and $a$ is the fiber attenuation coefficient.

In (1), it has been assumed that fiber dispersion causes just pulse walk-off and no pulse distortion or inter-pulse collision [14,25,30].

$$X_i(z,t) = \sum_{j=1}^{N} x_{ij}(z,t) \tag{2}$$

where $x_{ij}(z,t)$ is the power change in the $i$th channel due to remaining $(N-1)$ channels induced by SRS. In (2)

$$x_{ij}(z,t) = \sum_{k=0}^{\infty} b_k q_{ij} \left( t - z - kT \right) \tag{3}$$

where $b_k=\{0,1\}$ is random transmission data and thus $x_{ij}$ is considered to be a random variable. $v_i$ is group velocity of the $i$th channel and $T$ is bit period. In (3)

$$q_{ij}(t) = K \int_0^L p(t-dz) e^{-az} g_i(z') g_{j}(z') dz' \tag{4}$$

where

$$K = \frac{g_{jk} \int f_i - f_j |f_i - f_j| \leq 15 \text{ THz}}{\int_{f_j} f_i} \tag{5}$$

In (4), $p(t)$ is pulse shape and $d_{ij} (=1/v_i - 1/v_j)$ is propagation time difference between the two different channels $(ij)$ during a unit length transmission. In (5), the Raman gain function is approximated by triangular function and $g_{ik}$ is the average slope of the triangular profile [26]. The frequency of $i$th (jth) channel is $f_i$ ($f_j$), polarization constant $\psi$ is equal to 2 and $A_{eff}$ is effective area of cross-section. The parameter $K$ takes a positive (negative) value if $f_j$ is greater (less) than $f_i$ which implies that optical power $P_i$ is depleted (amplified) by the jth channel. If $f_i=f_j$ then $K$ becomes zero and there is no power exchange between the channels.

A typical DRA configuration is shown in Fig. 1. It shows a number of signals at different wavelengths multiplexed in a single DRA. Each wavelength represents an independent communication channel being amplified by the DRA and ultimately bounded on the minimum frequency spacing for stable transmission (XPM), Raman self frequency shift (SFS) and Raman cross frequency shift (XFS) and SRS crosstalk rather than SRS crosstalk alone [27,31]. Moreover perturbations like cross phase modulation (XPM), Raman self frequency shift (SFS) and Raman cross frequency shift (XFS) also contribute to signal degradation and set a bound on the minimum frequency spacing for stable transmission due to Raman induced interplay between amplitude and frequency dynamics [32]. The above effects in DRA are not considered in the present work but will be considered in future work.

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The paper is organized as follows. In Section 2 closed formed formulae have been developed to study crosstalk in WDM system employing bi-directional pumped DRA. Section 3 uses the closed formed formulae to evaluate crosstalk in a typical configuration of WDM system and analyze the performance of different modulation formats and pulse shapes. Section 4 concludes the paper.

![Fig. 1. Schematic diagram of bi-directional multi-pumped DRA.](image-url)
demultiplexed at the output. Forward amplification gain [35] is

\[ g_f(x') = \exp \left( \gamma \left( \sum \frac{\gamma' P_{so} + P_{po}}{A_{eff} x'} \right)(1 - e^{-x'}) \right) \]  \hspace{1cm} (6)

where peak Raman gain \( \gamma = 6.5 \times 10^{-14} \) m/W, \( P_{po} \) and \( P_{so} \) are the initial pump and signal power respectively, and \( x' \) is signal (pump) wavelength.

Backward amplification gain [35] is

\[ g_b(x') = \exp \left( \gamma \left( \sum \frac{\gamma' P_{so} + P_{po}}{A_{eff} x'} \right)e^{\gamma(x' + d)} - e^{-x'} \right) \]  \hspace{1cm} (7)

where \( P_{bo} \) is initial backward pump power at \( L \) where \( L \) is the length of DRA and other parameters are the same as in (6).

2.1. Variance of SRS crosstalk for bi-directional pumped DRA

The variance of SRS crosstalk is calculated for the bi-directional pumped DRA shown in Fig. 1.

The variance of SRS crosstalk is given as [30]

\[ \sigma_s^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_s(\omega)d\omega \]  \hspace{1cm} (8)

where \( S_s(\omega) \) is the power spectrum of \( x(t) \) (given in Eq. (1)) and is given as

\[ S_s(\omega) = \frac{\sigma_s^2}{T_b} (Q_{ij}(\omega))^2 + \frac{2\pi m_b^2}{T^2} \sum_{k = -\infty}^{\infty} \left| Q_{ij}(2k\pi/T) \right|^2 \delta(\omega - 2k\pi/T) \]  \hspace{1cm} (9)

where \( \sigma_s^2 = E[b_k^2] - \langle E[b_k] \rangle^2 = \frac{1}{4} \langle E[b_k] \rangle \) is the expected mean of \( b_k(0,1) = 1/2 \).

\( T_b = 2T \) for DPSK signal as one symbol duration (\( T_b \)) is equal to two bits duration (\( 2T \)) in DPSK modulation format.

\( T_b = T \) for OOK signal as one symbol duration (\( T_b \)) is equal to one bit duration (\( T \)) in OOK modulation format.

\( Q_{ij}(\omega) \) is the Fourier transform of \( q_j(t) \) and is given as

\[ Q_{ij}(\omega) = K \int_{-\infty}^{\infty} P(\omega)e^{-j\omega t}e^{-j\omega t}e^{i\omega t}e^{i\omega t}e^{j\omega t}e^{j\omega t} \]  \hspace{1cm} (10)

where

\[ K' = \left( \frac{\gamma' P_{so} + P_{po}}{2A_{eff}} \right) \]

and \( K'' = \left( \frac{\gamma' P_{so} + P_{po}}{2A_{eff}} \right) \)

Substituting \( \zeta' = \ln(t), \zeta'' = \frac{1}{t} \)

The equation becomes

\[ Q_{ij}(\omega) = K(\omega)e^{K'}e^{-K'x''} \int_{1}^{t} e^{-j\omega t}e^{K'x''}e^{-K'x'}e^{j\omega t} dt \]  \hspace{1cm} (11)

In (11) the exponential \( e^{-K'r^2 + K't^2}e^{-x''} \) is binomially expanded to be the sum of two terms \( (1 - K't^2 + K't^2) \) and the higher order terms are neglected which contribute insignificantly in the series expansion.

