Chapter 1

Do Wholesale Grain Markets Serve the Interests of Farmers? An Analysis of Reserve Price Setting and Farmers’ Revenues in a Paddy Auction Market in North India
1.1 Introduction

India is the second largest producer of rice in the world, contributing more than a fifth of the world’s rice output. The large government participation in the rice market is probably well known: the government purchases large quantities of rice for distribution to the rural and urban poor through subsidized food outlets in its ‘public distribution system’ (PDS), and also maintains a large buffer stock. Government procurement as a proportion of paddy output has increased from about 10% in 1978-79 to about 33% in 2010-11 (Ministry of Agriculture, Directorate of Economics and Statistics, Agricultural Statistics at a Glance, various issues). Also well known is the fact that it announces a minimum support price (MSP) for paddy at sowing time, and purchases paddy at the MSP from a large number of primary paddy markets.

While the government is a large buyer of grain, there is at the same time a considerable private trade in grain. Grain sales are primarily conducted in government-regulated wholesale markets that were set up under the Agricultural Produce Marketing Committee (APMC) Act. This Act mandates the use of ascending auctions to sell grain, and this is the principal mechanism through which paddy in North India is sold to millers. Thus the government has played a key role in instituting and regulating these auction markets that mediate the grain trade. However, there is little work on studying how these markets function, and how well they serve farmers.

In this chapter we analyze paddy auction data from one such market in an important rice-producing state in North India (district Panipat in the state of Haryana), in terms of a formal structural model of ascending auctions. The primary objective of this chapter is to investigate whether the auction market for paddy serves farmers’ interests. In particular, we evaluate whether the reserve prices set by the auctioneer in an ascending auction work to maximize the revenues they expect to receive from paddy sales.

Various factors determine the extent to which the auction mechanism affects prices and revenues in a given market. These include the number of buyers present (the greater the competition, the higher the price), the existence of collusion (which would depress prices) and the level of the reserve price. Among these, only the reserve price is directly under the control of the auctioneer; setting it well is a way of fetching good revenues from sales. The auctioneer faces a trade-off in
setting the reserve price: setting it at a high price risks the lot being unsold, and setting it at too a level may imply that the sale price is lower than it would have been otherwise. Thus, given the format of an ascending auction, the level of competition and the extent of collusion, there is an optimal level of the reserve price that maximizes the expected revenue for the seller. For the Panipat paddy market, we compute optimal reserve prices and compare them to those actually set by the auctioneer. We then go on to evaluate differences in revenues the farmer would expect to get at (a) the levels of reserve prices actually used by the auctioneer, compared to those that would obtain if (b) reserve prices were set optimally.

A second objective of this chapter is to assess the role of competition in mitigating the impact of any sub-optimally set reserve prices on expected revenues. This is set out in two simple propositions and also explored empirically.

An analysis of these issues necessitates knowledge of the underlying distributions of buyers’ valuations for the lots of paddy. While the auctioneer cannot know the specific valuations of each buyer, the theory proceeds on the basis that he does not know the distributions from which the valuations are drawn. Estimation of the underlying distributions requires that they be identified from the given data; auction theory can be used to map observed bids (sale prices) and other information into the latent values for which these bids are optimal. Taking advantage of some recent identification results as detailed subsequently, we estimate these value distributions, using semi-nonparametric methods, and incorporating bidder asymmetry. This is in contrast to earlier analyses that have relied on parametric approaches to estimating value distributions (see Meenakshi and Banerji, 2005). We find that the more general semi-nonparametric characterization represents a statistically significant improvement over parametric estimations.

The analysis and results in this chapter contribute to the literature in several ways. First, they enhance our understanding of the functioning of wholesale grain markets in India. These grain wholesale markets, or *mandis*, were mandated to be set up by Indian states by the APMC Act, proposed at the Centre, and legislated in the states, in the 1960s. The few studies of these markets have all by and large criticized their functioning; contending that they are inefficient (Ramaswami and Balakrishnan, 2002, Umali-Deininger and Deininger, 2001), that market integration is absent (Palaskas and Harriss-White, 1996), and that there is collusion among buyers (Banerji and Meenakshi, 2004).

However, many insights about efficient functioning of markets can be gained
from analyzing it in the context of the trading mechanism that is used; in some of these government-regulated markets, ascending auctions are employed, and in others, grain is traded through mutual negotiations between individual buyers and *katcha arhtias* or agents selling grain on behalf of farmers. But apart from Banerji and Meenakshi (2004, 2008), and Meenakshi and Banerji (2005), we are not aware of any studies that directly incorporate the trading mechanism in the analysis. A careful reading of these three papers suggests that in fact, in the grain markets where auctions are used, the negative impact of collusion on sale prices is quite limited; in that sense, these auction markets perform reasonably well. Our analysis in this chapter throws light on a complementary, and key, indicator of the performance of auction markets, viz., the setting of reserve prices. Our findings corroborate the good functioning of auction markets (see section 1.6).

The APMC Act has been modified in the past decade in various ways to change some of the regulations that imposed entry barriers on buyers and proscribed farmers from selling outside of these mandis; our study suggests that a key part of the functioning of these markets, viz. the choice of the trading mechanism and its impact on farmers’ revenues has not got the attention that it deserves.

Second, our study contributes to the fast growing literature on the structural estimation of auction models\(^1\). Structural estimation of auctions\(^2\) is crucial to asking policy questions of the above sort, as it enables us to simulate alternative states of the world. There is now a large and growing literature on such estimation, starting with Paarsch (1992), Laffont, Ossard and Vuong (1995), Athey and Levin (2001) among others. See Athey and Haile (2005) for a survey. One of the first papers to estimate optimal reserve prices at auctions is Paarsch (1997).

Semi-nonparametric estimation is increasingly popular (see Chen, 2007); our implementation for auctions is similar to that by Brendstrup and Paarsch (2006). Research on agricultural markets that uses the structural auction framework is recent and relatively limited (see Tostao, Chung and Brorsen, 2006, apart from the research by Banerji and Meenakshi, 2004, 2008, Meenakshi and Banerji, 2005).

Section 1.2 introduces the data set and discusses summary statistics. Section 1.3 provides a justification for the use of the independent private values framework

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\(^1\)Auction theory explains players’ bids as an equilibrium outcome that depends on their value distributions for the auctioned commodity. The structural approach to auctions uses data on bids and auction- and bidder-specific covariates and assumptions of some game-theoretic equilibrium to identify and estimate players’ unobserved value distributions.

\(^2\)Structural estimation of auction models uses data on bids and auction- and bidder-specific covariates
and outlines equilibrium bidding behavior in this setting. The identification results that enable estimation of distributions and the semi-nonparametric methodology employed for estimation are set out in Section 1.4, while Section 1.5 describes the distributions thus estimated. We derive the optimal reserve prices for our setting in Section 1.6, and demonstrate that these appear to be set sub-optimally. We show that despite this, the expected revenues for the farmers are not adversely affected. We argue that the level of competition in the market is the principal reason why sub-optimally set reserve prices do not adversely affect expected revenues, and set out two simple propositions to demonstrate this. Section 1.7 concludes.

1.2 Parmal Paddy Auctions in Panipat

We use data from auctions of parmal paddy at the Panipat market, in the state of Haryana (North India). The data are taken from a primary survey conducted in October 1999 by Meenakshi and Banerji (2005) during peak market arrivals. The survey was supplemented using market committee records and interviews with millers and farmers. Paddy is raw harvested grain which on milling is converted into rice (grain separated from chaff and most often polished subsequently). Parmal varieties are high-yielding and the rice milled from Parmal paddy is shorter in length and thicker in width than traditional varieties. As a consequence, it is not as preferred in much of India as longer-grained rice. This and its high-yields result in the parmal rice selling at about one-fifths the price of the premium Basmati varieties, making it the cheapest rice in the market. It is therefore the natural choice for government procurement and public distribution. The Parmal arrivals begin in October and the marketing season lasts for about a month. About 60,000 quintals of paddy arrives in the surveyed market over the entire season, of which more than 60% is concentrated in the first 3 weeks of October. The arrivals peak around the end of the first week, with about 3000 quintals arriving per day.

Panipat mandi is a regulated wholesale grain market set up by the Government under the APMC Act and run by a Market Regulation Committee. The mode of selling the grain is through auctions. The interacting players on the ground in this market include commission agents (katcha arhtias) who sell the grain on behalf of the farmers, and millers who purchase the grain. The government also purchases paddy at these auctions, through a commission agent (the participation of the
government in this and many other markets in Haryana was very limited in the given year, 1999, though). When it participates in the auction of a paddy lot, the government bids the minimum support price that it announced for paddy during sowing season. Commission agents are registered with the Market Committee and their license is renewed annually.

The buyers’ side of the market is rather concentrated. Though 25 distinct buyers were recorded over the entire marketing season, the combined market share of the two largest buyers (with large mills located within 5 km from the mandi) was about 45% of the total arrivals. The remaining buyers had smaller mills and picked up smaller shares of the market arrivals. It was observed that the two large buyers avoided competing with each other by alternating the days on which they made large purchases. Such collusion is expected, ceteris paribus, to depress the win price. Meenakshi and Banerji (2005) undertake parametric maximum likelihood estimation of both the noncooperative and the collusive models (for this market) and compare them using Vuong’s test. Their results support the hypothesis of collusion, in the form of simple bid rotation by the 2 largest buyers. In the present study, therefore, we work with the maintained assumption of bid rotation among the two large buyers.

The sellers side of the market by contrast is far from concentrated. A katcha arhtia typically serves between 100 and 500 farmers; and earns a commission of 2% of the total value of sales from the farmer. There were 49 katcha arhtias in this market with small individual shares (none exceeding 5%).

The parmal paddy lots are sold through oral ascending auctions. Based on a visual inspection of a lot for quality, the auctioneer announces a reserve price for the lot, following which the bidders whose valuation for the lot is less than this reserve price leave. The remaining bidders are the active bidders for the lot. The auctioneer then raises the price in small increments, as long as there are at least two interested bidders (active bidders keep exiting as the price goes past their valuation). The auction ends when only a single bidder is still interested. This bidder wins the object and pays an amount equal to the price at which the second-last bidder dropped out. The auctioneer receives 0.8% of the win price.

