Chapter 7

EOQ MODEL UNDER THE INFLUENCE OF MARKETING POLICIES WHEN RANDOM CHANCE OF DISCOUNT IS OFFERED
7.1 Introduction

Most of the inventory models are developed to minimize the total cost of the system. When the pricing policies considered are subject to the fixed markup of pricing and the system is independent of the price, minimization of the total variable cost of the system serves the purpose of maximization of the net profit. The net profit is defined as the difference between the generated revenue and the total cost of the model.

Wayland (1958) discussed the relation between economic order quantities and decision making based upon the break-even chart analysis and concluded that they are interrelated. Kotler (1971) showed the interaction between the marketing policies and the economic order quantities (EOQ). Ladaney and Sternlieb (1971) considered an EOQ model for a finite replenishment rate under the influence of the marketing policies. Two alternative types of quantity discounts were considered. The pricing policies considered were subject to the fixed markup of pricing and the varying but known price dependent deterministic demand. Brahmbatt and Jaiswal (1981) extended these models by including variable markup rate which is considered as a function of one of the decision variables.

Arcelus and Srinivasan (1987) developed a modified EOQ model assuming that an inventory management system designed to satisfy a known demand at a given price is not compatible with the treatment of inventories as an investment. The assumption of constant demand and constant price is relaxed and the demand is taken as a function of selling price. The selling price is considered to be a markup on unit cost which is either constant or dependent on the number of units purchased. The decision variables are the ordered quantity and the markup rates and objective is to
maximize the net profit.

Gor and Shah (1997) developed a deterministic demand inventory model with select individual reorder quantities to minimize the total cost during the scheduling period. They considered the discount which is not always available i.e., it is assumed that at reordering time there is a probability of having discount on purchase cost. The objective of the model was to estimate the optimal quantities to be ordered when discount is available and not available.

In this paper, an EOQ model is developed in which demand is dependent on the selling price and there is a probability of getting discount on purchase cost of the item under consideration at any instant. The optimal order quantities when discount is available and not available, and the optimal selling price are estimated. A hypothetical problem is worked out to illustrate the theory.

7.2 Mathematical Model

7.2.1 Assumptions

- The selling price per unit quantity is not fixed.
- The rate of demand is deterministic and is linearly dependent on the unit selling price.
- No shortages are allowed.
- There is a probability of getting discount at the time of placing the order. The probability of getting discount at any given time is a fixed constant.
No discount is available on the ordering cost. Inventory carrying cost is described as a constant fraction of the unit purchase cost and therefore when discount is available it applies to the inventory holding cost too.

7.2.2 Notations

- $R(p) = a - bp = \text{The rate of demand, where } a \text{ and } b \text{ are given constants.}$
- $\beta = \text{The probability of getting discount at any instant.}$
- $d = \text{The discount available as a fraction of the unit purchase cost.}$

7.2.3 Decision Variables

- $p = \text{The unit selling price.}$
- $Q_1 = \text{The quantity to be ordered if discount is not available at the time of placing the order.}$
- $Q_2 = \text{The quantity to be ordered if discount is available at the time of placing the order.}$

7.2.4 Determining The Objective Function

First we find out the total cost comprising of the fixed ordering cost, the purchase cost of the lot and the inventory carrying cost during a cycle when discount is not available, as in the standard EOQ model. Secondly we find the total cost during a cycle when discount is available and then using these two we find the expected total cost during a single cycle. Similarly we find the expected cycle length and lastly the
expected cost per unit time during a cycle.

The total cost during a single cycle when discount is not available is

\[ TC_1(Q_1, p) = A + CQ_1 + \frac{C_1Q_1^2}{2(a - bp)} \quad (7.1) \]

The total cost during a cycle when discount is available is

\[ TC_2(Q_2, p) = A + (1 - d)\epsilon'Q_2 + \frac{(1 - d)\epsilon'Q_2^2}{2(a - bp)} \quad (7.2) \]

The expected total cost during a cycle is

\[ TC(Q_1, Q_2, p) = (1 - \beta)TC_1(Q_1, p) + \beta TC_2(Q_2, p) \]

\[ = A + C((1 - \beta)Q_1 + \beta(1 - d)Q_2) \]

\[ + \frac{C_1}{2(a - bp)}((1 - \beta)Q_1^2 + \beta(1 - d)Q_2^2) \quad (7.3) \]

The expected cycle length is

\[ T'(Q_1, Q_2, p) = \frac{(1 - \beta)Q_1 + \beta Q_2}{(a - bp)} \quad (7.4) \]

From equations (7.3) and (7.4), we have the expected cost per unit time as follows.

\[ K(Q_1, Q_2, p) = \frac{TC(Q_1, Q_2, p)}{T'(Q_1, Q_2, p)} \]

\[ = \frac{(a - bp)(A + C(1 - \beta)Q_1 + \beta(1 - d)Q_2)}{(1 - \beta)Q_1 + \beta Q_2} \]

\[ + \frac{C_1(1 - \beta)Q_1^2 + C_1\beta(1 - d)Q_2^2}{2((1 - \beta)Q_1 + \beta Q_2)} \quad (7.5) \]
Generated revenue per unit time is:

\[ GR(p) = p(a - bp) \]  \hspace{1cm} (7.6)

