Chapter 7

Conclusion and Future Scope

7.1 Conclusion

In this thesis, we have carried out detailed linear and nonlinear studies of the shear driven sausage and kink instabilities in the framework of electron-magnetohydrodynamic (EMHD) [1] model of the plasma. The motivation behind these studies comes from the observations of the sheared electron current channels, which are susceptible to these shear driven instabilities, in a number of physical situations like plasma opening switches [2-4], collisionless magnetic reconnection [5-12], fast Z-pinches [13, 14], fast ignition concept of laser fusion [15, 16] etc. For example 3D PIC simulations [30, 31] in the context of fast ignition concept of laser fusion show the formation of sheared electron current channels. These channels disintegrate and develop sausage and kink like structures on EMHD time scales leading to the generation of electromagnetic turbulence. The development of sausage and kink instabilities and electromagnetic turbulence in these electron current channels have strong implications for the stopping of the energetic electrons in the dense core of the D-T pellet, which is crucial for the success of fast ignition scheme. In the context of collisionless magnetic reconnection also, two fluid simulations of Biskamp et al. [5, 6] show that the dissipation region, where frozen in condition of magnetic field is broken, develop two scale structure consisting of a narrow electron current layer of scale length $d_e = c/\omega_{pe}$ embedded in a broader current layer of scale length $d_i = c/\omega_{pi}$ (here $d_e$ and $d_i$ are electron and ion skin depths and $\omega_{pe}$ and $\omega_{pi}$ are electron and
ion plasma frequencies). The narrow electron current layer is susceptible to EMHD shear driven instabilities [37–40]. The presence of these shear driven instabilities and the generation of electromagnetic turbulence as a consequence of these instabilities may alter the mechanism by which the frozen in condition of the magnetic field is broken in the dissipation regime. Apart from this, these instabilities may play an important role in determining the partitioning of the magnetic energy into the flow and thermal energies.

Our investigations in this thesis on sausage and kink instabilities in EMHD reveal a number of important features (for example basic linear physics of destabilization, thresholds, behaviour in different parameter regimes, nonlinear development and saturation, nonlinear modification of mean quantities etc.) of these instabilities. In this chapter, we shall summarize the important results obtained in previous chapters.

In chapter-2 we have performed linear analysis of the azimuthally symmetric two dimensional(sausage mode) mode. An analysis of the linearized EMHD equations showed that the necessary condition for the instability is that the quantity \( v''_o - v'_e \), where \( v_o \) is equilibrium flow profile, should change sign in the region of interest. This condition is different from that of the Kelvin-Helmholtz instability [57,58] of sheared flow of neutral fluids because of the presence of \( \vec{e} \times \vec{B} \) force on the magnetized electron fluid. In this chapter we summarized the results and dispersion relations obtained by Das et al. [37]. These results are (i) in local analysis i.e. for the perturbations with scale lengths smaller than the equilibrium scale lengths there is no instability, (ii) in nonlocal analysis i.e. when perturbations scale lengths are of the order of or larger than the equilibrium scale length, there exists instability which is similar to Kelvin-Helmholtz instability of sheared neutral fluid in the hydrodynamic limit \( k_z d_e \gg 1 \) (\( k_z \) is the wave number parallel to the electron flow and \( d_e = c/\omega_{pe} \) is the electron skin depth) and shows the magnetized character of electron fluid in the opposite limit \( k_z d_e \ll 1 \). By solving numerically the dispersion relation obtained for piece-wise linear profile by Das et al. [37] we found that unlike the case of step-function profile, the growth rate does not increase monotonically with \( k_z \) but first increases up to a maximum value and then decreases to zero at a particular value of \( k_z = 0.639/\epsilon \) for a given value of shear width (\( \epsilon \)). It was also observed that the eigen values are either purely real or purely imaginary which means that the mode is either
purely growing or purely oscillatory. The threshold $k_z \epsilon = 0.639$ on the instability was then obtained analytically using principle of exchange of stability [57,58] i.e. by putting $\omega = 0$ in the dispersion relation. The inverse scaling of the maximum value of the growth rate with the shear width $\epsilon$, as observed in the numerical solution of dispersion relation, was obtained analytically also. We also studied the sausage instability for tanh-profile of equilibrium velocity using numerical techniques. We observed almost similar features as for piece-wise linear profile. The growth rate first increases with $k_z$ and then decreases to vanish beyond $k_z = 1/\epsilon$. In other words, the modes with the wavelengths shorter than the equilibrium scale length are stable. This result is in accordance with the physical intuition that the modes which require lesser bending of the flow lines (i.e. large wavelength modes) are easier to be excited as compared to those which require more bending of flow lines (short wavelength modes). The threshold $k_z \epsilon = 1$ on the instability was obtained analytically using the principle of exchange of stability. It was also observed that there exists only one growing mode for a given value of $k_z$ and $\epsilon$. The maximum value of the growth rate was again observed to scale inversely with the shear width ($\epsilon$). The growth rate was found to reduce with increasing value of $\epsilon$. In the opposite limit $\epsilon \to 0$, we recover the results of step-function profile [37]. The stabilization of the mode with increasing value of $k_z$ was explained on the basis of localization of eigen function with increasing $k_z$, which shifts the mode towards stable local regime.

