Chapter 2

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2.1 INTRODUCTION:

In this chapter, we develop the results by making very general assumption that the unit prices charged by two suppliers and the unit holding costs incurred for items purchased from the two suppliers are different. We develop the total cost expression as a function of the order quantities from each supplier when both the orders are placed simultaneously and when orders are placed one after the other to the two suppliers.

Our model is based on the model which determines the optimal order quantity by minimizing the sum of inventory holding cost and ordering costs. In the development of model, we will assume that lead time is zero. The benefits of diversification are achieved through possible reduction in the total annual costs which include ordering, holding and purchase costs. The chapter concludes with a discussion of a numerical example for each situations and a comparison of the single source versus diversification.

In Section 2.2, the model is developed with infinite replenishment, while Section 2.3 deals with finite production rate. We analyze these problems for EOQ models under two different situations.

SITUATION I: WHEN ORDERS ARE SIMULTANEOUSLY PLACED TO THE TWO SUPPLIERS.
2.2 DIVERSIFICATION IN THE EOQ MODEL WITH INFINITE REPLENISHMENT

There exist two suppliers to a customer. The customer needs to determine the amount that it should order from each supplier every period such that his holding and ordering costs will be minimum.

In this section, we assume that annual demand is \( R \) units and we may order from two independent sources in which case \( K \) is the total ordering cost when both sources are used. \( Q_i > 0, \ i = 1,2 \) units are ordered from source \( i \). It is also assumed that shortages are not allowed and the unit price charged by source \( i \) is \( C_i, i = 1,2 \) and inventory holding charge fraction is \( 1/ \) annum for both the sources.

**SITUATION I :** When orders are simultaneously, placed to the two suppliers.

When \( Q_i \) units are ordered from each supplier the total amount ordered is \( Q_{11} + Q_{21} \) which becomes the initial inventory at the beginning of each cycle. The cycle length is \( \frac{Q_{11} + Q_{21}}{R} \), the total cost incurred per time unit
\[
TC_I(Q_{1I}, Q_{2I}) = \frac{R(C_1 Q_{1I} + C_2 Q_{2I})}{Q_{1I} + Q_{2I}} + \frac{KR}{Q_{1I} + Q_{2I}} \\
+ \left[ \frac{C_1 I Q_{1I}^2}{2R} + \frac{C_2 I Q_{2I}^2}{2R} + \frac{C_2 I Q_{1I} Q_{2I}}{R} \right] \frac{R}{Q_{1I} + Q_{2I}}
\] (2.1)

Minimizing \( TC_I(Q_{1I}, Q_{2I}) \) subject to \( Q_{1I} > 0 \) and \( Q_{2I} > 0 \) (since diversification is in effect, i.e. both sources are being used) would give us the optimal quantity to order from each source. To minimize \( TC_I \), we perform partial differentiations and obtain the following:

\[
\frac{\partial TC_I}{\partial Q_{1I}} = 0
\]

\[
C_1 I Q_{1I}^2 + C_2 I Q_{2I}^2 + 2(C_1 - C_2)R Q_{2I} + 2C_1 I Q_{1I} Q_{2I} = 2KR
\] (2.2)

\[
\frac{\partial TC_I}{\partial Q_{2I}} = 0
\]

\[
C_1 I Q_{1I}^2 + C_2 I Q_{2I}^2 + 2(C_1 - C_2)R Q_{1I} + 2C_2 I Q_{1I} Q_{2I} = 2KR
\] (2.3)

Assuming (2.2) to be quadratic in \( Q_{2I} \), we get

\[
Q_{1I} = -Q_{2I}
\]

\[
+ \sqrt{Q_{2I}^2 - \frac{2}{C_1 I} \left[ R Q_{2I} (C_1 - C_2) + \frac{1}{2} C_2 I Q_{2I}^2 - KR \right]}
\]

substituting, this value of \( Q_{1I} \) in eq. (2.3), it can be solved for \( Q_{2I} \) using Newton-Raphson's method.
It can be seen that it is not possible to obtain the explicit form of the minimum value of the objective function $TC_i^*$ in terms of all the parameter values (Parikh et al. (1997)).

$$TC_i^*(Q^*_{1i}, Q^*_{2i}) = \frac{R(C_1Q^*_{1i} + C_2Q^*_{2i})}{Q^*_{1i} + Q^*_{2i}} + \frac{KR}{Q^*_{1i} + Q^*_{2i}}$$

$$+ \left[ \frac{C_1IQ^2_{1i}}{2R} + \frac{C_2IQ^2_{2i}}{2R} + \frac{C_2IQ^2_{2i}Q^*_{1i}}{R} \right] \frac{R}{Q^*_{1i} + Q^*_{2i}}$$

(2.4)

