Chapter 4

DIVERSIFICATION IN EOQ MODELS FOR DETERIORATING ITEMS
4.1 INTRODUCTION 49
4.2 MATHEMATICAL MODEL 50
4.3 CONCLUSIONS 59
4.1 INTRODUCTION:

In this chapter, we will show, the objective function to be convex for a wide range of parameter values and this would suggest that a similar pattern of diversification might emerge. An analysis of the resulting diversification issues may suggest that it would be beneficial to order small quantities from two suppliers, rather than one large quantities, when units in inventory subject to deterioration, from a single supplier.

In this model, we develop the results by making the vary general assumption that the unit prices charged by two suppliers and the unit holding costs incurred for items purchased from the two suppliers are different. In an inventory, units are subject to deterioration at a constrain rate (say) $\theta$ of on hand inventory. They can neither be repaired or replaced during the cycle time. We develop the total cost expression as a function of cycle times, cycle timed as a function of the order quantities from each supplier when both the orders are placed simultaneously and when orders are placed one after the other from the two suppliers. In the development of the model, we will assume that lead timed is zero. The benefits of diversification are achieved through possible reduction in total annual costs which included ordering, holding, deteriorating cost and purchase costs. The model concludes with a discussion of a numerical example for each situations and a comparison of the single source versus diversification.
4.2 MATHEMATICAL MODEL:

In this section, we assume that annual demand $R$ units per time unit. We may order from two independent sources in which case $K$ is the total ordering cost when both sources are used. $Q_1$ and $Q_2$ ($Q_1 > 0, Q_2 > 0$) units are ordered from source 1 and source 2 with purchase price $C_1$ and $C_2$ ($C_1 > C_2$) respectively. Inventory holding charge fraction $I$/annum for both the sources is constant. It is assumed that shortages are not allowed. A constant fraction $\theta$ of on hand inventory deteriorate. The deteriorated units can not be repaired or replaced during the cycle time.

**SITUATION -I**: When orders are simultaneously, placed from the two suppliers.

Let $T_1$ be time at which supplier 1 satisfy the order and $T$ is time at which supplier 2 satisfy the order. $Q(t)$ denotes number of units on hand at any instant $t$, $0 \leq t \leq T$, then the differential equation that governs the instantaneous state of units on hand is given by
\[ \frac{\partial Q(t)}{\partial t} + \theta Q(t) = -R, 0 \leq t \leq T_1 \]
\[ = -R, T_1 \leq t \leq T \]

with boundary conditions \( Q(0) = Q_1, Q(T_1) = Q_2 \) and \( Q(T) = 0 \)

Then, the solution of above differential equation is given by

\[ Q(t) = \frac{R}{\theta} (e^{\theta(T_1 - t)} - 1), \quad 0 \leq t \leq T_1 \]
\[ = \frac{R}{\theta} e^{\theta t} [e^{\theta(T - t)} - 1], \quad T_1 \leq t \leq T \]

and the number of units \( Q_1 \) purchased from supplier 1 and \( Q_2 \) purchased from supplier 2 are given by

\[ Q_1 = \frac{R}{\theta} [e^{\theta T_1} - 1] \]
\[ Q_2 = \frac{R}{\theta} [e^{\theta T} - e^{\theta T_1}] \]

respectively.

The different cost components associated with total cost of an inventory per time unit are as follows:

a) Purchase cost :

\[ PC = \frac{C_1 R}{\theta T} [e^{\theta T_1} - 1] + \frac{C_2 R}{\theta T} [e^{\theta T} - e^{\theta T_1}] \]

b) Cost due to deterioration of units :

\[ DC = \frac{C_1 R}{\theta T} [e^{\theta T_1} - \theta T_1 - 1] + \frac{C_2 R}{\theta T} [e^{\theta T} - e^{\theta T_1} - \theta(T - T_1)] \]
c) Inventory holding cost:

\[ IHC = \frac{C_1IR}{\theta^2T}[e^{\theta T_1} - \theta T_1 - 1] + \frac{C_2IR}{\theta^2T}e^{\theta T_1}[e^{\theta(T - T_1)} - \theta(T - T_1) - 1] \]

d) Ordering cost /time unit:

\[ OC = K/T \]

Using (a) - (d), total cost \( TC_I(T, T_1) \) of on inventory system per time unit is given by

\[ TC_I(T, T_1) = PC + DC + OC + IHC \] (4.1)

We want to minimize \( TC_I \) with respect to \( T \) and \( T_1 \). The optimum value of \( T \) and \( T_1 \) can be obtained by solving

\[ \frac{\partial TC_I}{\partial T} = 0 \]

\[ \frac{\partial TC_I}{\partial T_1} = 0 \]

Simultaneously by Gass-Seidal iterative method. Since \( T_1 \) and \( T \) takes only finite values, the iteration will coverage after finite number of steps. It can be seen that it is not possible to obtain the explicit form of the minimum value of the objective function \( TC_I^* \) in terms of all the parameter values.
SITUATION -II: When orders are placed one after the other, to two supplier.

