Chapter 7

Conclusion and Further Scope of Work

In this chapter we conclude the investigations carried through out this thesis and also give scope for further study which may be undertaken on the basis of the results reported.

7.1 Conclusion of the Work Reported

The concept of entropy $H(f)$ introduced by Shannon (1948) in the literature measures the average uncertainty associated with a random variable $X$ with probability density function $f(.)$. For a component, which has survived up to time $t$, $H(f; t)$ measures the uncertainty about the remaining lifetime $[X|X \geq t]$. Observing that highly uncertain components are inherently not reliable, Ebrahimi and Pellery [40] have used the Shannon’s residual entropy, as a measure of the stability of a component or a system. This approach seemed more realistic and has opened the applications of information-theoretic measures in the area of reliability.
Considering the importance of non-additive entropy measure we have proposed one parameter generalized residual entropy measure $H^\alpha(f;t)$ and have observed that the proposed measure determines the distribution function uniquely. Further we have seen that it characterizes three specific lifetime distributions.

Next, we have extended the scope of dynamic entropy measures to the concept of inaccuracy measure given by Kerridge (1961). The dynamic inaccuracy measures, both residual and past, can be employed respectively under proportional hazard model (PHM) and proportional reversed hazard model (PRHM) to characterize specific lifetime distributions.

The concept of weighted distributions and hence that of weighted information measures is of wide interest when a stochastic process is recorded with some weight function. The dynamic inaccuracy measures, both residual and past, find a natural extension to the corresponding length biased residual and past inaccuracy measures. These measures also characterize the underlying distribution uniquely.

The cumulative distribution function based measures of entropy $\xi(X)$ are in general more stable in comparison to probability density function based measure $H(f)$ given by Shannon in (1948). The concept of cumulative residual entropy (CRE) given by Rao et al. in (2004) has been extended to cumulative residual entropies with one parameter and two parameters and further to their dynamic versions viz. $\xi^\alpha(X;t)$ and $\xi^\beta_\alpha(X;t)$. The dynamic cumulative entropy functions determine the distribution function uniquely. The exponential, the Pareto and the finite range distributions which are commonly used in the reliability modeling have been characterized in terms of the proposed generalized dynamic cumulative entropy measures.

The distribution function based dynamic measures of cumulative residual inaccuracy and cumulative past inaccuracy have been considered as natural extension of distribution function based dynamic entropy measures. The proposed cumulative
inaccuracy measures determine the underlying distribution uniquely under PHM (for residual) and PRHM (for past) models; and also characterize certain specific probability distributions using relation between different reliability measure.

7.2 Scope For Future Study

During the present investigation several ideas have originated which have the potential to extend the study further. The work reported in this thesis can be extended to bivariate and multivariate domains. The problem of extending the concept of the cumulative residual entropy (CRE) function to higher dimensions is yet to be examined. Characterizations of some bivariate distributions based on the functional form of the bivariate cumulative residual entropy function can be obtained analogous to that of bivariate failure rate.

In comparison to the quantum of work done on cumulative residual entropy in the continuous case, a little work seems to have been done in discrete domain. We can consider the dynamic measure proposed further for discrete cases, since practically discrete cases are suitable from applications point of view. Further the discrete measures of the dynamic version proposed can possibly find applications in image processing and information retrieval etc.

Another domain which can be explored in this context is that of order statistics. A number of researchers like Wong and Chen [128], Ebrahimi et al. [41], Baratpour et al. [14], Aghahimi et al. [6] and Zarezadeh and Asadi [131] are working in the area of information theoretic measures in order statistics. We can study the dynamic information measures and dynamic inaccuracy measures in the context of order statistics; also we can study the scope of measures of cumulative residual entropy in order statistics.