\[ Q_{ij}(\omega) = K(\omega)e^{K'} \int_{1}^{t} \frac{1 - K't^2 + K't^2e^{-x''}}{rt^2 + j\omega t^2} dt \]  \hspace{1cm} (12)

where

\[ C = Ke^{K'}e^{-K'x''} \]

Integrating and substituting the limits we get

\[ Q_{ij}(\omega) = CP(\omega) \left[ 1 - e^{-j(\omega t^2)} \right] - CK P(\omega) \left[ 1 - e^{-j(\omega t^2)} \right] \]  \hspace{1cm} (13)

A rectangular NRZ pulse is given as

\[ p(t) = \begin{cases} 2P_0 & (|t| < T/2) \\ 0 & (|t| > T/2) \end{cases} \]

Fourier transform of \( p(t) \) is given as

\[ |P(\omega)|^2 = 2P_0 T \sin(\omega T/2)/(\omega T/2) \]

An RZ pulse of duty cycle \( \tau_b \) is given by the same equation as above but with \( T \) replaced by \( T/\tau_b \) where

\( \tau_b = 2 \) for 50% duty cycle

\( \tau_b = 3 \) for 33% duty cycle

The second term of \( S_s(\omega) \) is equal to zero for rectangular pulse shape at \( k = 1, 2, 3, \ldots \) and very small for nonrectangular pulse shape.

The expressions of \( Q_{ij}(\omega) \) and \( S_s(\omega) \) are substituted in \( S_s(\omega) \) and the resulting expression of \( S_s(\omega) \) is substituted in the equation of variance \( (\sigma_s^2) \). The infinite integral is performed using table of integral and series [36] and the result is given in Appendix I.

3. Results and discussion

Fig. 1 shows the DRA configuration considered for our analysis. The interfering channels of WDM system are assumed to transmit 0 dBm power in the wavelength range of 1515–1595 nm with inter-channel separation of 1 nm. The multi-pumps consisting of 8 pumps are in the wavelength range of 1420–1490 nm, separated by 10 nm and each having power of 20 dBm yielding wideband and flat gain spectrum in the signal wavelength range of 1515–1595 nm. Each pump provides peak Raman gain to signal at a shifted wavelength of 100 nm from pump wavelength and has a gain spectrum of 10 nm. Other parameters used for calculation are given in Table 1.

The configuration of bi-directional pumped DRA shown in Fig. 1 reduces to a single span of single mode fiber by taking forward and backward gain \( g_f \) and \( g_b \) as zero. In Fig. 2, the dashed curve shows the results obtained by integrating Eq. (4) for zero gain and coincides with that obtained by using equation in Ref. [37] for single span of single mode. This shows the validity of our approach. Pumps interact among themselves in a similar way as do signals propagating in a fiber i.e. pumps at lower wavelengths lose power to higher wavelengths. The interaction among pumps is neglected in our analysis. Our work focuses on crosstalk between signals as they are continuously pumped while propagating in the fiber.

Figs. 3 and 4 show the variation of SRS crosstalk with bit rate of the system for bi-directional pumped DRA for NRZ-OOK and NRZ-DPSK modulation format respectively. Other parameters are

<table>
<thead>
<tr>
<th>Parameters of SMF [37]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope of Raman gain curve</td>
</tr>
<tr>
<td>Length of fiber (effective area)</td>
</tr>
<tr>
<td>Zero dispersion wavelength</td>
</tr>
<tr>
<td>Dispersion slope</td>
</tr>
<tr>
<td>Attenuation co-efficient</td>
</tr>
<tr>
<td>Length (l)</td>
</tr>
</tbody>
</table>
the same as in the previous analysis. It can be observed from the figure that with the increase in bit rate of the system, SRS crosstalk decreases. This may be because the walk-off length \( L_W \) given as \( T/D \Delta \lambda \) varies inversely with fiber dispersion coefficient and bit rate of the system. Hence it is expected that in the WDM system employing DRA, variance of SRS crosstalk will decrease with the increase in bit rate of the system for constant fiber dispersion coefficient.

Figs. 5 and 6 show the variation of SRS crosstalk with respect to signal wavelength for different pulse shapes of OOK and DPSK modulation formats respectively for bi-directional pumped DRA at 40 Gb/s optical data transmission. Other parameters are the same as in the previous analysis. As seen from the figure, with the decrease in duty cycle of the pulse, crosstalk decreases. The amount of crosstalk suffered by the lowest wavelength (1515 nm) which is the case of worst crosstalk decreases by almost 30% from NRZ pulse shape to RZ pulse shape and almost 18% from RZ pulse shape with 50% duty cycle to RZ pulse shape with 33% duty cycle for both the modulation formats. It can be concluded that SRS crosstalk suffered by a pulse varies inversely with duty cycle \( \tau_b \) of the pulse. Hence RZ pulse with small duty cycle suffers minimum crosstalk. On comparing Figs. 3 and 4 it is found that all the pulse shapes suffer almost 30% decrease in crosstalk from OOK to DPSK modulation format. The reason for the DPSK modulation format suffering less crosstalk stems from the fact that in DPSK signal one symbol duration \( T_b \) is equal to two bit duration \( 2T \), while in OOK signal one symbol duration \( T_b \) is equal to one bit duration \( T \). Thus, it can be concluded that the variance of SRS crosstalk depends inversely on symbol duration.