To see whether diversification is worthwhile, one need to compare $TC_i^*$ with $TC_i^*$. the minimum cost incurred it source $i = 1, 2$ is used exclusively. Letting $K_i$ to be order cost of source $i$ is used, we have optimal order quantity $Q^*_i$ as

$$Q^*_i = \sqrt{\frac{2K_iR}{C_i}}, i = 1, 2$$

(2.5)

The corresponding minimum cost is

$$TC_i^* = C_iR + \sqrt{2C_i1K_iR}$$

(2.6)

A comparison of (2.4) and (2.6) reveals that the optimal solution to the diversification problem will depend on which of $TC_1^*$, $TC_1$, or $TC_2^*$ is the smallest, given the parameters $R$, $K$, $K_i$ and $C_i$. we will discuss numerical examples where the comparison is made.

**SITUATION II : WHEN ORDERS ARE PLACED ONE AFTER THE OTHER TO THE TWO SUPPLIERS.**
Arguing, as above, in this situation, the total cost of an inventory system per time unit is given by

\[
TC_{II}(Q_{1II}, Q_{2II}) = \frac{R(C_1Q_{1II} + C_2Q_{2II})}{Q_{1II} + Q_{2II}} + \frac{KR}{Q_{1II} + Q_{2II}}
+ \left[\frac{C_1IQ_{1II}^2}{2R} + \frac{C_2IQ_{2II}^2}{2R}\right] \frac{R}{Q_{1II} + Q_{2II}}
\]

(2.7)

To minimize \(TC_{II}\), we perform partial differentiation and obtain the following:

\[
\frac{\partial TC_{II}}{\partial Q_{1II}} = 0
\]

\[
C_1IQ_{1II}^2 - C_2IQ_{2II}^2 + 2(C_1 - C_2)RQ_{2II} + 2C_1IQ_{1II}Q_{2II} = 2KR
\]

(2.8)

\[
\frac{\partial TC_{II}}{\partial Q_{2II}} = 0
\]

\[
C_2IQ_{2II}^2 - C_1IQ_{1II}^2 + 2(C_2 - C_1)RQ_{1II} + 2C_2IQ_{1II}Q_{2II} = 2KR
\]

(2.9)

solving eq. (2.8) for \(Q_{1II}\) (i.e. assuming it to be quadratic in \(Q_{1II}\)) we get,

\[
Q_{1II} = -Q_{2II}
+ \sqrt{Q_{2II}^2 - \frac{2}{C_1I}\left[RQ_{2II}(C_1 - C_2) - \frac{1}{2}C_2IQ_{2II}^2 - KR\right]}
\]

(2.10)

substituting, this value of \(Q_{1II}\) from (2.10) in (2.9) the final expression of (2.9) can be solved for optimum \(Q_{2II}\) using Newton-Raphson's method and hence optimum
\[ TC_{II}^*(Q_{III}, Q_{2II}) = R(C_1Q_{III}^* + C_2Q_{2II}^*) + \frac{KR}{Q_{III}^* + Q_{2II}^*} \]

\[ + \left[ \frac{C_1IQ_{III}^*}{2R} + \frac{C_2IQ_{2II}^*}{2R} \right] \frac{R}{Q_{III}^* + Q_{2II}^*} \]

(2.11)

can be found out using (2.7).

Again, here, to see whether this diversification is worthwhile we compare \( TC_{II}^* \), with \( TC_i^* \), \( i = 1, 2 \) (as defined in situation I, eq. (2.6)).

Comparing (2.4) and (2.11) indicates that

\[ TC_I^* - TC_{II}^* = \frac{C_1IQ_{III}Q_{2II}}{Q_{III}^* + Q_{2II}^*} > 0 \]

Thus, it may be advantageous to place a second order once, first order gets depleted.

The above comparison is also discussed by a numerical example.

**NUMERICAL EXAMPLE**:

We will use this example to determine the optimal order quantities and total cost for both situation stated above.

We use the following parameter values:

Order cost, \( K = $ 25.00 \) per order
Demand rate, \(R = 1000\) units per year

Unit price charged by source 1, \(C_1 = \$20.00\) per unit

Unit price charged by source 2, \(C_2 = \$30.00\) per unit

Inventory carrying charge fraction, \(I = 10\%\) per annum

We assume that the second source charge more, \((C_2 > C_1)\)

Using this data, the optimal order quantities and the annual cost are found by the procedure describe in the last section.