Parmal paddy is heterogeneous in several quality characteristics, variations in which affect the valuation a bidder may have for a lot. Based on information from agricultural scientists, market committee officials and bidders at auctions, Meenakshi and Banerji (2005) focus on the following seven quality characteristics:
moisture content, uniformity of grain size, grain luster, absence of chaff, green and immature grains, broken grains and a category of other variables (encapsulating evidence of disease or pest infestation). The auctions proceed at a fast pace and so laboratory testing of a sample for quality at the auction site was not possible. The bidders perform visual and other simple on-the-spot tests (such as breaking the grain and looking at the cross-section for evidence of brittleness). To construct quality variables, for each lot, the quality characteristic was evaluated on a scale of either 1 to 3 (worst to best) or 1 to 2 (poor and good); the determination of quality was done using the same visual and other tests, by a trained enumerator.

The number of distinct winners on any given day was used as a proxy for the number of potential bidders for all auctions on that day\(^3\). The bidders who continued to participate in the auction once the start price was announced, constituted the set of active bidders for that auction.

Thus the information recorded for each auction included the seven covariate quality vector, start (or reserve) price, win price, identity of the winner, number of potential and active bidders and the date of the auction.

Data for the sample are summarized in Table 1.1. Notable is the large number of potential bidders, indicative of a good level of competition in the market. Also noticeable is the sharp drop between the number of potential bidders and active bidders: this indicates that the reserve price was set high enough to be greater than the values of about half the bidders on a lot, and yet, not so high as to exclude all bidders in this fashion. Note also that the grain quality appears to vary significantly; the fairly large variation in sale prices is actually reflective of this variation in quality.

To further describe the data, we regress the log of the win price on quality characteristics, number of potential bidders, and week dummies. This regression is reported in Table 1.2. The regression brings out the dependence of the win price on quality: moisture, uniformity, absence of chaff or broken grain, and lustre, all have a positive effect on win price, and are significant. The win prices are also positively and significantly affected by the number of bidders (or the level of competition) in the auctions of the different lots\(^4\). The week dummies are used to capture other influences on the market prices: these could range from

\(^3\)Bidders generally stay through the bidding all day, and it is unlikely that there are bidders who don’t win even a single lot.

\(^4\)Amongst other explanatory variables considered and found to be not significant was the size of the lot.
price fluctuations in other markets, to the effect on millers’ valuation for grain, of higher levels of grain inventory in successive weeks. In the following sections, we delve into what this summary indicates, using our knowledge of how bidders bid in ascending auctions.

1.3 Bidding in IPV Ascending Auctions

We study this market with oral ascending auctions as the selling mechanism under the independent private values (IPV) assumption. The framework used here is standard (see Krishna, 2002). Each bidder $i$ is assumed to have a valuation, or value $v_i$ for a given lot of paddy (the valuation is specific to the given lot, as discussed later). That is, the bidder’s payoff from winning this lot at a price of $P$ equals $v_i - P$. A key assumption in auction theory is that these values of bidders are not known to the auctioneer. This makes the auction a good mechanism for making a sale; if values were known, the auctioneer could simply select the bidder with the highest valuation, and negotiate a high price with him. It is also generally assumed that a bidder does not know the values of the other bidders. Thus auction theory is cast in the framework of games of incomplete information, or Bayesian games. Since the econometric analysis of auctions is closely tied to the economic theory of auctions, and the theory of Bayesian games more generally, we set out some of the basic apparatus of this theory in Appendix 1.1.

The private values auction model assumes that each bidder $i$’s value $v_i$ for a given lot of grain is privately known (to him) and that bidders do not know other bidders’ values. This uncertainty is modeled by assuming that these values are random variables with a joint distribution that is common knowledge, and further, in the independent private values (IPV) case, that these are independent random variables. We will denote $i$’s marginal distribution by $F_{V_i}(.)$.

In our paddy auction setting, the bidders are millers. A bidder’s valuation for a paddy lot depends upon the difference between the price he expects to receive from selling rice (paddy is processed into rice in mills) and the cost of processing paddy. Millers operating in this market are required to sell 75% of their rice to the government at Rs 913 per quintal\(^5\). The quantity and quality of rice (better

\(^5\)These were the levy percentage and levy price figures for kharif 1999-2000 in the state of Haryana.
quality rice fetches a higher price) that a lot of paddy produces depends on its observable quality characteristics. But the processing cost of paddy is mill-specific and privately known. Thus, an IPV specification for valuations, conditional on observed quality of paddy, is a reasonable assumption for this market.

1.3.1 Equilibrium Bidding Behavior

In the IPV setting, a bidder’s bid is the price at which he decides to drop out of the bidding. The strategy set of each bidder is the set of all (measurable) functions from the set of possible valuations to the set of possible bids; conditioning on his value, a strategy is just a bid or price at which the bidder plans to drop out of the bidding.

In an ascending auction, for bidder $i$ the payoff from bid $b_i$ is

$$\pi_i = \begin{cases} v_i - \max b_j & \text{if } b_i > \max b_j \\ 0 & \text{if } b_i < \max b_j \end{cases}$$

This is because if the bidder wins, he pays a price equal to that at which the last of the bidders, excluding him, dropped out; it is equal to the bid chosen by this last bidder who drops out. Given this payoff function, it is a weakly dominant strategy for each bidder $i$ to bid according to $b_i = v_i$ (see for instance, Krishna, 2002). If $b_i < v_i$, i.e., if the bidder chooses to drop out at a price lower than his value, there is positive probability that the winner will be some bidder $j$ who plans to drop out at a bid $b_j$ s.t. $b_i < b_j < v_i$. Then, bidder $i$ loses, whereas if he had planned to drop out at $v_i$, he would have won and got a payoff equal to $v_i - b_j$. On the other hand, bidding $b_i > v_i$ can cause bidder $i$ to win and pay a price higher than his value $v_i$, with positive probability. This is the essence of the optimality of $b_i = v_i$.

As a result, the win price or sale price in an ascending auction is essentially the second-highest bid, which coincides with the second-highest valuation in an IPV setting. This lies at the heart of uncovering the latent value distributions of the players in econometric analysis: if the price at which the last player dropped out in an auction is observed, when there are $p$ potential bidders, then under the assumption of equilibrium bidding, the realization of the second highest order

\[\text{With the large millers’ costs typically being lower than the smaller millers’ costs.}\]
statistic of the random variables $V_1, \ldots, V_p$ (the upper case is used here to denote random variables, with the lower case $v_1, \ldots, v_p$ denoting realized values of the bidders) is observed as the win price.

1.4 Nonparametric Identification and Semi-nonparametric Estimation of Value Distributions

In general, before attempting estimation of the bidders’ value distributions from the set of observables, one needs to check whether statistically, the former can be identified from the latter; i.e., whether there exists a unique inverse for the mapping from the latent distribution to the observable data. Certain economic assumptions and restrictions are typically imposed on the latent structure and the way in which that could have generated the sample data. For parametric estimation, identifiability is based on the premise that the functional form of the distribution from which the sample could have been drawn is known. Nonparametric identifiability results do not assume a functional form for the latent distribution and hence have more stringent data requirements.

For the independent private values framework, nonparametric identification results follow from Athey and Haile (2002). These build on earlier results by Arnold, Balakrishnan and Nagaraja (1992) and Meilijson (1981). We give a flavor of these results below for the symmetric case as derived by Arnold, Balakrishnan and Nagaraja (1992).

In the symmetric IPV model, the common distribution $F$ is identified from the win price (Athey and Haile, 2002, Theorem 1). Under symmetry, in an auction, the $p$ potential bidders’ values are a random sample $v_1, \ldots, v_p$ of size $p$ from the distribution $F$; and the win price is a realization of the second-highest order statistic $V^{(p-1:p)}$ from the random variables $V_1, \ldots, V_p$. The distribution $F^{(p-1:p)}(.)$ of the second-highest order statistic can be shown to satisfy

$$F^{(p-1:p)}(s) = p(p-1) \int_0^{F(s)} t^{(i-1)}(1-t)^{(p-i)} \, dt \quad \forall s.$$
Since the right-hand side is strictly increasing in $F(s) \in [0, 1]$, $F^{p-1}(s)$ uniquely determines $F(s)$ for every $s$. In other words, two alternative value distributions, say $F$ and $H$, cannot both result in the same distribution of the second-highest order statistic; for, $F(s) \neq H(s)$ for at least one $s$, and so the right-hand sides corresponding to $F(s)$ and $H(s)$ will be different. Thus in the symmetric IPV case, observing the distribution of the win prices, or the distribution of the second-highest order statistic, can yield the distribution from which bidders draw values.

The treatment of identification in the asymmetric case, where bidders draw values from different distributions is detailed in Appendix 1.2. In brief, in the asymmetric IPV model, assuming that each $F_V(.i)$ is continuous and that the support of the distributions $supp[F_V(.i)]$ is the same for all $i$, each $F_V(.)$ is identified if the win price and the identity of the winner are observed (Athey and Haile, 2002, Theorem 2). In this chapter, we use the asymmetric model, owing to the asymmetry in bidder values that we can infer from the market shares of the millers: recall that the 2 large millers won about 45% of the lots, while the others won uniformly small shares. Our data set contains information on the win price and the identity of the winner; thus the model is nonparametrically identified.

Estimation of value distribution through kernel density methods is not feasible since they require a substantial number of data points for each unique covariate vector of quality attributes and number of bidders. Our data set has only a few observations per such unique covariate vector.