On the basis of above analysis it can be observed that 40 Gb/s RZ-DPSK signal (33% duty cycle) suffers from minimum SRS crosstalk. Using the above observation, variation of SRS crosstalk with signal wavelength is studied for 40 Gb/s RZ-DPSK signal (33% duty cycle) for different pumping schemes. The expression of variance of SRS crosstalk can be used to investigate crosstalk in forward pumped DRA by making backward pump \( P_{B0} \) strength equal to zero. Similarly to study crosstalk in backward pumped DRA, the strength of forward pumps \( P_{F0} \) is made equal to zero.
controlling SRS crosstalk. Among the three pumping schemes, minimum crosstalk is observed for the WDM system employing backward pumping scheme. The results based on the statistical analysis show that a 40 Gb/s RZ-DPSK signal (33.3% duty cycle) offers the best SRS crosstalk performance in a WDM communication system. Though the crosstalk estimated is in the order of $10^{-3}$ dB, it is significant to affect the system performance. For our analysis we have taken SMF fiber of length 80 km. But practical long haul WDM systems run up to thousands of kilometers and have dispersion managed systems to minimize dispersion. When SMF and NDF are placed alternately, the correlation between SRS crosstalk in every individual fiber segment becomes large and SRS crosstalk increases. Undoubtedly crosstalk due to SRS is lower compared to XPM and FWM but nonetheless it is not insignificant. XPM and SRS interact among themselves either constructively or destructively, depending on the dispersion managed scheme and the relative wavelength location between pump and probe. Resonance effect between fiber spans lead to significant rise in SRS crosstalk. On the contrary, XPM can be minimized effectively by dispersion managed scheme due to weaker phase to intensity noise conversion through residual dispersion. Intensity modulation due to SRS gets converted to phase modulation due to GVD and will further debilitate the system. The study of impact of SRS on dispersion managed system employing DRA is proposed for future work.

4. Conclusion

SRS crosstalk in WDM system employing bi-directional pumped DRA has been evaluated using closed form formulae and performances of different modulation formats (DPSK and OOK) and pulse shapes (RZ and NRZ) have been compared. It is found that 40 Gb/s RZ-DPSK signal has the best crosstalk performance compared to 10 Gb/s NRZ DPSK, 40 Gb/s RZ-OOK and 40 Gb/s NRZ-DPSK signals. The duty cycle of pulse also helps in controlling SRS crosstalk. Among the three pumping schemes, minimum crosstalk is observed for the WDM system employing backward pumping scheme. The results based on the statistical analysis show that a 40 Gb/s RZ-DPSK signal (33.3% duty cycle) offers the best SRS crosstalk performance in a WDM communication system. Though the crosstalk estimated is in the order of $10^{-3}$ dB, it is significant to affect the system performance. For our analysis we have taken SMF fiber of length 80 km. But practical long haul WDM systems run up to thousands of kilometers and have dispersion managed systems to minimize dispersion. When SMF and NDF are placed alternately, the correlation between SRS crosstalk in every individual fiber segment becomes large and SRS crosstalk increases. Undoubtedly crosstalk due to SRS is lower compared to XPM and FWM but nonetheless it is not insignificant. XPM and SRS interact among themselves either constructively or destructively, depending on the dispersion managed scheme and the relative wavelength location between pump and probe. Resonance effect between fiber spans lead to significant rise in SRS crosstalk. On the contrary, XPM can be minimized effectively by dispersion managed scheme due to weaker phase to intensity noise conversion through residual dispersion. Intensity modulation due to SRS gets converted to phase modulation due to GVD and will further debilitate the system. The study of impact of SRS on dispersion managed system employing DRA is proposed for future work.

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Appendix I

$$|Q_{ij}(\omega)|^2 = |M + N + O|^2 = (M + N + O)^2$$
$$= |M|^2 + |N|^2 + |O|^2 + MN^* + M^*N + MO^* + M^*O + ON^* + O^*N$$
$$= |M|^2 + |N|^2 + |O|^2 + 2 \text{Re}(MN^*) + 2 \text{Re}(MO^*) + 2 \text{Re}(NO^*)$$

Substituting the values of $M$, $N$ and $O$ from the equation of $Q_{ij}(\omega)$ and simplifying we get the following terms:

$$|M|^2 = \frac{C^2}{\sigma^2 + (d_{ij}\omega)^2} \left[ 1 - e^{-zL} \right]^2 + 4e^{-zL} \sin^2 \left( \frac{d_{ij}\rho\omega L}{2} \right)$$

(A.1)

$$|N|^2 = \frac{C^2K^2}{4\sigma^2 + (d_{ij}\omega)^2} \left[ 1 - e^{-2zL} \right]^2 + 4e^{-2zL} \sin^2 \left( \frac{d_{ij}\rho\omega L}{2} \right)$$

(A.2)

$$|O|^2 = \frac{C^2K^2e^{-2zL}}{(d_{ij}\omega)^2} \sin^2 \left( \frac{d_{ij}\rho\omega L}{2} \right)$$

(A.3)

$$2\text{Re}(MN^*) = \frac{2C^2K^2}{\sigma^2 + (d_{ij}\omega)^2} \left[ 1 + e^{-zL} - (e^{-zL} + e^{-2zL}) \right]$$
$$\times \left[ 1 - 2\sin^2 \left( \frac{d_{ij}\rho\omega L}{2} \right) \right] + \frac{2C^2K^2d_{ij}\rho\omega\sin d_{ij}\rho\omega L}{\sigma^2 + (d_{ij}\omega)^2} \left[ 1 + e^{-zL} - (e^{-zL} + e^{-2zL}) \right]$$

(A.4)
\[2\text{Re}(M^*) = \left(\frac{2C^2K e^{-2t}}{\left(2x^2 + d_{q}'^2 \sigma^2 \right)} \right) \left(1 + e^{-x^2} \right) d_q^2 \sigma^2 (1 - \cos \theta_0 L) + 2d_q \cos \sigma_0 L (1 - e^{-x^2}) \right) \]
\[\text{(A.5)}\]

\[2\text{Re}(N^*) = \left[\frac{2C^2 K e^{-2x^2}}{\left(4x^2 + d_q'^2 \sigma^2 \right)} \right] \left(1 + e^{-x^2} \right) d_q^2 \sigma^2 (1 - \cos \theta_0 L) + 2d_q \cos \sigma_0 L (1 - e^{-x^2}) \right] \]
\[\text{(A.6)}\]

The expressions of \(Q_0(\sigma), \sigma_0^2, T_b\) are substituted in \(S_0(\sigma)\) and the resulting expression of \(S_0(\sigma)\) is substituted in equation of variance \(\sigma_Y^2\). On performing the infinite integration for \(T_b = T\), we get the expression of variance \(\sigma_Y^2\) which is the sum of Eqs. (A7)–(A12).