In chapter 3 we studied nonlinear evolution of the sausage like perturbations growing on sheared flow by developing a two dimensional fluid code. The code solves the two dimensional nonlinear EMHD equations for tanh-profile of equilibrium electron velocity. Simulations show the development of sausage like instability the growth rate of which matches with the one predicted by the linear theory [37,40]. The presence of two non-dissipative square invariants $\int \int B_y^2 (\nabla B_y)^2 dx dz$ and $\int \int (\nabla B_y)^4 + (\nabla^2 B_y)^2 dx dz$, where $B_y$ is the $y$-component of magnetic field, restricts the transfer of power from short scales to long scales and hence the final nonlinear state of the system was found to be coherent. Due to the back reaction of the growing modes, the flow profile flattens with time and hence a reduction in directed electron flow was observed. However the reduction in the flow is not very efficient. This happens because this instability saturates rather easily. The instability saturates
either by getting rid of the curvature in the directed flow profile or by violating, even for the minimum value of the \( k_z \) possible in the system, the instability criteria \( k_z \epsilon_{eff} < 1 \) because of the increase in the effective shear width \( \epsilon_{eff} \) of the flow profile flattened by the instability. A quasilinear analysis of the governing two dimensional EMHD equations showed that relaxation of the flow profile via nonlinear interactions can be mocked up by an effective viscous or higher order derivative dissipation.

In chapter 4, linear analysis of the three dimensional kink mode was performed. Unlike the case of sausage instability, the dispersion relation obtained in the local limit shows the existence of the instability. The local dispersion relation shows that this mode is also driven by electron inertia effect but is unstable even in the absence of the curvature of equilibrium flow profile and requires just transverse gradient in the equilibrium flow profile. It was also shown that the presence of \( k_z \) (wave number in the flow direction) or \( B_0 \) (local value of magnetic field) makes the task of exciting local instability difficult. The nonlocal dispersion relation for step-function profile of equilibrium velocity obtained analytically in the limit \( kd_e \gg 1 \), where \( k = \sqrt{k_y^2 + k_z^2} \), shows that in addition to the sausage instability (which requires \( k_z \) to be finite) there exist modes which are unstable even for \( k_z = 0 \) and hence are distinct from the sausage mode. For \( \tanh \)-profile of equilibrium velocity we solved linearized EMHD equations numerically. It was observed that, unlike the two dimensional case, there exist more than one growing radial eigen functions for a given \( k_y, k_z \) and \( \epsilon \), and the maximum growth rate (maximized over \( k_z \)) does not vanish beyond \( k_z > 1/\epsilon \) for a finite value of \( k_y \). Hence in three dimensional case, there is a significant widening of the wavenumber domain for which modes are unstable. The existence of finite growth rate for large value of \( k_z \) can be explained on the basis of the transition from nonlocal regime to local regime (which is unstable in three dimensional case) with the increasing value of \( k_z \). It was shown that the eigen functions get localized with increasing value of \( k_z \) and for large values of \( k_z \) the growth rate obtained from nonlocal analysis of the \( \tanh \)-profile matches with that obtained from local dispersion relation. A comparison between MHD kink and EMHD kink was made to show that the destabilizing term in case of MHD comes from the \( \vec{J} \times \vec{B} \) term while in case of EMHD it comes from the inertia term.