<table>
<thead>
<tr>
<th>(Q_1^*)</th>
<th>(Q_2^*)</th>
<th>(TC^*)</th>
<th>Cycle time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Situation - I</td>
<td>909.91</td>
<td>788.63</td>
<td>0.92</td>
</tr>
<tr>
<td>Situation - II</td>
<td>7.94</td>
<td>0.73</td>
<td>0.79</td>
</tr>
</tbody>
</table>

where

\[Cycletime = \frac{Q_1^* + Q_2^*}{R}\]

We suppose that the order can be placed at one source. If the order costs at individual suppliers are for example \(K_1 = \$13.00\) and \(K_2 = \$12.00\) with as above. We obtain

\[Q_1^* = 114.01\] per order \[TC_1^* = \$20228.04\]

\[Q_2^* = 89.44\] per order \[TC_2^* = \$30268.33\]
Comparing these $TC_i^*$ values with $TC_i$ and $TC_j$, obtained when two suppliers are used ($K = \$ 25.00$), we see that it would be preferable to use only supplier $i$ since $TC_i^* < TC_i < TC_j^*$.

It is interesting to note when the values of $K_1$ and $K_2$ are interchange i.e. for $K_1 = \$ 12.00$, $K_2 = \$ 13.00$, the optimal solution is to order from supplier 1, since in this case we get

$$Q_1^* = 109.54 \text{ per order } TC_1^* = \$ 30419.09$$
$$Q_2^* = 93.09 \text{ per order } TC_2^* = \$ 30279.28$$

2.3 DIVERSIFICATION IN THE EOQ MODEL WITH FINITE PRODUCTION RATE

In this section, we assume that annual demand is $R$ units, replenishment rate of sources 1 is $P_1$ units and that of source 2 is $P_2$ units. We also assume that $R < P_1 + P_2$.

We may order from two independent sources in which case $K$ is the total ordering cost when both sources are used. $Q_i > 0$, $i= 1,2$ units are ordered from source $i$. It is also assumed that shortages are not allowed and the unit price charged by source $i$ is $C_i$ and inventory holding charge fraction is $I$/annum for the sources.

SITUATION I: When orders are simultaneously, placed from the two suppliers.
When $Q_i$ units are ordered from each supplier the total amount ordered is $Q_{1i} + Q_{2i}$ which becomes the initial inventory at the beginning of each cycle. The cycle length is $\frac{Q_{1i} + Q_{2i}}{R}$. Let $T_{1i}$ denotes the time point at which supplier 1 terminates production, So, $Q_{1i} = P_{1i}T_{1i}$ and $T_{2i}$ denotes the time point at which supplier 2 terminates production, So, $Q_{2i} = P_{2i}T_{2i}$

Then, the component of the total cost incurred per time unit are as follows:

a) Purchase cost (PC) per time unit

$$PC = \frac{R(C_1Q_{1i} + C_2Q_{2i})}{Q_{1i} + Q_{2i}}$$

b) Inventory holding cost (IHC) per time unit

$$IHC = \frac{C_1IQ_{1i} + C_2IQ_{2i}}{2} - \left[ \frac{C_1IQ_{1i}^2}{2P_{1i}} + \frac{C_2IQ_{2i}^2}{2P_{2i}} \right] \frac{R}{Q_{1i} + Q_{2i}}$$

c) Ordering cost (OC) per time unit

$$OC = \frac{KR}{Q_{1i} + Q_{2i}}$$

Using eqs. (a) - (c), the total cost $TC_i(Q_{1i}, Q_{2i})$ per time unit is given by

$$TC_i(Q_{1i}, Q_{2i}) = PC + IHC + OC$$

Minimizing $TC_i(Q_{1i}, Q_{2i})$ subject to $Q_{1i} > 0$ and $Q_{2i} > 0$ (since diversification is in effect, i.e. both sources are being used) would give us the optimal quantity to order from each sources. To minimize $TC_i$, we perform partial differentiations and obtain
the following:

\[ \frac{\partial TC_i}{\partial Q_{1l}} = 0 \]

\[ \frac{C_1I}{2} - \frac{C_1IQ_{1l}R}{P_{1l}(Q_{1l} + Q_{2l})} + \frac{C_1IQ_{1l}^2R}{2P_{1l}(Q_{1l} + Q_{2l})^2} + \frac{C_2IQ_{2l}^2R}{2P_{2l}(Q_{1l} + Q_{2l})^2} \]

\[ + \frac{C_1R}{Q_{1l} + Q_{2l}} - \frac{(C_1Q_{1l} + C_2Q_{2l} + K)R}{(Q_{1l} + Q_{2l})^2} = 0 \]

(2.13)

\[ \frac{\partial TC_i}{\partial Q_{2l}} = 0 \]

\[ \frac{C_2I}{2} - \frac{C_2IQ_{2l}R}{P_{2l}(Q_{1l} + Q_{2l})} + \frac{C_2IQ_{2l}^2R}{2P_{2l}(Q_{1l} + Q_{2l})^2} + \frac{C_1IQ_{1l}^2R}{2P_{1l}(Q_{1l} + Q_{2l})^2} \]

\[ + \frac{C_2R}{Q_{1l} + Q_{2l}} - \frac{(C_1Q_{1l} + C_2Q_{2l} + K)R}{(Q_{1l} + Q_{2l})^2} = 0 \]