\[
\frac{P^2}{8\pi T} \int_{-\infty}^{\infty} \sin^2 \theta / 4 \pi^2 T^2 \left[|M|^2 = \frac{C^2 P^2}{2\sigma L_w} \left(1 - e^{-x^2} \right)^2 P_1(\sigma) + e^{-x^2}P_2(\sigma) \right] \]
\[\text{(A.7)}\]

\[
\frac{P^2}{8\pi T} \int_{-\infty}^{\infty} \sin^2 \theta / 4 \pi^2 T^2 \left[|N|^2 = \frac{C^2 P^2}{8\sigma L_w} \left(1 - e^{-x^2} \right)^2 P_1(\sigma) + e^{-x^2}P_2(\sigma) \right] \]
\[\text{(A.8)}\]

\[
\frac{P^2}{8\pi T} \int_{-\infty}^{\infty} \sin^2 \theta / 4 \pi^2 T^2 \left[|O|^2 = \frac{8C^2 K P^2 e^{-2x^2}}{3d_q L_w} \left(\min \left(T^2, \frac{d_q^2 L_w^2}{d_q^2 L_w^2} \right) \right) \right] \]
\[\text{(A.9)}\]

\[
\frac{P^2}{8\pi T} \int_{-\infty}^{\infty} \sin^2 \theta / 4 \pi^2 T^2 \left[2\text{Re}(M^*) = \frac{-C^2 P^2}{2\sigma L_w} \left[1 + e^{-x^2} \right] P_2(\sigma) - \left(1 - e^{-x^2} \right) \left(\min \left(T^2, \frac{d_q^2 L_w^2}{d_q^2 L_w^2} \right) \right) \right] \]
\[\text{(A.10)}\]

\[
\frac{P^2}{8\pi T} \int_{-\infty}^{\infty} \sin^2 \theta / 4 \pi^2 T^2 \left[2\text{Re}(N^*) = \frac{C^2 K e^{-2x^2} P^2}{8\sigma L_w} \left[1 + e^{-x^2} \right] P_2(\sigma) - \left(1 - e^{-x^2} \right) \left(\min \left(T^2, \frac{d_q^2 L_w^2}{d_q^2 L_w^2} \right) \right) \right] \]
\[\text{(A.11)}\]

\[
\frac{P^2}{8\pi T} \int_{-\infty}^{\infty} \sin^2 \theta / 4 \pi^2 T^2 \left[2\text{Re}(O^*) = \frac{C^2 K e^{-2x^2} P^2}{8\sigma L_w} \left[1 + e^{-x^2} \right] P_2(\sigma) - \left(1 - e^{-x^2} \right) \left(\min \left(T^2, \frac{d_q^2 L_w^2}{d_q^2 L_w^2} \right) \right) \right] \]
\[\text{(A.12)}\]

where

\[
P_1(\sigma) = \left(e^{-2x^2} + 2L_w - 1 \right)
\]

\[
p_2(\sigma) = 2 \left(e^{-2x^2} + e^{-x^2} - 1 \right) - \left(1 - e^{-x^2} + e^{-2x^2} \right) \left(\min \left(T^2, \frac{d_q^2 L_w^2}{d_q^2 L_w^2} \right) \right)
\]

\[
p_3(\sigma) = \left[\left(\min \left(T^2, \frac{d_q^2 L_w^2}{d_q^2 L_w^2} \right) \right) \right] \left[\left(\min \left(T^2, \frac{d_q^2 L_w^2}{d_q^2 L_w^2} \right) \right) \right]
\]

\[
r_4(T, d_q L) = \left[\left(\min \left(T^2, \frac{d_q^2 L_w^2}{d_q^2 L_w^2} \right) \right) \right] \left[\left(\min \left(T^2, \frac{d_q^2 L_w^2}{d_q^2 L_w^2} \right) \right) \right]
\]

For integration the following trigonometric formulae and integrals are used [32]:

\[
\int_0^\infty \frac{\sin^2 ax}{x^2} \sin bx \ dx = \frac{\pi}{8 \pi^2} \left[ \sin(b) \left( \frac{2 - e^{-2b}}{b} \right) + 2 \sin(2a - b) \left( \frac{1}{b^2} \right) \right]
\]

and

\[
\sin^2 \theta = \frac{1}{4} \left( 2 \sin^2 \theta + 2 \sin^2 \theta \right)
\]
1. Introduction

Optical communication systems can support Tb/s capacities over long distances making them an ideal technology for high capacity wireline networks. With the growing bandwidth demand, there is a tremendous interest in increasing the transport capacity and transmission distance of WDM system. Expanding network functionality into the optical domain is another aim of fiber-optic communication research. Optical networks with high spectral efficiency are being designed to have higher per fiber transport capabilities as well as low cost per transmitted bits. Modulation format such as intensity and phase modulation formats is a key technology that enables the design of such networks. ON–OFF keying (OOK), carrier suppressed return to zero (CSRZ), Modified duobinary (MDB), Alternate mark inversion (AMI) are some examples of intensity modulation format. Similarly, Differential Phase shift keying (DPSK), Quadrature phase shift keying (QPSK) and differential quadrature phase shift keying (DQPSK) are some examples of phase modulation formats. In this research article, analytical expressions have been derived for XPM and SPM induced crosstalk for two modulation formats—one intensity modulation format i.e. OOK and other phase modulation format i.e. DPSK. The reason for choosing the two formats stem from the fact that in literature a number of research work is available that deals with the performance of the two formats [1–4]. Similar study for other modulation formats such as CSRZ, MDB, AMI, QPSK and DQPSK will be pursued in future.

Coherent detection system has gained considerable attention over past few years because of improved receiver sensitivity and increased spectral efficiency of wavelength division multiplexed (WDM) system compared to Intensity Modulation Direct Detection (IM/DD) systems [5]. The advantage of coherent detection techniques is that the amplitude and phase of the detected optical signal can be measured and hence information can be transmitted in the form of amplitude, phase or frequency of optical signal. When OOK and DPSK modulation formats are implemented for coherent systems, it is required that phase of the electrical field associated with optical signals remains constant as the detector response depends on the phase of the received signal. When large number of signals simultaneously propagates in wavelength division multiplexed (WDM) system, due to high power confinement, nonlinear effect comes into play. Cross-Phase modulation (XPM) and Self-Phase modulation (SPM) are two such nonlinear effects also known as optical Kerr effects [6–10]. The change in phase of channel is proportional to its own intensity in SPM and to the intensity of other channels in XPM. The phase of the signal gets affected due to XPM and SPM is denoted as crosstalk or nonlinear phase shift and hence the detector response changes in coherent detection system.