Chapter 5 is devoted to the study of the nonlinear evolution of the general three
dimensional perturbations of small amplitudes growing on the sheared flow, with the help of a three dimensional code developed by us. This code solves three dimensional nonlinear EMHD equations for \( \text{tanh} \)-profile of equilibrium electron velocity. The simulations show the development of the kink instability the growth rate of which matches with that predicted by the linear theory [39]. The instability first develops in the shear layer leading to the generation of short scale structures in the shear layer and hence making the layer turbulent. This turbulence then spreads from shear region to otherwise stable regions and later occupies the whole region. As in the two dimensional case, mean flow profile flattens with time due to which the magnitude of the directed flow of electrons current reduces. However, the reduction in the current is much more efficient in this case as compared to two dimensional case. This happens because saturation of the instability in three dimensions occurs at amplitudes higher than those in two dimensions because the conditions for the instability in three dimensional case are rather relaxed. In three dimensions, there is no such condition as \( k_z \epsilon < 1 \); the mode is unstable in the local limit also and driven by current gradient which means mean flow profile needs to be not only curvature free but also gradient free. Therefore saturation in this case occurs because the direct nonlinear cascade provides coupling of the unstable modes to damped short scale structures and not for the reason that the system is no longer unstable as was the case in the two dimensions.

In chapter 6 we have applied the results of our fluid simulations to fast ignition concept of laser fusion [15,16] and discussed the importance of shear driven instability in the stopping of energetic electrons. In our three dimensional simulations, we observed the generation of electromagnetic turbulence in the shear layer and an associated reduction of the directed electron current. The reduction in the directed flow of electrons via the generation of electromagnetic turbulence is directly related to the loss of directed kinetic energy of electrons. This indicates that the stopping of the electrons observed in 3d PIC simulations of Sentoku et al. [31] and the stopping in our 3d fluid simulations may have common Physics (stopping via electromagnetic turbulence generated by the shear driven instabilities). In fact we estimated the stopping distance of the electrons using the results of our fluid simulations and found that it matches well with that obtained from the PIC simulations. In fast
ignition, the density of the beam of fast electrons propagating towards the core is smaller than the density of the slow background plasma electrons which provide the return shielding current in response to the fast electron current. But our simulations do not distinguish between these two species of electrons, namely fast electrons and slow electrons. To investigate the dependence of the instability on the ratio \( n_b/n_0 \), where \( n_b \) is the beam density and \( n_0 \) is the background plasma density, we have derived a dispersion relation using two fluid model in which we distinguish between the ingoing and outgoing electrons. We have found that the growth rate of the instability depends very weakly on the ratio \( n_b/n_0 \) for the shear driven modes. It was shown that the stopping distance of the electrons is inversely proportional to the growth rate of the instability, which depends very weakly on the ratio \( n_b/n_0 \). This makes this mode very attractive for efficient stopping in fast ignition scenario. Also the stopping distance of the electrons was found to be directly proportional to the shear width \( \epsilon \) of the \( \tanh \)-profile.