(2.14)

Equation (2.13) and (2.14), we get

\[ Q_{1l} = \frac{\alpha_0 - \alpha_1Q_{2l}}{\alpha_2} \]

(2.15)

where

\[ \alpha_0 = 2(C_2 - C_1)R \]
\[ \alpha_1 = (C_1 - C_2)I + \frac{2Rh_2}{P_{2l}} \]
\[ \alpha_2 = (C_1 - C_2)I + \frac{2Rh_1}{P_{1l}} \]

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with value of $Q_1$ as given in (2.15), (2.13) can be solved for $Q_2$. Once $Q_2$ is known, $Q_1$ and $TC(Q_1, Q_2)$ can be evaluated. Also

$$\frac{\partial Q_1}{\partial Q_2} = -\frac{\alpha_1}{\alpha_2} > 0$$

(because $C_1 > C_2$, gives $h_1 > h_2$ implies $h_1 - h_2 > 0$. Also $\alpha_1 > 0$, $\alpha_2 < 0$).

To see whether diversification is worthwhile, one need to compare $TC_i^*$ with $TC_i^*$, the minimum cost incurred if source $i = 1, 2$ is used exclusively. Letting $K_i$ to be order cost if source $i$ is used, we have optimal order quantity $Q_i^*$ as

$$Q_i^* = \sqrt{\frac{2K_i R(1 - \frac{R}{P_i})}{C_i}}, i = 1, 2$$

(2.16)

The corresponding minimum cost is

$$TC_i^* = C_i R + \sqrt{2C_i K_i R(1 - \frac{R}{P_i})}$$

(2.17)

A comparison of (2.15) and (2.17) reveals that the optimal solution to the diversification problem will depend on which of $TC_1^*, TC_2^*$, or $TC_2^*$ is the smallest, given the parameters $R$, $K$, $K_i$ and $C_i$. The above comparison is discussed by a numerical example.

**NUMERICAL EXAMPLE :**

We will use this example to determine the optimal order quantities and total cost for the situation stated above.
We use the following parameter values:

Order cost, $K = \$ 25.00$ per order

Demand rate, $R = 1000$ units per year

Unit price charged by source 1, $C_1 = \$ 35.00$ per unit

Unit price charged by source 2, $C_2 = \$ 30.00$ per unit

Inventory carrying charge fraction, $I = 33 \%$ per annum

Replenishment rate of source 1, $P_{1l} = 12000$ per year

Replenishment rate of source 2, $P_{2l} = 15000$ per year

We assume that the first source charges more, $(C_1 > C_2)$

Using this data, the optimal order quantities and the annual cost are found by the procedure describe in the last section.
TABLE

<table>
<thead>
<tr>
<th></th>
<th>Situation - 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1^*$</td>
<td>7710.14 units</td>
</tr>
<tr>
<td>$Q_2^*$</td>
<td>2529.29 units</td>
</tr>
<tr>
<td>$TC^*$</td>
<td>365470.30</td>
</tr>
<tr>
<td>Cycle time</td>
<td>1.024 Yrs.</td>
</tr>
</tbody>
</table>

where Cycle time = $\frac{Q_1^* + Q_2^*}{R}$

We suppose that the order can be placed at once source. If the order costs at individual suppliers are for example $K_1 = $ 13.00, $P_{11} = 12000$ units and $K_2 = $ 12.00, $P_{21} = 15000$ units with as above. we obtain

$$Q_1^* = 61.25 \text{ per order } TC_1^* = $ 350707.46$$

$$Q_2^* = 89.89 \text{ per order } TC_2^* = $ 300889.94$$

Comparing these $TC_i^*$ with the $TC^*$ obtained when two suppliers are used i.e. for $K = $ 25.00), we see that it would be preferable to use only supplier 2 since $TC_2^* < TC_i^*$. $TC_1^*$

It is interesting to note when the values of $K_1$ and $K_2$ are interchanged i.e. for $K_1 = $ 12.00, $P_1 = 12000$ units, $K_2 = $ 13.00, $P_2 = 15000$ units the optimal solution is to order from supplier 1, since in this case we get

$$Q_1^* = 58.85 \text{ per order } TC_1^* = $ 34104.55$$

$$Q_2^* = 93.56 \text{ per order } TC_2^* = $ 37390.33$$
2.4 CONCLUSIONS:

In this chapter, we have provided a analysis of the diversification situation in the EOQ model under two different situations. The two situations those we modeled are both the orders are placed simultaneously and when second order is placed one by one, once the first is depleted. We obtained analytical solution for the optimal order quantities from two sources and the minimum average cost in term of all parameter values for the EOQ model.