Chromatic dispersion is referred to broadening of input signal as it travels down the fiber length. It is the second derivative of optical phase with respect to optical frequency. The interaction between nonlinearity and dispersion is an important issue in the design of Lightwave system. Phase modulation of signals in
WDM system due to SPM and XPM gets converted to intensity modulation through dispersion and thus results in waveform distortions. Depending on fiber chromatic dispersion and its management, XPM induced nonlinear phase shift may become very detrimental for WDM signals [11–13]. In WDM transmission systems, XPM induces a broadening of the signal spectrum and so wider optical filter bandwidth is required at the receiver. This degrades the system performance, because more spontaneous emission noise enters the receiver [14]. Moreover, the damaging effects of dispersion become more dominant on wider spectrum of signal and hence system performance degrades further. It has been found that XPM-induced signal broadening is similar to the one induced by laser phase noise. In the decision circuit of the receiver [5], the phase fluctuations cause error in decision making. The phase fluctuation due to laser phase noise is minimized by using semiconductor laser whose line width is a small fraction of bit rate. But phase fluctuation due to XPM induced signal broadening still remains and causes error in the signal detection.

Electronic predistortion (EPD) of chromatic dispersion using digital signal processing is a cost-effective alternative to conventional optical dispersion compensation (ODC) using inline dispersion compensating fiber [15]. The inline dispersion compensation in ODC systems is replaced by an EPD transmitter that precompensates the individual WDM channels for the chromatic dispersion of the entire transmission distance. EPD has been experimentally demonstrated for 10 Gb/s single channel and WDM system [16,17]. Research experiments [18,19] and simulations [20] have demonstrated that EPD systems are strongly degraded by SPM and XPM compared to ODC systems.

Optical amplifiers (OA) such as Erbium-doped fiber amplifiers (EDFA) and distributed Raman amplifier (DRA) are used to compensate attenuation of signals which is a major limiting factor in optical communication. In presence of OA, the nonlinear effects XPM and SPM are greatly enhanced. In OOA high power pump co- and counter propagate with the signal providing continuous gain all along the transmission line. Raman pumps also interact among themselves but the spacing between them is in the present work has been considered to be around 10 nm. At such large separation only stimulated Raman scattering (SRS) will be the dominant nonlinear effects and will cause power transfer between the pumps. XPM dominate at smaller interchannel spacing and will not be a detrimental effect. Another nonlinear effect FWM between pump–pump and pump-signal causes generation of new wave-lengths under the condition of phase matching. These new wave-lengths can be in the range of both signal wave-lengths and pump wave-lengths. But appropriate selection of fiber dispersion helps in minimizing FWM. Moreover, the effect has been reported to be more dominant in broad band amplifiers made from NZDF with zero dispersion wave-length located between pump and signal bands [21–24].

It has been found in research that when multiple pumps at different wave-length are launched into the fiber, the gain at shorter wave-length decreases whereas at longer wave-length increases. In present analysis this difference in gain is simplified by assuming equal gain for all pumps. Similar approach can be found in some previous works [25–29] in which when distributed Raman amplifier was analyzed, undepleted pump approximation was assumed as the intensity of pump is much higher than that of signal. This assumption is generally true for many practical cases and allows one to solve the analytical equation of evolution of pump power and signal power for a variety of pumping configuration. In the present work, undepleted pump approximation has been assumed for the derivation of analytical expression of crosstalk.

In our analysis noise power evolution and interpulse collision has not been considered. In Ref. [24], the author gives a detailed discussion of different types of noise occurring in DRA such as amplified spontaneous noise, signal-spontaneous beat noise, multi-path interference (MPI) noise, transfer of relative intensity of noise (RIN) from pump to signal. Some work has been done in this field such as Ref. [25] that gives an analytical study of RIN, Ref. [26] where MPI in DRA has been discussed. Again in Ref. [30], the author gives an analytical characterization of evolution of signal power and noise figure in forward pumped DRA. As can be seen in literature, consideration of noise requires detailed analysis and has hence been deferred by the authors for future. It has been found in research that for complete collisions, the collision-induced frequency shift of a pulse is negligible, whereas position shift is significant [31,32]. For strong dispersion management it has been found that incomplete collisions can be neglected, whereas for weak dispersion management system the contribution of the incomplete collisions can be significant [31,32]. As the collision induced frequency shift is negligible, there is no significant shift in frequency of signal. Moreover dispersion management is not considered in the analysis hence the impact of collision rate on crosstalk performance is neglected without loss of significant change in result.

There has been considerable research work on study of nonlinear effects and its effect on signal degradation [33–35] and interplay between them [36–40]. XPM effect in WDM system for OOK and DPSK modulation format has been studied theoretically [41], experimentally [42] and numerically [43]. Due to random temporal waveform alignments of WDM channels, large variations of XPM degradation have been predicted for OOK and DPSK signals [4,42]. The effect of nonlinear stimulated Raman scattering has been previously studied by the present authors in WDM system employing EDFA [44] and DRA [45]. In continuation with the previous work, in this paper using statistical methods a comprehensive study of XPM and SPM induced crosstalk or nonlinear phase shifts has been done for WDM system employing DRA for DPSK and OOK signal in NRZ- and RZ-modulation formats. On the basis of the detailed study we have shown that 40 Gb/s RZ-DPSK signal with 33.3% duty cycle is most tolerant to XPM and SPM induced crosstalk in coherent detection systems. In WDM system employing DRA, backward pumped DRA is most tolerant to XPM and SPM induced crosstalk among the three pumping scheme i.e. backward, forward and bi-directional.

The paper is organized as follows. In Section 2 closed form formulae are derived to study XPM and SPM induced crosstalk in WDM system employing DRA. Section 3 uses the closed formed formulae to evaluate crosstalk in a typical configuration of WDM system. Performance of different modulation formats at different data rates and pulse shape used in optical data transmission system is investigated. After choosing the best performing pulse shape and data rate, crosstalk has been investigated for the three pumping scheme of DRA i.e. forward, backward and bi-directional. Section 4 concludes the paper.