We assumed that $C_1 < C_2$. WLOG, we assume that initial demand in the cycle is satisfied by units received from supplier 2 and then from supplier 1.

Let $Q_1(t)$ denotes the on hand inventory of the system at any instant $t$ for the units received from supplier 1 and $Q_2(t)$ denotes the on hand inventory of the system at any instant $t$ for units received from supplier 2. The instantaneous states of units on hand at any instant $t$ is given by following differential equations:

\[
\frac{\partial Q_1(t)}{\partial t} + \theta Q_1(t) = 0, \ 0 \leq t \leq T_1 \\
\frac{\partial Q_2(t)}{\partial t} + \theta Q_2(t) = -R, \ 0 \leq t \leq T_1 \\
\frac{\partial Q_1(t)}{\partial t} + \theta Q_1(t) = -R, \ T_1 \leq t \leq T \\
\frac{\partial Q_2(t)}{\partial t} = 0, \ T_1 \leq t \leq T
\]

with boundary conditions

$Q_2(0) = Q_2, Q_2(T_1) = 0, Q_1(0) = Q_1$ and $Q_1(T) = 0$

The solution of above differential equation is given by

\[
Q_1(t) = \frac{R}{\theta} e^{\theta T_1} [e^{\theta t} - e^{\theta T_1}] e^{-\theta t}, 0 \leq t \leq T_1 \\
Q_1(t) = \frac{R}{\theta} e^{\theta T_1} [e^{\theta(T_1 - t)} - 1], T_1 \leq t \leq T \\
Q_2(t) = \frac{R}{\theta} [e^{\theta(T_1 - t)} - 1], 0 \leq t \leq T_1 \\
Q_2(t) = 0, T_1 \leq t \leq T
\]
Then the number of units purchased from each supplier is given by

\[ Q_1 = \frac{R}{\theta} [e^{\theta T} - e^{\theta T_1}] e^{\theta T_1} \]

\[ Q_2 = \frac{R}{\theta} [e^{\theta T_1} - 1] \]

The different component of the cost function per time unit are as follows:

e) Cost due to deterioration of units per time unit:

\[ DC = C_1 R e^{\theta T_1} [e^{\theta T} - e^{\theta T_1}] + \frac{C_2 R}{\theta T} [e^{\theta T_1} - 1] + (C_1 - C_2) \frac{RT_1}{T} - C_1 R \]

f) Inventory holding cost per time unit:

\[ IHC = \frac{C_1 I R}{\theta^2 T} [e^{\theta T} - e^{\theta T_1}] [e^{\theta T_1} - 1] \]

\[ + \frac{C_2 I R}{\theta^2 T} [e^{\theta T_1} - \theta T_1 - 1] \]

\[ + \frac{C_1 I R}{\theta^2 T} e^{\theta T_1} [e^{\theta (T - T_1)} - \theta (T - T_1) - 1] \]

Using (d) - (f), total cost \( TC_{II}(T, T_1) \) of an inventory system per time unit is given by

\[ TC_{II}(T, T_1) = DC + OC + IHC \]

The optimal value of \( T \) and \( T_1 \) can be obtained by solving

\[ \frac{\partial TC_{II}}{\partial T} = 0 \] and \[ \frac{\partial TC_{II}}{\partial T_1} = 0 \]

solve both the equations simultaneously for \( T \) and \( T_1 \) using Gauss-seidal iterative method.
To see whether the diversification is worth while one need to compare $TC_i^*$ and $TC_i^{**}$ with $TC^*$, the minimum cost incurred, if sources $i = 1, 2$ is used exclusively respectively.

Let $K_i$ be the ordering cost if source $i (i=1, 2)$ is used, we have

$$TC_i^* = \frac{K_i}{T} + \frac{C_i(\theta + I)R}{\theta^2T}[e^{\theta T} - \theta T - 1] + \frac{C_iR}{\theta T}[e^{\theta T} - 1] \quad (4.3)$$

and $T = T_i^*$ $(i = 1, 2)$ is given by Hwang and Sojn (1983)

$$T = T_i^* = \sqrt{\frac{2K_i}{C_i(2\theta + I)R}}, \; i = 1, 2 \quad (4.4)$$

and hence

$$Q_i^* = RT_i^*, \; i = 1, 2$$

A comparison of (4.1) and (4.3) ( or (4.2) and (4.3)) reveals that the optimal solution to the diversification problem will depend on which $TC_i^*$ and $TC_i^{**}$, $TC^*$ and $TC_2^*$ is the smallest, given the parameters $R, K_i, K_i$ and $C_i (i = 1, 2)$. We will discuss numerical examples where the comparison of costs is made between above mentioned two situations and that of single supplier.

**NUMERICAL EXAMPLE:**

We describe numerical examples for both the situations.

We use following parametric values .