We therefore use the semi-nonparametric technique to estimate the value distributions using a recently proposed strategy by Brendstrup and Paarsch (2006); see Chen (2007) for a general survey. We assume that at the $t^{th}$ auction for a lot with covariate vector $z_t$, the valuation of player $i$ of type $j$ is given by

$$\ln v_{it}^j = z_t \beta + \mu + u_{it}^j \quad (1.1)$$

The 10-tuple covariate vector $z_t$ is composed as follows: the first term is to allow for a baseline week 1 valuation and carries a value of ‘one’. The terms two

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7Following Chen (2007) and others, we define a semi-nonparametric model as one that has both finite and infinite dimensional parameters of interest $(\beta, \mu)$ and $u_{it}^j$. In contrast, the term semi-parametric model is used if the parameter of interest is finite dimensional and the nuisance parameter is infinite dimensional. In our application, the vector $(\beta, \mu)$ in Eq.(1.1) that follows is a finite-dimensional parameter and the density of $u_{it}^j$ is an infinite-dimensional parameter.
through eight record the quality of the lot with respect to the following seven characteristics: (i) moisture content, (ii) uniformity of grain, (iii) presence of chaff, (iv) presence of brokens, (v) lustre of grain, (vi) others and (vii) green and immature grain. The ninth and the tenth terms of $z_t$ are dummies to record whether the lot was sold in the second or third week respectively of the sample. The parameter vector $\beta$ that captures the marginal effect of each lot-specific characteristic, is unknown and needs to be estimated. By type or class of bidder we refer to two bidder classes: ‘1’ refers to the two large bidders; ‘2’ refers to the rest of the bidders. Owing to bid rotation, only one large bidder is present at each auction, the rest being the small bidders. So, $\mu$ is the parameter that captures the asymmetry between the two bidder classes

$$
\mu = \begin{cases} 
\neq 0 & \text{for the large bidders}(j = 1) \\
= 0 & \text{for small bidders}(j = 2) 
\end{cases}
$$

$u_{it}^j$ which is the idiosyncratic component of bidder $i$’s (of type $j$) valuation at auction $t$ is assumed to

1. be independently and identically distributed (i.i.d.) for all bidders with distribution function $F_U(.)$,

2. have $E[u_{it}^j | z_t] = 0$,

3. be independent of $z_t$.

Thus we will simply call $u_{it}^j$ as $u_t$.

### 1.4.1 Specifying the Valuation Density

We denote the large bidders’ valuation density and distribution functions by $f_{V_1}(.)$ and $F_{V_1}(.)$ and small bidders’ valuation density and distribution functions by $f_{V_2}(.)$ and $F_{V_2}(.)$ respectively.

Note that the auction data is a three-week long series; for simplicity, we assume the $u$ term to be i.i.d., while admitting temporal variation in values through dummies for the weeks. In part, this modeling springs from the possibility that over time, accumulation of inventory can affect millers’ values for the grain.

We approximate the density function $f_U(.)$ of $u$ by a Hermite series expansion. Gallant and Nychka (1987) show that a density with mean zero, support
can be estimated using a Hermite series. The Hermite series is in the form of a polynomial squared times a normal density function (with mean zero) with the coefficients of the polynomial restricted so that the series integrates to one. The rule for determining series length is data-dependent: the greater the sample size, the longer the length of the series (Chen, 2007).

Hermite polynomials are a class of orthogonal polynomials that have support over the entire real line and the Gaussian function \( \exp(-x^2/2) \) as the weighting function. The advantage of polynomial approximations using orthogonal polynomials (rather than ordinary polynomials) has to do with efficiency. The \( n^{th} \) order Hermite polynomial is defined by (Paarsch and Hong, 2006):

\[
H_n(x) = (-1)^n \exp\left(\frac{x^2}{2}\right) \frac{d^n}{dx^n}(\exp(-x^2/2))
\]

where \( \frac{d^n}{dx^n} \) refers to the \( n \)th derivative. The \( n^{th} \) order normalized Hermite polynomial is defined by \( h_n(x) = \frac{H_n(x)}{\sqrt{n!\sqrt{2\pi}}} \). Given the size of our sample (275 data points), a Hermite series of order two is reasonable; this gives ordinary polynomials of up to the 4th degree.

The first three Hermite polynomials are \( H_0(x) = 1, H_1(x) = x, H_2(x) = x^2 - 1 \). The first three normalized Hermite polynomials are \( h_0(x) = \frac{H_0(x)}{\sqrt{\sqrt{2\pi}}} \), \( h_1(x) = \frac{H_1(x)}{\sqrt{\sqrt{2\pi}}} \), \( h_2(x) = \frac{H_2(x)}{\sqrt{2\sqrt{2\pi}}} \).

Also, we employ as weighting function, a normal density with mean zero and standard deviation \( \sigma = 0.4 \). The Hermite polynomials must then also be suitably modified so that they are orthonormal with respect to the weighting function \( \exp(-x^2/(2\sigma^2)) \). The first three modified normalized Hermite polynomials are \( h_0^*(x) = \frac{H_0(x)}{\sqrt{1.0026}}, h_1^*(x) = \frac{H_1(x)}{\sqrt{0.5013}}, h_2^*(x) = \frac{H_2(x)}{\sqrt{0.752}} \).

The density function \( f_U(.) \) of \( u_t \) is approximated as

\[
\hat{f}_U(s) = \left[ \sum_{k=0}^{2} \gamma_k h_k^*(s) \right]^2 \exp\left[ \frac{-s^2}{2(0.4)^2} \right]
\]  

---

\(^8\)This ensures that we have a density function.

\(^9\)See Paarsch and Hong (2006). We are projecting a function onto a lower dimensional space. So they draw an analogy with OLS, where we project the dependent vector onto the range space of the data matrix \( X \). The efficiency analogy between orthogonal polynomials is with the case of OLS estimates when \( X \) is an orthogonal matrix.

\(^{10}\)To circumvent some computational problems, we performed the optimization routine for MLE over a grid of standard deviations for the weighting function, and the likelihood was maximized at a standard deviation of 0.4.
where $h_k^*(\cdot)$ is the $k^{th}$ order normalized (and modified) Hermite polynomial. $\gamma_k$ are the coefficient parameters to be estimated\textsuperscript{11}.

Let $G(\cdot|F_{V_1, V_2})$ be the joint distribution of the win price (second-highest order statistic) and the winner identity (the last remaining bidder). We need to estimate $F_{V_1}(\cdot), F_{V_2}(\cdot)$ using $G(\cdot|F_{V_1, V_2})$. This is implemented using the method of maximum likelihood.

Maximum likelihood estimation requires determination of $\hat{f}_{V_1}(\cdot), \hat{f}_{V_2}(\cdot)$ such that

$$
(\hat{f}_{V_1}, \hat{f}_{V_2}) = \arg\max_{f_{V_1}, f_{V_2}} \frac{1}{T} \sum_{t=1}^{T} \ln g(y_t|F_{V_1, V_2})
$$

where $g(y_t|F_{V_1, V_2})$ is the joint probability density function of the win price and the winner identity for lot $t$; we give details of this density shortly. From Equation (1.1), we see that the estimated densities $\hat{f}_{V_1}$ and $\hat{f}_{V_2}$ are related to $\hat{f}_{U}$, the estimate of the idiosyncratic component of value, as follows.

$$
\hat{f}_{V_1}(v) = \frac{1}{v} \hat{f}_{U}(\ln v - z_t \hat{\beta} - \hat{\mu})
$$

$$
\hat{f}_{V_2}(v) = \frac{1}{v} \hat{f}_{U}(\ln v - z_t \hat{\beta})
$$

To ensure that $\hat{f}_{V_1}, \hat{f}_{V_2}$ are in fact densities, the following constraints must be imposed

$$
\int_{0}^{+\infty} \hat{f}_{V_1}(y) \ dy = 1
$$

$$
\int_{0}^{+\infty} \hat{f}_{V_2}(y) \ dy = 1.
$$

This can be implemented through the following restriction on the parameter space

$$
\int_{-\infty}^{+\infty} \hat{f}_{U}(u) \ du = 1.
$$

From the definition of the Hermite series, this implies the following restriction on the Hermite coefficients:

\textsuperscript{11}Subject to the restriction $\sum_{k=0}^{2} \gamma_k^2 = 1$, in order that $\hat{f}_{U}(\cdot)$ is actually a density function (i.e., such that it integrates to one). See the next section.
\[ \gamma_0^2 + \gamma_1^2 + \gamma_2^2 = 1. \quad (1.9) \]

Thus to obtain \( \hat{f}_{V_1}, \hat{f}_{V_2} \), we maximize \( \sum_i^T \ln \hat{g}(\cdot) \) with respect to \( \hat{\beta}, \hat{\mu}, \hat{\gamma}_k \) \((k = 0, 1, 2)\) subject to the restriction that the norm of the Hermite coefficients equals one.

### 1.4.2 Specifying the Joint Density of the Win Price and Winner’s Identity

The number of potential bidders \( p \) at an auction is assumed to be the same as the number of distinct winners on the day of that auction. But assuming there is collusion among the two large bidders, the effective number of potential bidders at an auction reduces to \( p - 1 \).