2. Theory

In N-channel WDM systems employing distributed Raman amplifier for bi-directional pumping scheme, the phase modulation of nth channel induced by nth channel due to XPM is given as [46]

\[ \phi_{mn}(t) = 2\pi \int_0^t P_e(0, t - d_mz)e^{-\alpha z}g_F(z')g_B(z')dz' \]

where \( P_e(z, t) \) is the power of nth channel as a function of length z and time t, \( P_e(0, t) \) is the launched power at the transmitter, \( \gamma \) is the nonlinear co-efficient, \( \alpha \) is the fiber attenuation co-efficient, \( d_m = 1/\nu_n - 1/\nu_m \) is propagation time difference between the two different channels (m, n) during a unit length transmission. In Eq.
it has been assumed that fiber dispersion causes just pulse walk-off and no pulse distortion [46–48].

In DRA, forward amplification gain [49] is

\[ g_f(x') = \exp \left( g \left( \sum \frac{\Delta P_m + P_n}{A_{eff} z} \right) (1 - e^{-x'}) \right) \]  \hspace{1cm} (2)

where peak Raman gain \( g = 6.5 \times 10^{-14} \text{m/W} \), \( P_m \) and \( P_n \) are the initial pump and signal power respectively, \( x \) is Signal (Pump) Wavelength.

Backward amplification gain [49] is

\[ g_b(x') = \exp \left( g \left( \sum \frac{\Delta P_m + P_n}{A_{eff} z} (e^{x'} - 1) \right) \right) \]  \hspace{1cm} (3)

where \( P_m \) is initial backward pump power, \( L \) is the length of DRA and other parameters are same as (2).

In the expression of forward and backward gain following assumptions has been made:

(a) The attenuation constant of pump and signal wavelength is same and equal to 0.2 dB/km. Variation in attenuation constant with wavelength has been neglected.

(b) In forward gain pump depletion has been taken into consideration since the optical signal at the Stokes frequency exists from the beginning.

(c) In backward gain pump depletion is neglected as the system is designed not to induce pump depletion because the signal level will fluctuate if pump depletion occurs.

Power spectral density of \( \varphi_{mn} \) is found by taking the Fourier transform of auto-correlation function and is given as

\[ S_{\varphi_{mn}}(\omega) = S_{Pmn}(\omega) |Hmn(\omega)|^2 \]  \hspace{1cm} (4)

where \( S_{\varphi_{mn}}(\omega) \) is the power spectral density of \( \varphi_{mn}(t) \) and \( S_{Pmn}(\omega) \) is the power spectral density of \( P_m(0, t) \) which depends on pulse shape \( p(t) \).

A rectangular NRZ pulse is given as

\[ p(t) = \begin{cases} 2P_0 & (|t| < \frac{T}{2}) \\ 0 & (|t| > \frac{T}{2}) \end{cases} \]  \hspace{1cm} (5)

Fourier transform of \( p(t) \) is given as

\[ |P(\omega)| = 2P_0 \sin(\frac{\pi \omega T}{2}) \]  \hspace{1cm} (6)

Similar representation for RZ pulse of duty cycle \( r_b \) is obtained by replacing \( T \) with \( \frac{T}{r_b} \) in the expression for \( |P(\omega)| \), where \( r_b \) assumes value 2 and 3 for 50% and 33.3% duty cycle respectively. Hence power spectral density

\[ S_{\varphi_{mn}}(\omega) = \frac{1}{4T^2} |P(\omega)|^2 + \frac{1}{4T^2} \sum_{k=-\infty}^{\infty} |P(\omega)|^2 \delta(\omega - k) \]  \hspace{1cm} (7)

For Eq. (4), the transfer function is obtained as

\[ H_{mn}(\omega) = 2^\gamma \int_0^1 e^{-j\beta_m z} e^{-\omega x} e^{i(K(\tau - x' + \tau'))} e^{i(K' x' - x')} \, dz' \]  \hspace{1cm} (8)

where \( K' = \frac{(\sum \Delta P_m + P_n)}{A_{eff}} \) and \( K'' = \frac{(\sum \Delta P_m + P_n)}{A_{eff}} \)

Substituting \( z' = T \ln x \).

The equation becomes

\[ H_{mn}(\omega) = 2^\gamma e^{i\frac{K' x}{T}} \int_0^T e^{-j\beta_m T} \frac{1}{x} e^{-K' x + K' x'} \, dx \]  \hspace{1cm} (9)

The exponential \( e^{-K' x + K' x'} \) is binomially expanded to be the sum of two terms (1 – \( K' x + K' x' \)) and the higher order terms are neglected which contribute insignificantly in the series expansion.

\[ H_{mn}(\omega) = 2^\gamma \left[ e^{-e^{-j\beta_m (\omega T)}} - 2^\gamma K' e^{-j\frac{K' (\omega T)}{T}} + 2^\gamma K' e^{-j\frac{K' (\omega T)}{T}} \right] \]  \hspace{1cm} (10)

2.1. DPSK modulation format

At DPSK [46] receiver, the differential nonlinear phase shift \( \Delta \varphi_{mn}(L, t) = \varphi_{mn}(L, t) - \varphi_{mn}(L, t - T) \) adds to the differential phase of the signal. Here \( T \) is the bit period. Hence power spectral density of \( \Delta \varphi_{mn}(L, t) \) is

\[ S_{\Delta \varphi_{mn}}(\omega) = 4S_{\varphi_{mn}}(\omega) |H_{mn}(\omega)|^2 \sin^2(\omega T/2) \]  \hspace{1cm} (11)

The variance of XPM induced crosstalk is given by [13, 46]

\[ \sigma_{\Delta \varphi_{mn}}^2(m, n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{\Delta \varphi_{mn}}(\omega) \, d\omega \]  \hspace{1cm} (12)

Substituting the expression of power spectral density of DPSK-NRZ signal i.e. \( S_{\varphi_{mn}}(\omega) \) in the expression of \( S_{\Delta \varphi_{mn}}(\omega) \) and performing the infinite integration of Eq. (12) using Tables of Integral and Series [50] and is given in Appendix A.