Ordering cost, $K = $ 25.00 per order.

Demand rate, $R = 1000$ units per year.
Unit price charged by supplier 1, $C_1 = $20.00 per unit.

Unit price charged by supplier 2, $C_2 = $30.00 per unit.

Inventory holding charge fraction, $I = 10\%$ per annum.

(Note: we assume that second source charge more $C_1 < C_2$)

In TABLE 1 and TABLE 2 establish, the effect of deterioration an optimal order quantities procured from each sources and total minimum cost per time unit of an inventory system in two different situations are studied.
TABLE : 1

Effect of deterioration an order quantities and total cost (situation I)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$Q_{1I}$</th>
<th>$Q_{2I}$</th>
<th>$TC_{I}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.015</td>
<td>3552.84</td>
<td>1500.52</td>
<td>286377.0</td>
</tr>
<tr>
<td>0.020</td>
<td>3619.03</td>
<td>1037.19</td>
<td>28727.90</td>
</tr>
<tr>
<td>0.025</td>
<td>3611.75</td>
<td>0684.18</td>
<td>28824.13</td>
</tr>
<tr>
<td>0.030</td>
<td>3762.19</td>
<td>0396.49</td>
<td>28924.19</td>
</tr>
<tr>
<td>0.035</td>
<td>3839.46</td>
<td>0143.83</td>
<td>29012.50</td>
</tr>
</tbody>
</table>

TABLE : 2

Effect of deterioration an order quantities and total cost (situation II)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$Q_{1II}$</th>
<th>$Q_{2II}$</th>
<th>$TC_{II}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.015</td>
<td>7936.76</td>
<td>6158.02</td>
<td>20034.99</td>
</tr>
<tr>
<td>0.020</td>
<td>6843.79</td>
<td>6566.84</td>
<td>21436.60</td>
</tr>
<tr>
<td>0.025</td>
<td>3732.25</td>
<td>7816.92</td>
<td>23723.26</td>
</tr>
<tr>
<td>0.03</td>
<td>2135.70</td>
<td>8485.96</td>
<td>26224.73</td>
</tr>
<tr>
<td>0.035</td>
<td>1964.94</td>
<td>8732.82</td>
<td>28605.98</td>
</tr>
</tbody>
</table>

From TABLE : 1, it can be observed that as rate of deterioration of units in inventory increases, the number of units to be procured at lower units cost (i.e. from supplier 1) increase whereas those from the supplier 2 decreases. Because of deterioration of units in an inventory total cost of an inventory system increases with increase in deterioration of units.

TABLE : 2 depicts that because of ordering by one, the retailer will like to put an
order to second supplier first because of higher unit cost. It can be seen that of units to be purchased from supplier 2 increases and those to be purchased from Supplier 1 decreases with increase in deterioration rate of units in inventory. The total cost of an inventory system increase with increase in deterioration rate units.

It can be seen that total cost of an inventory system in situation II is much lower than of situation I. Next table gives sensitivity of optimal procurement quantities from each supplier and total inventory cost of the system, of situation I with respect to situation II.

TABLE : 3
Sensitivity analysis of situation II w. r. t. situation I.

<table>
<thead>
<tr>
<th>θ</th>
<th>(\frac{Q_{II} - Q_{II}}{Q_{II}})</th>
<th>(\frac{Q_{II} - Q_{II}}{Q_{II}})</th>
<th>(\frac{TC_{II} - TC_{II}}{TC_{II}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.015</td>
<td>0.552</td>
<td>0.756</td>
<td>-0.429</td>
</tr>
<tr>
<td>0.20</td>
<td>0.471</td>
<td>0.842</td>
<td>-0.340</td>
</tr>
<tr>
<td>0.025</td>
<td>0.012</td>
<td>0.912</td>
<td>-0.216</td>
</tr>
<tr>
<td>0.03</td>
<td>-0.761</td>
<td>0.953</td>
<td>-0.103</td>
</tr>
<tr>
<td>0.035</td>
<td>-0.953</td>
<td>0.984</td>
<td>-0.014</td>
</tr>
</tbody>
</table>

We suppose that the orders can be placed at the one source. If the order costs of individual suppliers are e.g. \(K_1 = $11.00\) and \(K_2 = $10.00\), we obtain

\[Q_1^* = 85.72579\] per order \(TC_1^* = $26149.79\)

\[Q_2^* = 66.72383\] per order \(TC_2^* = $30175.72\)
Comparing these $TC_i^*$ ($i = 1, 2$) values with $TC_1$ and $TC_2^*$ obtained when two suppliers are used ($K = \$25.00$) we see that it would be preferable to use only supplier 1 since $TC_1^* < TC^* < TC_2^*$ when rate of deterioration $\theta$ is 0.025.

4.3 CONCLUSIONS:

In this chapter, an extension of the chapter 2, to two suppliers when units in inventory are subject to deteriorate and determined the optimal order quantities from each supplier is studied.