The joint probability density of the win price and a specific small bidder winning is given by (Banerji and Meenakshi, 2004):

\[
\begin{align*}
\left( \begin{array}{c} p-3 \\ n-1 \end{array} \right) F_{V_1}(r)(F_{V_2}(r))^{p-n-2} & (1 - F_{V_2}(w))(n-1)(F_{V_2}(w) - F_{V_2}(r))^{n-2} f_{V_2}(w) \\
+ \left( \begin{array}{c} p-3 \\ n-2 \end{array} \right) (F_{V_2}(r))^{p-n-1} & (1 - F_{V_2}(w))[(n-2)(F_{V_2}(w) - F_{V_2}(r))^{n-3} f_{V_2}(w) \\
(F_{V_1}(w) - F_{V_1}(r)) + (F_{V_2}(w) - F_{V_2}(r))^{n-2} f_{V_1}(w)]
\end{align*}
\]

The first term in this equation corresponds to the case where the large bidder’s valuation is less than the start price \( r \). The probability that the large bidder’s valuation is less than \( r \) is \( F_{V_1}(r) \); the probability that a specific small bidder’s valuation is more than \( w \) is \( (1 - F_{V_2}(w)) \). The realized set of remaining \((n-1)\) bidders (all small) could be any one of \( \left( \begin{array}{c} p-3 \\ n-1 \end{array} \right) \) different possibilities. The probability that one of these has valuation equal to \( w \) while the rest have valuations between \( r \) and \( w \) is \( (n-1)f_{V_2}(w)(F_{V_2}(w) - F_{V_2}(r))^{n-2} \). Finally, the probability that the valuations of all the (remaining) \((p-n-2)\) potential bidders are less than the reserve price is \( (F_{V_2}(r))^{p-n-2} \).
The second term consists of two possibilities, the large bidder’s valuation being between $r$ and $w$, and it being exactly $w$. In either case, the probability that a specific small bidder’s valuation is greater than $w$ is $(1 - F_{V_2}(w))$, the realized set of non-winning active small bidders can be one of $\binom{p - 3}{n - 2}$ different combinations, and the probability that the $(p - n - 1)$ remaining small bidders are inactive is $(F_{V_2}(r))^{p-n-1}$. Given this, the probability that the large bidder’s valuation is exactly $w$ and that the $(n - 2)$ small bidders with valuations between $r$ and $w$ is $f_{V_1}(w)(F_{V_2}(w) - F_{V_2}(r))^{n-2}$; while the probability that the win price valuation belongs to one of $(n-2)$ small bidders and that the large bidder’s and $(n-3)$ small bidders’ valuations lie between $r$ and $w$ is $(n-2)f_{V_2}(w)(F_{V_1}(w) - F_{V_1}(r))(F_{V_2}(w) - F_{V_2}(r))^{n-3}$.

The joint probability density of the win price and a specific large bidder winning is given by
\[
\binom{p - 2}{n - 1} F(r)^{p-n-1}(1 - G(w))(n-1)(F(w) - F(r))^{n-2} f(w)
\]

The probability that a specific large bidder’s valuation is greater than the win price is $(1 - F_{V_1}(w))$. The realized set of the other $(n-1)$ active bidders who are all small is akin to a random draw from the set of $(p-2)$ small potential bidders and could be one of $\binom{p - 2}{n - 1}$ different combinations. The probability that one of these $(n-1)$ small bidders has valuation equal to the win price while the rest $(n-2)$ small non-winning active bidders’ valuations are between the reserve price and the win price is $(n-1)f_{V_2}(w)(F_{V_2}(w) - F_{V_2}(r))^{n-2}$. Finally, the probability that $(p-n-1)$ small bidders’ valuations are less than $r$ is $(F_{V_2}(r))^{p-n-1}$.

### 1.5 Estimated Value Distributions

We now present the results from our constrained maximum likelihood estimation using semi-nonparametric approximation to the true density functions. The estimation was performed by writing code in the programming language GAUSS. Constrained optimization routines in GAUSS based on quasi-Newton methods were used to estimate the parameters and Hermite coefficients that characterize the value distributions.
We have estimated the $10-tuple$ covariate coefficient vector $\beta$, the parameter capturing the difference in means $\mu$ and the coefficients in the Hermite series expansion $\gamma_0, \gamma_1, \gamma_2$. The estimates for $\beta$ and $\mu$ are listed in Table 1.3. The Hermite coefficients as estimated in our exercise are $\hat{\gamma}_0 = 0, \hat{\gamma}_1 = 0, \hat{\gamma}_2 = 1$.

Thus the estimated densities are

$$\hat{f}_{V_1}(v) = \frac{1}{v} \hat{f}_U(ln v - z_t \hat{\beta} - \hat{\mu})$$

$$\hat{f}_{V_2}(v) = \frac{1}{v} \hat{f}_U(ln v - z_t \hat{\beta})$$

where,

$$\hat{\beta} = (5.9481, 0.0336, 0.0185, 0.0203, 0.0205, 0.0263, -0.0003, 0.0049, -0.0559, 0.0039)$$

$$\hat{\mu} = -0.1468$$

$$\hat{f}_U(s) = \frac{(s^2-1)^2}{0.16} \exp[-s^2/2(0.4)^2].$$

Since all the seven paddy characteristics are measured on a scale that is increasing in quality (either 1 to 2 or 1 to 3), they are expected to have positive signs. Our results are consistent with this expectation. Moisture content has the largest coefficient (also highly significant) suggesting that a substantial quality premium is associated with this characteristic. Three of the characteristics, viz., brokens, green and immature grain and other are found to be not significant in the sample. A negative coefficient for week 2 dummy implies that a lot of a specific quality has a lower valuation in week 2 than in week 1. One reason for this could be that anxiety to accumulate stocks is greater in week 1 and thus increases the valuations (the two large bidders for instance, had made 80% of their purchases by the end of week 1). There could be other reasons as well, such as the prices in this market get influenced by those prevailing in the other bigger markets.

A negative sign on the parameter estimate (highly significant) for difference in means implies that the large bidders draw their valuations from a higher distribution than the small bidders, i.e., given a lot with a particular quality vector, the large bidders have a greater valuation for it than the small bidders.

The semi-nonparametrically estimated density $\hat{f}_U$ (Figure 1.1) is essentially truncated (i.e., has positive support on a limited interval), implying that the probability of the valuation of a bidder for a lot being too far from the expected valuation for that quality is zero. This is in contrast to popular parametrizations such as the lognormal density, the log of which never cuts the axes.

Indeed, the valuation distributions of bidders in this market have previously been estimated parametrically, using a lognormal distribution, see Meenakshi and
Lognormal functional forms are quite popular with researchers seeking to approximate positive (or right) skewed distributions because they allow flexibility on two accounts - location and variance. We therefore test whether for the asymmetric bidders specification, the semi-nonparametrically estimated model is indeed an improvement over the lognormal approximation.

Let \( g_{lnrml}(.) \) denote the log-likelihood of a data point from estimation assuming the lognormal model and let \( g_{snp}(.) \) denote the log-likelihood of the data point from the semi-nonparametric model. Using the vectors of the maximized log-likelihood for all data points from the above two estimated specifications, we construct the following vector

\[
m_t = \ln \left( \frac{g_{lnrml}(y_t | z_t)}{g_{snp}(y_t | z_t)} \right)
\]

where \( t \) refers to the \( t \)'th data point in the sample. Vuong’s statistic (Vuong, 1989) for testing the non-nested hypothesis of the lognormal model versus the semi-nonparametric model is

\[
vstat = \sqrt{T} \left[ \frac{1}{T} \sum_{t=1}^{T} m_t \right] \sqrt{\frac{1}{T} \sum_{t=1}^{T} (m_t - \bar{m})^2}
\]

Vuong’s statistic for comparing non-nested models has a limiting standard normal distribution. It is a bidirectional statistic.

\[
|vstat| < 2 \Rightarrow \text{the test does not favor one model or the other}
\]

\[
vstat > 2 \Rightarrow \text{the lognormal is more plausible}
\]

\[
vstat < -2 \Rightarrow \text{the semi-nonparametric model is more plausible}
\]

Following our estimation of the two models, the value of the Vuong’s statistic for testing the lognormal versus the semi-nonparametric model is computed to be \(-10.21046\), which strongly suggests that the semi-nonparametric model is closer to the actual data-generating process.

### 1.6 Optimal Reserve Prices

As noted in Section 1.3, in an IPV ascending auction, bidders’ bids are the prices at which they plan to drop out of the bidding. The reserve price set by an auctioneer has the following implication: if all bids for the lot are lower than the reserve price, then the lot is unsold; if one bid is higher than the reserve price and the others are all lower, then in an ascending auction the sale price of the lot equals the reserve price; and if more than one bid exceeds the reserve price, the reserve
price has no effect on the sale or the sale price, which is the second highest of the bids; this is where the last but one bidder drops out of the auction, the bidding stops, and this dropout price becomes the sale price. Thus the choice of reserve price in an ascending auction affects the revenue that the seller expects from it. Raising the reserve price can increase revenue if the reserve price is higher than the second highest bid; but if it is set too high, the risk is that the bidders’ bids will turn out to be too low and the lot will go unsold.

The choice of the reserve price (the threshold price $r$ below which the seller does not sell the object) constitutes an important instrument with the seller to take advantage of his monopoly power to increase his profits from an auction. Let $Y_i$ denote the $i^{th}$ highest order statistic or bid, from the values drawn by the bidders. Selecting a reserve price to maximize expected profits (such a reserve price is known as an ‘optimal reserve price’) balances the tradeoff between not selling the object (in the event that $r > Y_1$), with the possibility of a higher revenue ($= r - Y_2$) in case the reserve price lies between the highest and the second-highest valuations (in the event $Y_2 < r < Y_1$).

1.6.1 Optimal Reserve Prices under Collusion and Asymmetry

With collusion between the two large bidders taking the form of bid-rotation (both never participate in any one auction together), the analysis and derivation of optimal reserve price follows the assumption of non-cooperative behavior among the bidders with the number of large bidders at an auction being equal to 1.

Suppose there are $N_i$ bidders of type $i, i = 1, 2$ (large and small bidders respectively). If a specific bidder of the $i^{th}$ type wins the auction, the distribution of the max of the values of all other bidders is given by $G_i(.)$ below:

$$G_1(x) = F_2(x)^{N_2} F_1(x)^{N_1 - 1}$$

$$G_2(x) = F_1(x)^{N_1} F_2(x)^{N_2 - 1}$$

The expected payment of a bidder of type $i$ with value $x \geq r$ is
\[ m_i(x, r) = r G_i(r) + \int_r^x y g_i(y) dy. \] (1.10)

Note that in this section, we use the symbol \( x \) to represent a bidder’s value for a lot. The first term captures the expected payment of a bidder if none of the other bidders’ values is greater than \( r \), the probability of which happening is \( G_i(r) \). If on the other hand, there is (are) other bidder(s) with values exceeding \( r \), but less than \( x \), the object is sold to our bidder at the highest of other values; so as the auctioneer raises the price from \( r \), the probability of our bidder winning at a price \( y \), is \( g_i(y) \); thus, the second component of the expected payment integrates from \( r \) through \( x \), the product of each value with the probability of our bidder winning at that value.