Therefore, from equations (7.5) and (7.6), we have the net profit per unit time as follows.

\[ NP(Q_1, Q_2, p) = K(Q_1, Q_2, p) - GR(p) \]

\[ = (a - bp) \left( p - \frac{A + C(1 - \beta)Q_1 + \beta(1 - d)Q_2}{(1 - \beta)Q_1 + \beta Q_2} \right) - \frac{C_1(1 - \beta)Q_1^2 + C_1\beta Q_2^2}{2((1 - \beta)Q_1 + \beta Q_2)} \]  \hspace{1cm} (7.7)

### 7.2.5 Determining The Optimal Solution

Now we find the optimum values of \( Q_1, Q_2 \) and \( p \) which will give the maximum net profit per unit time. The optimal \( Q_1, Q_2 \) and \( p \) are one of the solutions of the following system of equations.

\[ \frac{\partial NP(Q_1, Q_2, p)}{\partial Q_1} = 0 \]

\[ \frac{\partial NP(Q_1, Q_2, p)}{\partial Q_2} = 0 \]

\[ \frac{\partial NP(Q_1, Q_2, p)}{\partial p} = 0 \]

Simplifying the third equation of the above system, we have

\[ p = \frac{a}{2b} + \frac{A + C(1 - \beta)Q_1 + \beta(1 - d)Q_2}{2((1 - \beta)Q_1 + \beta Q_2)} \]  \hspace{1cm} (7.8)

which gives \( p \) as a function of \( Q_1 \) and \( Q_2 \). Using this in equation (7.7) we have \( NP \) as a function of \( Q_1 \) and \( Q_2 \). The optimal \( Q_1 \) and \( Q_2 \) which gives the maximum net
profit per unit time constitute a solution of the following system of equations.

\[
A_i Q_i^3 + B_i Q_i^2 Q_j + D_i Q_i Q_j^2 + E_i Q_j^3
+ F_i Q_i^2 + G_i Q_i Q_j + H_i Q_j^2 + I_i Q_i + J_i Q_j + K_i = 0, \quad i = 1, 2
\]

where

\[
A_1 = C_1(1 - \beta)^2, \\
B_1 = 3C_1 \beta(1 - \beta), \\
D_1 = C_1 \beta(1 + 3\beta + d - \beta d), \\
E_1 = -C_1 \beta^2(1 - d), \\
F_1 = 0, \\
G_1 = C\beta(1 - \beta)d(a - bC), \\
H_1 = C\beta^2d(a - bC(1 - d)), \\
I_1 = -A(1 - \beta)(a - bC), \\
J_1 = -A\beta(a - bC + 2bdC), \\
K_1 = bA^2, \\
A_2 = C_1(1 - \beta)^2, \\
B_2 = C_1(1 - \beta)(3\beta + 2d - 2\beta d - 2), \\
D_2 = -3C_1 \beta(1 - \beta)(1 - d), \\
E_2 = -C_1 \beta^2(1 - d), \\
F_2 = C(1 - \beta)^2d(a - bC), \\
G_2 = C\beta(1 - \beta)d(a - bC(1 - d)), \\
H_2 = 0, \\
I_2 = A(1 - \beta)(a - bC(1 + d)), \\
J_2 = A\beta(a - bC(1 - d)) \quad \text{and} \\
K_2 = -bA^2
\]
and satisfies the following condition.

\[
\frac{\partial^2 NP}{\partial Q_1^2} \frac{\partial^2 NP}{\partial Q_2^2} - \frac{\partial^2 NP}{\partial Q_1 \partial Q_2} < 0
\]  

(7.10)

7.2.6 Special Cases

By assuming that there is no probability of having discount at reordering time (i.e., by putting \( \beta = 0 \)) in the present mathematical model, we get the model developed by Arcelus and Srinivasan (1987).

7.2.7 Observations

It is expected that with an increase in the probability of having discount at reordering time, in the discount fraction and in the rate of demand, there will be an increase in the net profit. A hypothetical problem is worked out to illustrate the observations. The effect of variations in \( a, b, \beta \) and \( d \) on optimal \( Q_1, Q_2, p \) and \( NP(Q_1, Q_2, p) \) is studied in the following example.