2.2. OOK modulation format

At OOK receiver, the nonlinear phase shift \( \varphi_{mn}(L, t) \) adds to the phase of the signal. Hence power spectral density of \( \varphi_{mn}(L, t) \) is

\[ S_{\varphi_{mn}}(\omega) = S_{Pmn}(\omega) |H_{mn}(\omega)|^2 \]  \hspace{1cm} (13)

The variance of XPM induced crosstalk is given by

\[ \sigma_{\Delta \varphi_{mn}}^2(m, n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{\varphi_{mn}}(\omega) \, d\omega \]  \hspace{1cm} (14)

Substituting the expression of power spectral density of OOK-NRZ signal i.e. \( S_{\varphi_{mn}}(\omega) \) in the expression of \( S_{\Delta \varphi_{mn}}(\omega) \) and performing the infinite integration of Eq. (14) using Tables of Integral and Series [50] and is given in Appendix A.

In N-channel WDM system, each channel is modulated independently by random data. Thus, the variance can be approximated as

\[ \sigma_{\Delta \varphi_{mn}}^2(m) = \sum_{n=1}^{N} \sigma_{\Delta \varphi_{mn}}^2(m, n) \]  \hspace{1cm} (15)

XPM and SPM are nonlinear Kerr effects that originate from intensity dependence of the refractive index. While SPM refers to self induced phase shifts experienced by an optical signal during its propagation in optical fiber, XPM refers to nonlinear phase shift of optical field induced by co-propagating field at different wavelength. From the equation of nonlinear phase shift [5], it can be said that for equally intense optical field, the contribution of XPM to the nonlinear phase shift is twice compared to that of SPM. The variance of SPM induced nonlinear phase shift can be calculated directly from the expression of XPM by making inter-channel separation equal to zero as is given in Ref. [46] (Eqs. (5) and (6)). This is due to the fact that in SPM change in phase of the signal is proportional to its own intensity. Though SPM causes XPM induced PM (Phase Modulation) to PM conversion and IM
Intensity modulation conversion, the analytical characterization of which has been done in Ref-[38], this conversion is negligible for XPM characterization in WDM system. In the final calculation, the total nonlinear phase shift of a signal in an N-channel WDM system is calculated by the sum of XPM induced phase shift from remaining N-1 channels and SPM induced phase shift from the same channel. The total variance of nonlinear effect is given by the sum of variance of XPM and SPM and is in accordance with earlier approach [7, Eq. (2)].

The variance of SPM induced crosstalk is calculated by taking the interchannel separation ($\Delta \lambda$) equal to zero in the expression of variance of XPM [46]. Hence, for $\Delta \lambda = 0$,

$$\sigma_{XPM}^2(m) = \frac{1}{4} \sigma_{SPM}^2(m)$$

The factor of $1/4$ comes from the fact that phase shift induced by XPM is twice as large as SPM for the same intensity of the signal [46]. Thus total variance of XPM and SPM induced crosstalk in $m$th channel is given as [7]

$$\sigma_x^2(m) = \sigma_{XPM}^2(m) + \sigma_{SPM}^2(m)$$

The crosstalk standard deviation is evaluated on decibel scale by using the formulæ

$$\sigma_x(dB) = -10 \log_{10} e^{-\sigma_x} = \sigma_x \cdot 10 \log_{10} e$$

3. Results and discussion

Fig. 1 shows the DRA configuration considered for our analysis. The N-channels of WDM system are assumed to transmit 0 dBm power in wavelength range of 1515–1575 nm with inter–channel separation of 1 nm. As the gain spectrum of Raman amplifier depends on pump wavelength, it gives flexibility to achieve gain at any desired signal wavelength. The multi-pumps consisting of 6 pumps are in wavelength range of 1420–1470 nm, separated by 10 nm and each having power of 20 dBm yielding wideband and flat gain spectrum in signal wavelength range of 1515–1575 nm. This is done in order to achieve broadband and flat gain which is known as “WDM pumping” scheme [51,52]. Each pump thus provides peak Raman gain to signal at a shifted wavelength of 100 nm from pump wavelength and has a gain spectrum of 10 nm.

The generalized formulæ are derived for WDM system employing bi-directional pumped DRA. The forward and backward pumping are considered to be special cases of bi-directional pumping by taking into account the launched pump power. Furthermore, the interaction among pumps is neglected in current analysis and spotlight is on crosstalk between signals as they are continuously pumped while propagating in the fiber. For analysis purpose standard SMF fiber is used with following parameters: $\gamma$ (Nonlinear Coefficient) = 1.18 W$^{-1}$ km$^{-1}$; $A_{eff}$ (Effective Area) = 80 $\mu$m$^2$; Zero Dispersion Wavelength = 1265.5 nm; Dispersion Slope = 0.058 $\times$ 10$^3$ ps/nm/nm/km; $\alpha$ (attenuation co-efficient) = 0.2 dB/km; $L$ (length) = 80 km.

First the effect of bit rate on XPM and SPM induced crosstalk has been investigated. Figs. 2 and 3 show the variation of XPM and SPM induced crosstalk with signal wavelength for bit rate of 2.5 Gb/s, 10 Gb/s, 40 Gb/s for NRZ-OOK and NRZ-DPSK signal. We have not included the results for 100 Gb/s as at such high bit rates intrachannel XPM and FWM i.e. IXPM and IFWM becomes dominant and ignoring such effects might result in large inaccuracy in results. The authors propose to include IXPM in future works to take into account intrachannel nonlinearities. It can be seen from the figure that with the increase in bit rate of the signals, crosstalk decreases and minimum crosstalk is observed for 40 Gb/s signal. This is because walk-off length $L_w = |T|/|d_{\lambda_{MN}}|$, the distance at which two $m$th and $n$th pulses of length $T$ becomes completely walkoff [46], varies inversely with fiber dispersion co-efficient and bit rate of the system. Neglecting the impact of collision rate between the pulses and having only considered walk-off length (which is actually the inter collision distance), it can be said from the analysis that in DRA variance of crosstalk varies with $L_w$. Thus, crosstalk decreases with the increase in bit rate of the system, other parameters remaining constant. We have neglected the impact of
collision rate between the pulses and have only considered walk-
off length (which is actually the inter collision distance).

After selecting the best performing bit rate, effect of variation in
duty cycle of pulse was investigated. Figs. 4 and 5 show the vari-
ation of XPM and SPM crosstalk with signal wavelength for different
duty cycle of the pulse for both OOK and DPSK signal at 40 Gb/s

data rate. Other parameters assumed are same as the previous
analysis and configuration of fiber link is as shown in Fig. 1. It
can be seen from the figures that with the decrease in duty cycle
of the pulse, XPM and SPM crosstalk decreases for both OOK and
DPSK signal. The reason for RZ-pulse suffering less crosstalk com-
pared to NRZ-pulse is because in RZ-pulse, the ‘1’ bit occupy only

Fig. 3. Variation of crosstalk standard deviation (dB) with signal wavelength (nm) for DPSK at bit rate (B) = 2.5 Gb/s, 10 Gb/s, 40 Gb/s.