Let \( w \) denote the win price. Then the \textit{ex-ante}\textsuperscript{12} expected payment of a bidder of type \( i \) is

\[
E[m_i(X, r)] = \int_r^w m_i(x, r) f_i(x) dx
\]

\[ = r(1 - F_i(r)) G_i(r) + \int_r^w y(1 - F_i(y)) g_i(y) dy \] (1.11)

The overall expected payoff of the seller from setting a reserve price \( r \geq x_0 \) is

\[
\Pi = N_1 E[m_1(X, r)] + N_2 E[m_2(X, r)] + F_1(r)^{N_1} F_2(r)^{N_2} x_0 \] (1.12)

where \( x_0 \) denotes the seller’s reservation utility. In this context it refers to the price that the farmer can expect to get elsewhere for a lot that goes unsold in the present auction.

Differentiating \( \Pi \) with respect to \( r \)

\[
\frac{d\Pi(r)}{r} = N_1 \frac{d}{dr} E[m_1(X, r)] + N_2 \frac{d}{dr} E[m_2(X, r)]
\]

\[ + N_1 F_1(r)^{N_1-1} f_1(r) F_2(r)^{N_2} x_0 \]

\[ + N_2 F_1(r)^{N_1} F_2(r)^{N_2-1} f_2(r) x_0 \] (1.13)

where

\textsuperscript{12}Before the value is drawn.
\[
\frac{d}{dr} E[m_i(X,r)] = [1 - F_i(r) - r f_i(r)] G_i(r) \quad (1.14)
\]

Thus

\[
\frac{d\Pi(r)}{dr} = N_1 [1 - F_1(r) - r f_1(r)] G_1(r) + N_2 [1 - F_2(r) - r f_2(r)] G_2(r)
\]

\[+ N_1 f_1(r) G_1(r) x_0 + N_2 f_2(r) G_2(r) x_0 \quad (1.15)\]

Since

\[
f_i(r) = \lambda_i(r) [1 - F_i(r)] \quad (1.16)
\]

where \(\lambda_i(.)\) is the hazard-rate function, we have

\[
\frac{d\Pi(r)}{dr} = N_1 [1 - F_1(r) - r \lambda_1(r) (1 - F_1(r))] G_1(r)
\]

\[+ N_2 [1 - F_2(r) - r \lambda_2(r) (1 - F_2(r))] G_2(r)
\]

\[+ N_1 \lambda_1(r) [1 - F_1(r)] G_1(r) x_0
\]

\[+ N_2 \lambda_2(r) [1 - F_2(r)] G_2(r) x_0
\]

\[= N_1 [1 - (r - x_0) \lambda_1(r)] (1 - F_1(r)) F_2(r)^{N_2} F_1(r)^{N_1 - 1} \]

\[+ N_2 [1 - (r - x_0) \lambda_2(r)] (1 - F_2(r)) F_1(r)^{N_1} F_2(r)^{N_2 - 1} = 0 \quad (1.17)\]

Dividing throughout by \(F_1(r)^{N_1 - 1} F_2(r)^{N_2 - 1}\) we get

\[
N_1 [1 - (r^* - x_0) \lambda_1(r^*)] [1 - F_1(r^*)] F_2(r^*)
\]

\[+ N_2 [1 - (r^* - x_0) \lambda_2(r^*)] [1 - F_2(r^*)] F_1(r^*) = 0 \quad (1.18)\]

The above equation gives for a lot of a specific quality, the first order condition for profit maximization of the seller. Putting \(N_1 = 1\), we get the expression (in implicit form) for the optimal reserve price \(r^*\) under simple bid rotation by the 2 large bidders in our data. Given our parameter estimates and the lot-specific covariates, Eq.(1.18) is solved for \(r^*\) for every lot in the data. These are our optimal
reserve price estimates. Confidence intervals around these are constructed using the Delta Method (see Appendix 1.3).

### 1.6.2 Calculating the Farmer’s Reservation Utility

The farmer’s reservation utility $x_0$ is unknown to us, but based upon our knowledge of the functioning of the market (gained by interviewing the farmers) we can impute a value to $x_0$. We describe below our procedure for doing this; we note though that our results are quite robust to varying this reservation utility in a band around the value that we actually ascribe to $x_0$. Part of the reason is that with a large number of bidders, the probability of a lot going unsold is low, so the third term in Eq. (1.12) is small; indeed lots in this auction market go unsold quite infrequently - possibly less than 2% of the time.

In the event of a lot going unsold at the formal auction, it goes back to the shop of the *katcha arhtia* through whom the farmer sells his grain. Typically, it gets sold to some private miller later through mutual negotiations. The private miller is free to opt out of this possibility of a purchase and not get anything; the farmer is free to opt out of the sale and take his grain to some other market to sell. Thus it is useful to think of these negotiations in terms of bilateral bargaining with outside options.

We use a stylized model here, namely the complete information bargaining model of Rubinstein (1982), augmented with outside options (see for example Muthoo, 1997 for a textbook exposition). Let the miller’s (buyer’s) valuation for the lot be $v$. We denote by $s$, the seller’s (farmer’s) use value for the lot, which is the worth that the farmer attaches to a lot that does not get sold anywhere and is eventually privately used. In the absence of outside options, the subgame perfect equilibrium shares of the players in Rubinstein’s model are $r_B \frac{s}{r_B+r_S} (v-s)$ (buyer), and $r_S \frac{r_B}{r_B+r_S} (v-s)$ (seller), where $r_B$ and $r_S$ are the buyer’s and seller’s respective discount rates. In the presence of outside options, the unique subgame perfect equilibrium division of the surplus $(v-s)$ can be either the Rubinstein division, or a division in which one of the players gets a payoff equal to his outside option and the other gets the residual surplus. The idea is that if an outside option payoff is larger than what a player gets as his Rubinstein payoff, then the other player is forced to concede this payoff in the bargaining.

We assume the buyers (millers) to have a discount rate equal to 15% per annum.
(which corresponds to the rate at which they could have borrowed from banks at that time), while the sellers (farmers) were able to borrow from the co-operative societies or the *katcha arhtias* at about 2% per month (i.e., 24% per annum). In the course of our interviews with them, farmers stated that if a lot goes unsold, then transporting it and selling it elsewhere (possibly at another market where auctions are not employed) can mean a discount of up to Rs 100 compared to the price obtainable in this market through auctions. We therefore estimate the outside option of the farmer, for each lot, as the expected second-highest valuation for that quality minus a penalty amount of Rs 100. On the other hand, if the miller opts out of the bargaining, his payoff is zero; in effect, the model then is one of alternating offers bargaining with an outside option for the farmer/seller.

The farmer’s equilibrium payoff in the bargaining model with an outside option available to him is the larger of his Rubinstein share \( s + \frac{r_B}{r_B + r_S} (v - s) \) and his outside option. This equilibrium payoff is the \( x_0 \) that we plug into Eq.(1.12). With \( v \) fixed at a small miller’s expected valuation for the lot and \( s \) being allowed to vary from zero to a small miller’s expected valuation discounted by Rs 100, we find that this reservation utility (\( x_0 \)) for the lot equals the farmer’s payoff from the outside option.

### 1.6.3 Observed and Optimal Reserve Prices, and Expected Revenues

The mean of the estimated optimal reserve prices is Rs 517.08, which is about Rs 33 higher than the mean of the observed reserve prices of Rs 483.86 (Table 1.4). The observed reserve prices are also seen to lie below the 45 degree line when plotted against the optimal reserve prices (Figure 1.2). The mean absolute difference between the two series is Rs 35.21. The t-statistic for difference in means of the two series is -14.28 and the confidence intervals around the optimal reserve prices are within Rs 2, so there is a significant difference between these and the observed reserve prices (see Table 1.4).

The optimal reserve prices also closely reflect the quality of the paddy lots, unlike the observed reserve prices. To see this, we present a plot of the optimal and the observed reserve prices of lots arranged in increasing order of quality (Figure 1.3); we use the expected second-highest valuation as a proxy for quality. There is a monotonically increasing relationship between quality and optimal reserve prices.
This is to be expected given the theoretical relationship between the expected second-highest value and the optimal reserve price, as both vary positively with the quality of a lot. The optimal reserve price in the asymmetric bidders specification is a weighted average of the inverse hazard rates of the distributions of the two types of bidders; as the quality improves, inverse hazard rates increase, and so does the the optimal reserve price.

The plot of the observed reserve prices by contrast, has a lot of noise, though the overall relationship with quality is positive (Figure 1.3). Moreover, there appears to be clumping around certain reserve prices such as 480, 500, 520. These are probably salient reserve prices in the auctioneer’s mind, corresponding to ‘quality grades’. Finally, we note that varying the level $x_0$ of the reservation utilities of the seller leads to variation in the levels of the optimal reserve prices, but not the degree of monotonicity of the optimal reserve prices with respect to the quality and the tight relationship between the two (Figure 1.4). Irrespective of the level, the discrepancy between this close relationship between optimal reserve prices and the absence of it in the case of the observed reserve prices becomes obvious.

Having established that the reserve prices set by the auctioneer are on average significantly lower than the optimal reserve prices, it is natural, and of greater importance, to ask: by how much would farmers’ expected revenues increase if the reserve prices are optimally set, as compared with expected revenues that accrue if the reserve prices are as in the data set? To evaluate this, we compute the expected revenue for each lot under the alternative scenarios that the reserve price equals (i) the observed reserve price and (ii) the optimal reserve price. We do this by estimating the expression for expected revenue (Eq. (1.12)) for each lot, at the two alternative reserve prices. The mean difference between the two series of expected revenues at Rs 2.40 is very small in magnitude and not significant at a level of 5%. Thus the non-optimality of the reserve prices does not make a significant difference to farmers’ revenues.