7.3 A Hypothetical Problem

\( C = \$40 \) per unit
\( C_1 = \$7.5 \) per unit per unit time
\( A = \$250 \) per order

For various values of \( a, b, \alpha \) and \( d \) the optimal values of \( Q_1, Q_2, p \) and corresponding \( NP(Q_1, Q_2, p) \) (i.e., \( Q_{10}, Q_{20}, p_0 \) and \( N P_0 \)) are tabulated below.
Table 7.1: Effect of variations in $\beta$

\[
\begin{array}{|c|c|c|c|c|}
\hline
\beta & Q_{10} & Q_{20} & p_0 & NP_0 \\
\hline
0.1 & 93 & 233 & 91.9 & 3588 \\
0.5 & 31 & 152 & 89.8 & 4552 \\
\hline
\end{array}
\]

From the above data, it can be seen that with an increase in the probability of having discount, $\beta$,

- the optimal order quantities in both cases along with the unit selling price decrease but the difference between $Q_{10}$ and $Q_{20}$ increases (fig 7.1 and fig 7.2)

- there is an increase in the optimal net profit per unit time. (fig 7.3 and fig 7.4)

- It is also noted that changes in $Q_{10}, Q_{20}$ and $NP_0$ corresponding to the changes in $\beta$ are wide ranging where as there is little change in $p_0$ due to change in $\beta$.

Table 7.2: Effect of variations in $d$

\[
\begin{array}{|c|c|c|c|c|}
\hline
\beta & Q_{10} & Q_{20} & p_0 & NP_0 \\
\hline
0.08 & 67 & 281 & 91.8 & 3611 \\
0.10 & 49 & 326 & 91.6 & 3645 \\
\hline
\end{array}
\]

The above data shows the effect of changes in the discount fraction $d$ clearly.

- $Q_{10}$ decreases and $Q_{20}$ increases with increase in $d$. (fig 7.5 and fig 7.6)

- There is a small decrease in $p_0$ with increase in $d$. (fig 7.8)

- $NP_0$ increases with increase in the value of $d$. This can be explained by increase in $Q_{20}$ due to increase in $d$. (fig 7.9)
Table 7.3: Effect of variations in $a$

<table>
<thead>
<tr>
<th>$a$</th>
<th>$Q_{10}$</th>
<th>$Q_{20}$</th>
<th>$p_0$</th>
<th>$NP_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>93</td>
<td>223</td>
<td>92</td>
<td>3588</td>
</tr>
<tr>
<td>1200</td>
<td>101</td>
<td>273</td>
<td>102</td>
<td>10493</td>
</tr>
</tbody>
</table>

- Increase in $a$ results in increase in the lot-sizes when discount is available and not available. (fig 7.9 & fig 7.10)

- We can also observe that due to increase in $a$, the optimal unit selling price and the corresponding net profit per unit time also increase. (fig 7.11 & fig 7.12)

Table 7.4: Effect of variations in $b$

<table>
<thead>
<tr>
<th>$b$</th>
<th>$Q_{10}$</th>
<th>$Q_{20}$</th>
<th>$p_0$</th>
<th>$NP_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>99</td>
<td>255</td>
<td>113</td>
<td>14353</td>
</tr>
<tr>
<td>10</td>
<td>93</td>
<td>223</td>
<td>92</td>
<td>3588</td>
</tr>
</tbody>
</table>

From above data, it is observed that with an increase in $b$

- there is a decrease in $Q_{10}$ and $Q_{20}$. (fig 7.13 & fig 7.14)

- there is a decrease in $p$. (fig 7.15)

- and there is a drastic decrease in $NP_0$. (fig 7.16)

7.4 Concluding Remarks

In this chapter, an EOQ model with a selling price dependent deterministic demand rate is developed, when there is a probability of getting discount on unit purchase cost at the time of reordering. The rate of demand is taken as $R(p) = a - bp$ where
The constants $a, b$ are taken. The probability of getting discount at any instant, and, the discount as a fraction of the unit purchase cost, are taken as constants. A mathematical model is developed to determine the expected cost per unit time and thereby the optimal unit selling price and the order quantities (when discount is available and is not available). It is verified that the present mathematical model reduces to the model in Arcelus and Srinivasan (1987) when there is no probability of having discount at the time of reordering. A hypothetical problem is worked out. It is observed that with an increase in the probability of getting discount, in the discount as the fraction of the unit purchase cost, and in the value of $a$, there is increase in the net profit; but with an increase in the value of $b$ there is a decrease in the net profit.
Fig 7.1: Effect of changing $\alpha$ on $Q_{10}$
Fig 7.2: Effect of changing $\alpha$ on $Q_{20}$
Fig 7.3: Effect of changing $\alpha$ on $p_0$
Fig 7.4: Effect of changing $\alpha$ on $NP_0$
Fig 7.5: Effect of changing $d$ on $q_{10}$
Fig 7.6: Effect of changing \( d \) on \( Q_{20} \)
Fig 7.7: Effect of changing $d$ on $p_0$
Fig 7.8: Effect of changing $d$ on $NP_0$
Fig 7.9: Effect of changing $a$ on $Q_{10}$
Fig 7.10: Effect of changing $\alpha$ on $Q_{20}$
Fig 7.11: Effect of changing $a$ on $p_0$
Fig 7.12: Effect of changing $a$ on $NP_0$
Fig 7.13: Effect of changing $b$ on $Q_{10}$
Fig 7.14: Effect of changing $b$ on $Q_{20}$
Fig 7.15: Effect of changing $b$ on $p_0$
Fig 7.16: Effect of changing b on $NP_0$