Fig. 4. Variation of crosstalk standard deviation (dB) with signal wavelength (nm) for NRZ-OOK, RZ-OOK (50% duty cycle) and RZ-OOK (33% duty cycle).

Fig. 5. Variation of crosstalk standard deviation (dB) with signal wavelength (nm) for NRZ-DPSK, RZ-DPSK (50% duty cycle) and RZ-DPSK (33% duty cycle).
a fraction of the time slot. Hence the probability of overlap of ‘1’ bit in neighbouring channels decreases. On comparing Figs. 4 and 5, it is found that for both RZ- and NRZ-modulation format there is a decrease of almost 20% in crosstalk from OOK to DPSK signal. The reason for the DPSK signal suffering less crosstalk stems from the fact that in the DPSK signal only differential phase of the signal is affected. On the contrary in OOK signal the total phase of the signal gets affected due to XPM induced phase shift. Thus detector response gets more affected in case of OOK signal compared to DPSK signal. Hence RZ-DPSK signal with 33.3% duty-cycle suffers minimum crosstalk compared to NRZ-DPSK signal, NRZ-OOK signal, RZ-DPSK and RZ-OOK signal with 50% duty cycle.

On the basis of above analysis it can be observed that 40 Gb/s RZ-DPSK signal (33.3% duty cycle) minimum SPM and XPM crosstalk was used to investigate crosstalk in forward pumped DRA and backward pumping scheme. The expression of variance of SPM and XPM induced crosstalk with signal wavelength is studied for 40 Gb/s RZ-DPSK signals (33.3% duty cycle) for different modulation format. It is found that for both RZ- and NRZ-modulation format there is a decrease of almost 20% in crosstalk from OOK to DPSK signal.

**4. Conclusion**

XPM and SPM induced crosstalk in WDM system employing bi-directional pumped DRA has been evaluated using closed form formulae for DPSK and OOK signal in RZ and NRZ modulation format. It is found that 40 Gb/s RZ-DPSK signal has best crosstalk performance compared to 10 Gb/s NRZ DPSK, 40 Gb/s RZ-OOK and 40 Gb/s NRZ-DPSK signal. Among the three pumping schemes, minimum crosstalk is observed for WDM system employing backward pumping scheme. The results based on the statistical analysis shows that a 40 Gb/s RZ-DPSK signal (33.3% duty cycle) offers the superlative crosstalk performance in a WDM communication system.

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**Appendix A**

$$|H_{ij}(\omega)|^2 = |M + N + O|^2 = (M + N + O) \times (M + N + O)$$

$$= |M|^2 + |N|^2 + |O|^2 + MN' + M'N + MO' + M'O + ON'$$

$$= (M)^2 + |N|^2 + |O|^2 + 2 \times \text{Re}(MN') + 2 \times \text{Re}(MO') + 2 \times \text{Re}(NO')$$

Substituting the values of $M$, $N$ and $O$ from equation of $H_{mol}(\omega)$ and simplifying we get the following terms

$$|M|^2 = \frac{(2\gamma)^2}{\omega^2 + (d_{\text{mol}}\omega)^2} \left[ (1 - e^{-2\gamma})^2 + 4e^{-2\gamma} \sin^2 \left( \frac{d_{\text{mol}}\omega L}{2} \right) \right] \quad (A.1)$$

$$|N|^2 = \frac{(2\gamma)^2}{4\omega^2 + (d_{\text{mol}}\omega)^2} \left[ (1 - e^{-2\gamma})^2 + 4e^{-2\gamma} \sin^2 \left( \frac{d_{\text{mol}}\omega L}{2} \right) \right] \quad (A.2)$$

$$|O|^2 = \frac{(2\gamma)^2}{(d_{\text{mol}}\omega)^2} \left[ 4 \sin^2 \left( \frac{d_{\text{mol}}\omega L}{2} \right) \right] \quad (A.3)$$

$$2\text{Re}(MN') = \frac{(2\gamma)^2 K^2 (2\omega^2 + d_{\text{mol}}^2 \omega^2)}{(\omega^2 + d_{\text{mol}}^2 \omega^2) (4\omega^2 + d_{\text{mol}}^2 \omega^2)} \times \left[ 1 + e^{-2\gamma} - (e^{-2\gamma} + e^{-2\gamma}) \left( 1 - 2 \sin^2 \left( \frac{d_{\text{mol}}\omega L}{2} \right) \right) \right] + \frac{2\gamma K' \omega d_{\text{mol}} \omega \sin d_{\text{mol}}\omega L (e^{-2\gamma} - e^{-2\gamma})}{(\omega^2 + d_{\text{mol}}^2 \omega^2) (4\omega^2 + d_{\text{mol}}^2 \omega^2)} \quad (A.4)$$

$$2\text{Re}(MO') = \frac{(2\gamma)^2 K' e^{-2\gamma}}{(\omega^2 + d_{\text{mol}}^2 \omega^2) (d_{\text{mol}}^2 \omega^2)} \left[ (1 + e^{-2\gamma})d_{\text{mol}}^2 \omega^2 (1 - \cos d_{\text{mol}}\omega L) + 2\omega d_{\text{mol}} \omega \sin d_{\text{mol}}\omega L (1 - e^{-2\gamma}) \right] \quad (A.5)$$
$$2\text{Re}(\text{NO}^*) = \left[\frac{(2\gamma)^r K_r K_m e^{-2\Delta}}{4\pi^2 + d_{mn}^2 \omega^2}\right] \left\{ (1 + e^{-2\Delta}) d_{mn} \omega^2 (1 - \cos d_{mn} \omega L) + 2d_{mn} \omega \sin d_{mn} \omega (1 - e^{-2\Delta}) \right\} \quad (A.6)$$

The integration has been performed using formula 3.826.1, 3.824.1, 3.725.1 and 3.725.3 of Ref [50] and basic trigonometric and algebraic functions.

References