So in a substantial sense, the auction market functions well: expected revenue to the seller is close to the maximum expected revenue. One reason is that the reserve prices that the auctioneer sets, although quite a bit lower than what is optimal, are still not really out of the ball park, relative to the reserve price that maximizes expected revenue. The experience of the auctioneer, and the auctioneer’s payoff being linked to the sale price of the lot, provide incentives and reason for this good market performance.
1.6.4 Impact of Competition on Expected Revenues

Given the arguments of the expected revenue function in Eq. (1.12), it is natural to ask if the choice of $x_0$ makes a difference to expected revenues. To assess this, we compute the optimal reserve prices under alternative penalty amounts of Rs 110 and Rs 120; the results are presented in Table 1.5. The means of the corresponding optimal reserve prices are estimated to be Rs 508.84 and Rs 500.69, both significantly different from the mean of the observed reserve prices\(^{13}\). The expected revenues under these alternative (relatively lower) optimal reserve prices are even closer to those under observed reserve prices (Table 1.5). We thus find that variations in $x_0$ do not affect expected revenue significantly given the level of bidder competition in the market. This is because the probability of all of the bidders’ values falling short of a reasonable reserve price is very low, given the extent of market competition.

However, the level of competition itself is an important factor that affects how expected revenue varies with the reserve price. Consider the expected revenue curve as a function of the reserve price. This attains a maximum at the optimal reserve price; choosing a sub-optimal reserve price results in lower expected revenue. We find that under conditions satisfied by the data, a higher number of bidders results in a flatter expected revenue curve; and consequently, a smaller revenue loss from a sub-optimal reserve price. We explore here one sufficient condition under which the expected revenue becomes flatter with a larger number of bidders. We consider first a symmetric setting. Suppose $N$ is the number of potential bidders and $r^*$ is the optimal reserve price.

**Proposition 1** Consider an ascending auction with $N$ symmetric bidders in the independent private values setting. Suppose $(1 + N \log F(r^*)) < 0$. Then expected revenue as a function of the reserve price becomes less concave around $r^*$ as the number of bidders $N$ increases.

**Proof.** See the Appendix. Note that in a symmetric setting, $r^*$ does not depend on $N$ (Krishna, 2002). So, even with an optimal reserve price around the median of $F$, the condition $(1 + N \log F(r^*)) < 0$ is met whenever there are at least 2 bidders.

\(^{13}\) $t$-statistics (Welch two-sample t-test) for differences in means being -10.75 and -7.26 for the optimal reserve prices with penalty amounts 110 and 120 respectively compared to the reserve prices set by the auctioneer.
The proposition for an asymmetric market similar to that in our data is stated next.

**Proposition 2** Consider an ascending auction with asymmetric bidders such that one bidder’s value is drawn from $F_1(.)$ while the remaining $N$ bidders’ values are drawn from $F_2(.)$, and values are independently distributed. The concavity of the expected revenue as a function of the reserve price decreases as $N$ increases provided \[ \frac{d}{dr} \left( (1 + N \log F_2(r^*))(1 - F_2(r^*) - (r^* - x_0)f_2(r^*)) \right) > 0, \] where $r^*$ is the optimal reserve price.

**Proof.** See the Appendix. While the propositions provide sufficient conditions for expected revenue to respond weakly to changes in reserve price as the number of potential bidders increases, we also find the expected revenue curve to behave similarly in our data. This is demonstrated using simulations. Since those simulations have direct bearing on the subject of Chapter 2, they are discussed there. For now, suffice it to say that with one large bidder and six to eight small bidders, as in our data, the expected revenue curve turns out to be rather flat: so that, moderately well-targeted reserve prices achieve close to maximal expected revenues.

### 1.7 Conclusions

In this chapter, we have analyzed a paddy auction market using the structural estimation approach to investigate how well it functions in terms of maximizing the expected revenue of the paddy selling farmers? We find that although the reserve or start prices that the auctioneer chooses are not optimal, the differences from the optimal reserve prices are small enough that the expected revenues from the auctions are close to the optimum. The following factors appear to be responsible for this. First, the auctioneer has several years of market experience and this could translate into a reasonable idea about the value distributions of bidders, as well as of the relationship between reserve prices and expected revenues. Second, a relatively large number of bidders tends to make the expected revenue curve (as a function of the reserve price) somewhat flat around the maximum; so, errors within bounds, in finding the optimal reserve price, do not result in large departures of
expected revenue from the maximum. Third, the auctioneer is paid a (small)
percentage of the sale price; so his incentives are aligned with maximizing revenue
from each sale. Finally, the presence of the sellers (farmers) in the marketplace
possibly disciplines market transactions.

This is a time of debate and reform of the APMC Act, and many of the
reforms are positive changes on existing practices and institutions. However, it
is surprising that in this environment, the basic trading mechanism of auctions
that the original act espoused is nowhere in the discussions. A significant factor
behind this is likely the lack of systematic studies of these auction markets; how
well they function, how well they serve sellers (farmers) and buyers. Our findings
in this chapter contribute not just by expanding our knowledge of the functioning
of such an auction market for paddy, but by pointing to its success in terms of
expected revenue outcomes for farmers. Together with earlier studies that use
structural estimation of auctions for grain markets (e.g. Meenakshi and Banerji,
2005) and find that the downward impact of collusion on prices is limited, this
leads us to suggest that grain auctions as practiced under the original APMC act
have strengths that have gone unnoticed, and that should be part of the discourse
and design of reforms.
1.8 Tables and Figures

Table 1.1 Summary Statistics of the Sample

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserve price (Rs/quintal)</td>
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<td>30.01</td>
<td>350</td>
<td>580</td>
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<tr>
<td>Win price (Rs/quintal)</td>
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<td>23.64</td>
<td>400</td>
<td>611</td>
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<td>No. of potential bidders per lot</td>
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<td>0.66</td>
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<td>No. of active bidders per lot</td>
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<td>0.70</td>
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<td>Moisture content</td>
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<td>0.57</td>
<td>1</td>
<td>3</td>
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<tr>
<td>Uniformity in grain size</td>
<td>2.43</td>
<td>0.57</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Presence of chaff</td>
<td>2.07</td>
<td>0.56</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Presence of brokens</td>
<td>1.46</td>
<td>0.50</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Grain lustre</td>
<td>1.59</td>
<td>0.49</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Green and immature grain</td>
<td>1.17</td>
<td>0.38</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Others</td>
<td>1.37</td>
<td>0.48</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Notes

1. Sample size = 275

2. Potential bidders are those interested in buying a paddy lot before its reserve price is announced

3. Active bidders are those whose valuation for a paddy lot exceeds the reserve price and hence stay on at the auction after the reserve price is announced

4. Quality attributes are categorical variables taking integer values as follows

   (a) Quality in terms of moisture content, uniformity in grain size and presence of chaff improving on a scale from 1 to 3

   (b) Quality in terms of presence of brokens, grain lustre, green and immature grain and others improving on a scale from 1 to 2
Table 1.2 Regressing Log Win Price on Quality Characteristics, Week Dummies and Number of Bidders

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>6.0640</td>
<td>571.7332***</td>
</tr>
<tr>
<td>Moisture content</td>
<td>0.0222</td>
<td>8.4927***</td>
</tr>
<tr>
<td>Uniformity of grain</td>
<td>0.0158</td>
<td>5.8711***</td>
</tr>
<tr>
<td>Presence of chaff</td>
<td>0.0095</td>
<td>3.4803***</td>
</tr>
<tr>
<td>Presence of brokens</td>
<td>0.0090</td>
<td>2.7540***</td>
</tr>
<tr>
<td>Lustre of grain</td>
<td>0.0137</td>
<td>4.1303***</td>
</tr>
<tr>
<td>Others</td>
<td>0.0020</td>
<td>0.6745</td>
</tr>
<tr>
<td>Green and immature grain</td>
<td>0.0085</td>
<td>2.2149**</td>
</tr>
<tr>
<td>Week 2 dummy</td>
<td>-0.0298</td>
<td>-7.7864***</td>
</tr>
<tr>
<td>Week 3 dummy</td>
<td>-0.0044</td>
<td>-1.2213</td>
</tr>
<tr>
<td>Number of active bidders</td>
<td>0.0038</td>
<td>1.8303*</td>
</tr>
</tbody>
</table>

Notes

1. Sample size = 275
2. Sample size by week
   (a) week 1 : 95
   (b) week 2 : 70
   (c) week 3 : 110
3. *, **, *** indicate significance at 10%, 5%, 1% levels respectively
4. t-statistics are calculated using robust standard errors.
### Table 1.3 Semi-nonparametric Maximum Likelihood Estimates for the Asymmetric Bidders Specification

<table>
<thead>
<tr>
<th>Parameter estimate</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>5.9481</td>
</tr>
<tr>
<td>Moisture content</td>
<td>0.0336</td>
</tr>
<tr>
<td>Uniformity of grain</td>
<td>0.0185</td>
</tr>
<tr>
<td>Presence of chaff</td>
<td>0.0203</td>
</tr>
<tr>
<td>Presence of brokens</td>
<td>0.0205</td>
</tr>
<tr>
<td>Lustre of grain</td>
<td>0.0263</td>
</tr>
<tr>
<td>Others</td>
<td>-0.0003</td>
</tr>
<tr>
<td>Green and immature grain</td>
<td>0.0049</td>
</tr>
<tr>
<td>Week 2 dummy</td>
<td>-0.0559</td>
</tr>
<tr>
<td>Week 3 dummy</td>
<td>0.0039</td>
</tr>
<tr>
<td>Difference in means</td>
<td>-0.1468</td>
</tr>
<tr>
<td>Mean log-likelihood</td>
<td>-3.32904</td>
</tr>
</tbody>
</table>

#### Notes

1. Sample size = 275
2. Model estimated:

\[
(\hat{f}_{V_1}, \hat{f}_{V_2}) = \text{argmax}_{f_{V_1}, f_{V_2}} \frac{1}{T} \sum_{t=1}^{T} \ln g(y_t|F_{V_1, V_2})
\]

where \(g(y_t|F_{V_1, V_2})\) is the joint probability density function of the win price and the winner identity for lot \(t\)

while \(\hat{f}_{V_1}\) and \(\hat{f}_{V_2}\) are the estimated densities of the large and small bidders’ values given by

\[
\hat{f}_{V_i}(v) = \frac{1}{v} \hat{f}_U(ln v - z_i\beta - \mu)
\]
\[ \hat{f}_{V_2}(v) = \frac{1}{v} \hat{f}_U(\ln v - z_t\hat{\beta}) \]

where \( z_t = \) covariate vector, \( \beta = \) covariate coefficient vector, \( \mu = \) difference in means.