### 7.2 Future Directions

In this thesis, we investigated various basic aspects of the shear driven sausage and kink instabilities [38–40] in the framework of EMHD. We have analyzed these instabilities in slab geometry and in the approximation of collisionless uniform plasma using non-relativistic EMHD equations. However, in various physical situation of interest the approximations (uniform and collisionless plasma, slab geometry, non-relativistic treatment etc.) involved in our studies may not be valid. For example, in fast ignition concept of laser fusion there exists a large density gradient (density changes by five order of magnitude in few tens of micrometer) between the critical density surface, where the fast electrons are generated, and the dense core region, where the fast electrons are supposed to deposit their energy. This gradient is along the direction of current and fast electrons have to pass through this gradient in order to reach the dense core region. In the dense core region the collisions may also be important. The current channels formed in fast ignition concept of laser fusion are cylindrical in shape and the electron flow inside these channels is relativistic. Also the density and speed of ingoing and outgoing electrons are also different. Therefore,
further extension of our linear and nonlinear studies of the shear driven instabilities in the framework of EMHD including the below mentioned effects will further improve the understanding not only of the basic physics of these instabilities but also of their role in stopping the fast electrons in fast ignition concept of laser fusion.

1. Cylindrical geometry

2. Non-uniform plasma along the flow direction

3. Relativistic treatment

4. Effects of collisions

5. Different densities of ingoing and outgoing electrons and hence different inflow and outflow speeds

The sheared electron current channels observed in 3-D PIC simulations are formed when Weibel, tearing and current filamentation instabilities operate on an initial configuration of two counter streaming and inter-penetrating electron currents. Some researchers [29, 74, 75] have pointed out that these instabilities also contribute to the stopping of electrons. However the role of these instabilities in stopping the electrons is not clear till date. The relative role of these instabilities and shear driven instabilities studied in this thesis may depend on various parameters like ratio of beam to plasma density, density gradient scale lengths etc. An integrated fluid simulation of an initial configuration of two counter streaming and inter-penetrating electrons current which is susceptible to all the above said instabilities is desirable. The parametric studies of the relative importance of various instabilities in stopping the electrons can be investigated with the help of such simulations.

In two fluid simulation of collisionless magnetic reconnection, it has been observed that the dissipation region, where frozen-in condition of magnetic field is broken, develop two scale structures. Within the distance $d_e$ (electron skin depth) of the x-line there exists a narrow electron current layer embedded in a broader layer of scale length $d_i$ (ion skin depth). It is anticipated that the narrow electron current layer may broaden out to ion skin depth as a result of shear driven instabilities [8]. On the scale length of the electron skin depth only electron dynamics is dominant
while on the scale length of the ion skin depth ions also take part in the dynamics. However, within the distance $d_i$ of x-line, ions remain demagnetized. The character of these instabilities may be changed by ion dynamics as the current layer broadens up to ion skin depth. Therefore studies of these instabilities including the inertial response of the ions would be of importance in the context of collisionless magnetic reconnection.

In the presence of the guide magnetic field (parallel to the out of reconnection plane current) the narrow electron current layer further thins down to the electron larmor radius \[76,77\]. On such scales kinetic effects become important. Therefore, kinetic treatment of sausage and kink instabilities is also desirable.

In recent years, the phenomena of turbulence spreading has attracted a lot of interest \[78,79\]. We have also observed this phenomena in our simulations. The electromagnetic turbulence is generated by the shear driven instabilities first in the localized sheared region and then it spreads to other stable regions. Investigations of various features (for example characterization of the front that separates the laminar and turbulent regions, the mechanism of spreading, parameter dependence of the front speed, time scale of spreading etc.) of this turbulence spreading could be of fundamental interest. These studies also have implications both for fast ignition and magnetic reconnection. In particular, in fast ignition, time scale of spreading will play an important role in determining how fast the whole electron current channel will be influenced by the turbulence generated initially in a localized shear region. This in turn will determine how fast most of the hot electron can be stopped by the action of shear driven instabilities.

Another important study that is of interest is the test particle simulations of the particle diffusion in turbulent electric and magnetic fields. For this purpose one needs to solve the particles equation of motion in the presence of space and time dependent turbulent electric and magnetic field obtained from the simulations of shear driven instabilities. Then the diffusion rates can be obtained by tracking the particle trajectory.