The 10-tuple covariate vector \( z_t \) is composed as follows: the first term is to allow for a baseline week 1 valuation and carries a value of ‘one’. The terms two through eight record the quality of the lot with respect to the following seven characteristics: (i) moisture content, (ii) uniformity of grain, (iii) presence of chaff, (iv) presence of brokens, (v) lustre of grain, (vi) others and (vii) green and immature grain. The ninth and the tenth terms of \( z_t \) are dummies to record whether the lot was sold in the second or third week respectively of the sample.

\( \hat{f}_U \) the density function of the random component is estimated as

\[ \hat{f}_U(s) = \left[ \sum_{k=0}^{2} \gamma_k h_k^*(s) \right]^2 \exp\left[-\frac{s^2}{2(0.4)^2}\right] \]

where \( h_k^*(,) \) is the \( k^{th} \) order normalized (and modified) Hermite polynomial, \( \gamma_k \) are the coefficient parameters to be estimated

3. Mean log-likelihood= \( \frac{1}{T} \sum_{t=1}^{T} \ln g(y_t|F_{V_1, V_2}) \)

4. *, **, *** indicate significance at 10\%, 5\%, 1\% levels respectively
Table 1.4 Optimal versus Observed Reserve Prices

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Expected Revenue Mean</th>
<th>Expected Revenue Std Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed start price</td>
<td>483.86</td>
<td>1.81</td>
<td>350</td>
<td>580</td>
<td>517.61</td>
<td>1.36</td>
</tr>
<tr>
<td>Optimal reserve price</td>
<td>517.08</td>
<td>1.46</td>
<td>449.33</td>
<td>556.64</td>
<td>520.01</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Notes

1. Sample size = 275

2. Expected revenues and optimal reserve prices are computed assuming a baseline penalty of Rs 100. Penalty refers to the discount at which a farmer would have to sell the lot (elsewhere, in another market) if it remains unsold at the auction as well as in mutual negotiations with a miller in this mandi.

3. Welch two sample t-statistic for difference in means of start prices and optimal reserve prices = -14.28.

Table 1.5

Reserve Prices and Expected Revenues - Various Penalties

<table>
<thead>
<tr>
<th>Penalty (r$S$)</th>
<th>Observed Reserve Price (r$S$)</th>
<th>Optimal Reserve Price (r$^*$)</th>
<th>Expected Revenue with r$S$</th>
<th>Expected Revenue with r$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>483.86</td>
<td>517.08</td>
<td>517.61</td>
<td>520.01</td>
</tr>
<tr>
<td>110</td>
<td>483.86</td>
<td>508.84</td>
<td>516.39</td>
<td>518.02</td>
</tr>
<tr>
<td>120</td>
<td>483.86</td>
<td>500.69</td>
<td>515.16</td>
<td>516.29</td>
</tr>
</tbody>
</table>

Note: Refer to Note (2) in Table 1.4 for the meaning of ‘penalty’.
Note: Figure 1.1 provides a plot of $u$, estimated using Hermite polynomials, where $u$ is the additive stochastic term in the specification of bidder value in Eq.(1.1).
Figure 1.2: Observed Reserve Prices against Optimal Reserve Prices
Figure 1.3: Observed and Optimal Reserve Prices against Quality

- Observed reserve prices
- Optimal reserve prices with penalty = 100

Quality (expected second-highest valuation)

Reserve prices (Rs/quintal)
Figure 1.4: Optimal Reserve Prices (various penalties) against Quality

- ▲ optimal reserve prices with penalty=100
- ■ optimal reserve prices with penalty=110
- ○ optimal reserve prices with penalty=120

quality (expected second-highest valuation)
reserve prices (Rs/quintal)
1.9 Appendix

Appendix 1.1. A Brief Description of Auctions as Bayesian Games

Bayesian Games

A Bayesian game can in general be described as a strategic game consisting of:

1. a set $\mathcal{N}$ of players
2. for each player $i \in \mathcal{N}$ a nonempty set $\mathcal{A}_i$ of actions
3. for each player $i \in \mathcal{N}$ a set of signals $\mathcal{X}_i$
4. for each player $i \in \mathcal{N}$ a payoff function $u_i : \times_j \mathcal{A}_j \times_j \mathcal{X}_j \rightarrow \mathbb{R}$
5. a probability distribution $f$ over the product set of signals $\times_j \mathcal{X}_j$.

A pure strategy for player $i$ is a function $\alpha_i : \mathcal{X}_i \rightarrow \mathcal{A}_i$ mapping signals into actions. Once the signals $X$ have been drawn according to $f$, each player $i$, based on the realization $X_i = x_i$ of his signal, chooses an action $a_i$. Payoffs to players depend on the profile of signals $x = (x_1, \ldots, x_N)$ and profile of actions $a = (a_1, \ldots, a_N)$ of all players. So, a pure strategy for a player $i$ specifies what he would do in the event of receiving every possible signal $x_i \in X_i$.

A pure strategy Bayesian-Nash equilibrium of a game of incomplete information is a vector of strategies $\alpha^*$ such that $\forall i, \forall x_i \in \mathcal{X}_i, \forall a_i \in \mathcal{A}_i$,

$$E[u_i(\alpha^*(X), X)|X_i = x_i] \geq E[u_i(a_i, \alpha^*_{-i}(X_{-i}), X)|X_i = x_i]$$

where $\alpha^*(x) = (\alpha^*_i(x_i))_{i \in \mathcal{N}}$ denotes the vector of actions of all players and $\alpha^*_{-i}(X_{-i}) = (\alpha^*_j(x_j))_{j \neq i}$ denotes the vector of actions of the other players. The expectation here is taken over the signal space of all other players: so in a Bayesian game, Player $i$ knows his signal but not that of the others.
Auctions as Bayesian Games

The general formulation of auction models as Bayesian games is considerably more general than the independent private values (IPV) model that we use; we will only describe the latter, briefly. In an IPV model, the signals of the \( N \) players are called values, and the set of values player \( i \) is \( V_i \). \( V_i \) is interpreted to be a random variable, and \( V_1, \ldots, V_N \) are independent random variables with densities \( f_1, \ldots, f_N \). In an ascending auction, a pure strategy for player \( i \) assigns, to each possible value \( v_i \in V_i \), a bid \( b_i \); \( b_i \) is the price at which player \( i \) plans to drop out of the bidding, given that his value is \( v_i \).

Appendix 1.2. Nonparametric Identification with Asymmetric Bidders

In the asymmetric IPV model, assuming that each \( F_j \) is continuous and that the support of the distributions \( \text{supp}[F_j] \) (the support of the distribution) is the same for all \( j \), each \( F_j \) is identified if the win price and identity of the winner are observed (Athey and Haile 2002, Theorem 2).

Consider an ascending auction with \( J \) bidders, and assume that their value distributions are distinct. This is just for simplicity; everything below holds if there are \( N \) bidders, of \( J \) distinct types, in the sense of their being \( J \) distinct value distributions, \( J < N \); i.e. if some bidders draw values from identical distributions. The valuation distribution of bidder \( j \) is given by \( F_j(.) \). Assume that each \( F_j(.) \) is continuous and that \( \text{supp}[F_j(.)] \) is the same for all \( j \). For each bidder \( j \), let \( G_j(.) \) be the joint distribution of the win price and the winner being bidder \( j \); let \( g_j(.) \) be the corresponding density.

Consider bidder \( J \). Bidder \( J \) wins at some price less than or equal to \( x \) with probability \( \int_0^x \frac{g_J(t)}{1-F_J(t)} dt \). The numerator within the integral is the joint density of the win price being equal to \( t \), \( 0 \leq t \leq x \) and the winner being equal to \( J \); the denominator says that bidder \( j \)'s value is greater than \( t \). On the other hand, bidder \( J \) wins at some price less than or equal to \( x \) only if bidders \( 1, \ldots, J-1 \) all have drawn values less than or equal to \( x \), the probability of which equals
\( F_1(x) \ldots F_{J-1}(x) \). In fact, it can be shown that these two probabilities are the same:
\[
F_1(x) \ldots F_{J-1}(x) = \int_0^x \frac{g_j(t)}{1-F_j(t)} \, dt.
\]
Or taking logs,
\[
\log F_1(x) + \ldots + \log F_{J-1}(x) = \log \left( \int_0^x \frac{g_j(t)}{1-F_j(t)} \, dt \right).
\]

This is the first equation in the system of equations below; the subsequent equations are for the cases of winners being bidders 1 to \( J-1 \).

\[
\begin{pmatrix}
1 & 1 & 1 & \ldots & 1 & 0 \\
1 & 1 & 1 & \ldots & 0 & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 1 & 1 & \ldots & 1 & 1 \\
\end{pmatrix}_{J \times J} \begin{pmatrix}
\log F_1(x) \\
\log F_2(x) \\
\vdots \\
\log F_{J}(x) \\
\end{pmatrix}_{J \times 1} = \log \begin{pmatrix}
\int_0^x \frac{g_j(t)}{1-F_j(t)} \, dt \\
\vdots \\
\int_0^x \frac{g_{J-1}(t)}{1-F_{J-1}(t)} \, dt \\
\end{pmatrix}_{J \times 1}
\]

Meilijson (1981) shows that given the joint densities \( g_1, \ldots, g_J \), this system has a unique solution \( F_1(x), \ldots, F_J(x) \). So the interpretation is that if we observe the joint distributions of win price and winner identities, we can infer the underlying distributions of values of the bidders.

**Appendix 1.3. Confidence Intervals for Optimal Reserve Prices**

We use the Delta method to derive these confidence intervals.

Recall that for a lot with quality vector \( z \), the valuation of player \( i \) of type \( j \) is given by
\[
\ln v^{ij} = z\beta + \mu + u^i
\]  

The first-order condition for revenue-maximization (equation from section), which gives the optimal reserve price \( r^* \) can be rewritten as an implicit function of \( r^* \) and the parameter vector \( \beta \)

\[
\gamma(r^*, \beta) = 0
\]

where
\[
\gamma(r^*, \beta) = N_1 \left[ 1 - F_1(r^*, \beta) - (r^* - x_0)f_1(r^*) \right] F_2(r^*) \\
+ N_2 \left[ 1 - F_2(r^*, \beta) - (r^* - x_0)f_2(r^*) \right] F_1(r^*)
\] (1.21)

We now obtain the asymptotic distribution of the optimal reserve price based on the semi-nonparametric estimates.


Consider an estimator \( \hat{\beta} \) of \( \beta \) that is consistent and distributed normally asymptotically. Thus,

\[
T^{1/2} (\hat{\beta} - \beta) \rightarrow^d N(0, V),
\] (1.22)

where \( V/T \) is the variance-covariance matrix of \( \hat{\beta} \). Then \( \hat{r}^* \), an estimator of \( r^* \) solves

\[
\gamma(\hat{r}^*, \hat{\beta}) = 0.
\] (1.23)

Expanding \( \gamma(\hat{r}^*, \hat{\beta}) \) in a Taylor’s series about \( (r^*, \beta) \)

\[
\gamma(\hat{r}^*, \hat{\beta}) = 0 = \gamma(r^*, \beta) + \gamma_r(r^*, \beta)(\hat{r}^* - r^*) \\
+ \nabla_\beta \gamma(r^*, \beta)'(\hat{\beta} - \beta) + U.
\] (1.24)

Ignoring \( U \), as it will be negligible in the neighborhood of \( (r^*, \beta) \), we obtain

\[
(\hat{r}^* - r^*) = \frac{-\nabla_\beta \gamma(r^*, \beta)'(\hat{\beta} - \beta)}{\gamma_r(r^*, \beta)} \equiv m'(\hat{\beta} - \beta).
\] (1.25)

Thus,

\[
T^{1/2} (\hat{r}^* - r^*) \rightarrow^d N(0, m'V m).
\] (1.26)
In practice, we work with approximations, so let

\[
m = \frac{-\nabla_{\hat{\beta}} \gamma(\hat{r}^*, \hat{\beta})(\hat{\beta} - \beta)}{\gamma_{\hat{r}^*} (\hat{r}^*, \hat{\beta})}.
\]

(1.27)

We use the parameter estimates (\hat{\beta}) to compute the optimal reserve price (\hat{r}^*) and the value of \( m \) at \( \hat{r}^* \). Then we construct 95% confidence intervals around \( \hat{r}^*, \hat{\beta} \) as follows.

\[
\left( \hat{r}^* - (1.96) \ast \left( \frac{m' \nabla m}{T} \right)^{1/2}, \hat{r}^* + (1.96) \ast \left( \frac{m' \nabla m}{T} \right)^{1/2} \right)
\]

(1.28)

Appendix 1.4. Proofs of Propositions 1 and 2

Proposition 1

The \textit{ex ante} expected payment of a bidder is

\[
E[m(X, r)] = \int_r^w m(x, r) f(x) dx
\]

\[
= r(1 - F(r))G(r) + \int_r^w y(1 - F(y))g(y)dy
\]

(1.29)

The overall expected payoff of the seller from setting a reserve price \( r \geq x_0 \) is

\[
\Pi = N \left[ E[m(X, r)] + F(r)^N x_0 \right]
\]

(1.30)

\[
\Pi = N \left[ r(1 - F(r))G(r) + \int_r^w y(1 - F(y))g(y)dy \right] + F(r)^N x_0
\]

(1.31)

Differentiating \( \Pi_0 \) with respect to \( r \)

\[14\text{for each covariate vector}\]
To study the curvature of \( \Pi(r) \) with respect to \( r \) we need the second derivative

\[
\frac{d^2 \Pi(r)}{dr^2} \bigg|_{r=r^*} = N (N - 1) F^{N-1}(r^*) f(r^*) \left[ 1 - F(r^*) + (x_0 - r^*)f(r^*) \right]
\]

\[+ N F^{N-1}(r^*) \left[ -2f(r^*) + (x_0 - r^*)f'(r^*) \right] \]

Also since \( \frac{d \Pi(r^*)}{dr} = 0 \)

we have

\[ [1 - F(r^*) + (x_0 - r^*)f(r^*)] = 0 \]

and therefore

\[
\frac{d}{dr} \left( \frac{d \Pi_0(r)}{dr} \right) \bigg|_{r=r^*} = NF^{N-1}(r^*)C
\]

where

\[
C = -2f(r^*) + (x_0 - r^*)f'(r^*)
\]

To see how the curvature of \( \Pi(r) \) responds to changes in the number of bidders, we differentiate \( \left( \frac{d \Pi(r)}{dr} \right) \bigg|_{r=r^*} \) with respect to \( N \)

\[
\frac{\partial}{\partial N} \left( \frac{d^2 \Pi(r)}{dr^2} \right) \bigg|_{r=r^*} = CF^{N-1}(r^*) (1 + N \log F(r^*))
\]

Now \( f(r^*) > 0 \) and \( (x_0 - r^*) < 0 \); also for this market and the value distribution as estimated, \( f'(r^*) > 0 \). So \( C < 0 \). Also again, given the competition in this market and the value distribution, the condition \( (1 + N \log F(r^*)) < 0 \) is met quite easily.\(^{15}\)

We now consider the asymmetric setting of our data.

\(^{15}\)While the average number of potential bidders in this market is 8.51; the minimum number is 5, this corresponds to 1 large bidder and 4 small bidders. With 5 symmetric bidders, the condition implied is \( F(r^*) < 0.818731 \) which is not binding since for the average quality vector and the small bidders value distribution, \( F(r^*) = \).
Proposition 2

\[
\frac{d}{dr} \Pi(r) = N_1 \left(1 - F_1(r) - (r - x_0) f_1(r)\right) G_1(r)
+ N_2 \left(1 - F_2(r) - (r - x_0) f_2(r)\right) G_2(r)
\]

(1.35)

where

\[
G_1(r) = F_2^{N_2}(r)
\]

\[
G_2(r) = F_1(r) F_2^{N_2-1}(r)
\]

Second derivative of \(\Pi(r)\) with respect to \(r\) is thus given by

\[
 \left. \frac{d^2\Pi(r)}{dr^2} \right|_{r=r^*} = C_1 N_1 F_2^{N_2}(r) + C_2 N_2 F_1(r) F_2^{N_2-1}(r)
+ \left(1 - F_1(r) - (r - x_0) f_1(r)\right) N_1 N_2 F_2^{N_2-1}(r) f_2(r)
\]

(1.36)

where

\[
C_1 = -2 f_1(r) - (r - x_0) f_1'(r)
\]

\[
C_2 = -2 f_2(r) - (r - x_0) f_2'(r)
\]

For further analysis, we put \(N_1 = 1\) and \(N_2 = N\). To see the impact of change in \(N\) on \(\Pi(r)\), we differentiate \(\frac{d^2\Pi(r)}{dr^2}\) with respect to \(N\).

\[
\frac{d}{dN} \left(\frac{d^2\Pi(r)}{dr^2}\right) = C_1 F_2^N(r) \log F_2(r)
\]

44
\[ \begin{align*}
+ & \; C_2 F_1(r) F_2^{N-1}(r) \left( 1 + N \log F_2(r) \right) \\
+ & \; \left( 1 - F_1(r) - (r - x_0) f_1(r) \right) F_2^{N-1}(r) f_2(r) \left( 1 + N \log F_2(r) \right) \\
+ & \; \left( 1 - F_2(r) - (r - x_0) f_2(r) \right) F_1(r) F_2^{N-1}(r) \left[ \left( (N-1) \frac{f_2(r)}{F_2(r)} \right) (1 + N \log F_2(r)) + N \frac{f_2(r)}{F_2(r)} \right]
\end{align*} \]

that gives
\[
\frac{F_2(r)}{f_2(r)} \frac{d}{dr} \left( \frac{d^2 \Pi(r)}{d^2 r^2} \right) = C_1 G_2(r) \log F_2(r) \frac{F_2(r)}{f_2(r)}
\]
\[ + \; C_2 G_2(r) \left( 1 + N \log F_2(r) \right) \frac{F_2(r)}{f_2(r)} \]
\[ + \; \left( 1 - F_2(r) - (r - x_0) f_2(r) \right) G_2(r) \left( 1 + N \log F_2(r) \right) \left( \frac{F_2(r)}{f_2(r)} \frac{f_1(r)}{F_1(r)} - 1 \right) \]
\[ + \; \left( 1 - F_2(r) - (r - x_0) f_2(r) \right) G_2(r) \frac{N}{r} \]

Now clearly the first three terms in the expression on the right hand side are positive while the fourth term is negative. Combining the second and the fourth terms we get
\[
\left[ \frac{d}{dr} \left( 1 - F_2(r) - (r - x_0) f_2(r) \right) \right] G_2(r) \left( 1 + N \log F_2(r) \right) \frac{F_2(r)}{f_2(r)}
\]
\[ + \; \left( 1 - F_2(r) - (r - x_0) f_2(r) \right) G_2(r) \frac{N}{r} \]
\begin{equation*}
\begin{aligned}
&= G_2(r) \frac{F_2(r)}{f_2(r)} \left[ \left(1 + N \log F_2(r)\right) \frac{d}{dr} \left(1 - F_2(r) - (r - x_0)f_2(r)\right) \right] \\
&\quad + N \frac{f_2(r)}{F_2(r)} \left(1 - F_2(r) - (r - x_0)f_2(r)\right) \\
&= G_2(r) \frac{F_2(r)}{f_2(r)} \frac{d}{dr} \left[ \left(1 + N \log F_2(r)\right) \left(1 - F_2(r) - (r - x_0)f_2(r)\right) \right]
\end{aligned}
\end{equation*}

Thus if \( \frac{d}{dr} \left[ \left(1 + N \log F_2(r)\right) \left(1 - F_2(r) - (r - x_0)f_2(r)\right) \right] \) is positive, we have increasing convexity of the revenue function in \( N \).
1.10 References


