chapter-4
Kant’s philosophy-An overall evaluation of transcendental aesthetics:

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4.1 Introduction:

In the previous chapters of this research work, we have examined and evaluated two main traditions of western philosophy in the form of empiricism and rationalism. In this chapter, we have to study and examine the main views of Kant's transcendental aesthetics with closer attention. Therefore in this chapter we want to investigate Kant's position in transcendental aesthetics as it is represented in the critique of pure reason. What is more important is this that Kant has much impact on contemporary philosophizing and entire logical positivistic movement is being reinterpreted in the light of Kantian position and problem. Therefore we start this chapter by indicating some of important dimensions where Kant's relevance is predominant.

4.1.1 Kant's contemporary significance:

It is a general remark about Kant that one can be either with Kant or against Kant but not without Kant. The major impact of Kant which has been seen throughout the 20th century on major disciplines of philosophy and methodology of science indicate that the relevance of Kant, and significant of his position, is still very much important. The questions which are raised by Kant either in methodology, epistemology or science are quite relevant and duly recognized. We start the indication of this relevance with the example of Stephen Hawking’s note about the position of Kant regarding the nature of basic philosophical questions and aims of philosophizing. In his famous book, ‘A brief History of time’, Stephen Hawking observes the current state of philosophizing and importance of Kant’s position about the questions which he raised in ‘antinomies of pure reason’. Hawking remarks in the following word.

"Up to now, most scientists have been too occupied with the development of new theories that describe what the universe is to ask the question why. On the other hand, the people whose business it is to ask why, the philosophers, have not been able to keep up with the advance of scientific theories. In the eighteenth century, philosophers considered the whole of human knowledge, including science, to be their field and discussed questions such as; did the universe have a beginning? However, in the nineteenth and twentieth century’s, science became too technical and
mathematical for the philosophers, or anyone else except a few specialists. Philosophers reduced the scope of their inquiries so much that Wittgenstein, the most famous philosopher of this century, said, "The sole remaining task for philosophy is the analysis of language." What a comedown from the great tradition of philosophy from Aristotle to Kant!"

In this observation of Stephen Hawking he putting emphasis on the why-questions regarding the universe as a whole. He thinks that Kant had rightly pointed out to the riddle of cosmological question. And this should be the subject matter of philosophy. His remarks about Wittgenstein and analytic philosophy, are not very much relevant here, as in the next section, it is to be indicated that Kant's influence on this tradition is very much there, and there is a continuous revival of this influence particularly in the field of semantics and methodology. But it can be said that Kant questions about the origin of universe are still relevant and current cosmological trends are indicating their significance.

Apart from Kant's relevance to contemporary cosmological questions and particularly with reference to big bang cosmology, there is still greater relevance of Kant which is arisen, and still being arisen, in the latter half of 20th century. The recognition of Kantian scholarship among international philosophical community can be seen by this fact that in the series of companions, Cambridge University has published three different companions on Kant. There are many contemporary scholars who are contributing much in the interpretation of Kant within contemporary discourse. An excellent example of Michel Friedman may be taken here who, in as important and influential work 'Foundations of space time theories-relativistic physics and philosophy of science' has put the following remark on Kant with reference to the methodology of philosophy of Kant.5

"The use which we can make in philosophy, of mathematics, consists either in the imitation of its methods or in the real application of its propositions to the objects of philosophy. It is not evident that the first has to have been of much use, however much advantage was originally promised from it... The second use, on the contrary, has been so much the more advantageous for the parts of philosophy concerned, which by the fact that they applied the doctrines of mathematics for their purposes, have raised themselves to a heights to which otherwise they could make no claim."

Michael Friedman has contributed much to Kantian scholarship, he contributed research articles in the volumes of Cambridge companion to Kant and
moreover in his work ‘reconsidering logical positivism’, he attempts to clarify the Kantian background for the entire logical positivistic movement. This is an important point in the historical interpretation of Kant. Particularly in the philosophy of geometry and apriori concept Carnap's famous work ‘Der logische Aufbau der welt’ is interpreted in the light of Kant and neo Kantian stand points.6 This entire movement was particularly started with the highly important work of J. Alberto Coffa whose most important book ‘The semantic tradition from Kant to Carnap’ has produced a revival of reinterpretation of Kant, particularly in the light of its connection with analytic philosophy.7 In all these current interpretations the point is this that the entire discussion of a apriori knowledge is currently being reinterpreted in the methodology of science. The point is this that Kant's discussion, as we shall see in this chapter, in the transcendental and metaphysical exposition of space and time, the entire framework requires a completely new method of interpretation which can serve as the basis of new Kantian dimensions in analytic philosophy and philosophy of science. Apart from these currently investigated matters, there is a close historical relationship between analytic philosophy and Kantian stand point. The famous works of P. F. Strowsan ‘The bounds of senses’ is an excellent example of Kant's influence on contemporary analytic philosophy. Apart from Strowsan other philosophers like Sellar and Lewies contributed much to Kant's relationship with analytic tradition. It may be said that post logical positivist scenario also contains much which can be recognized as indicating a predominant impact of Kant, particularly in philosophy of mathematics, Kant's problem still has some relevance. And some contemporary philosophers like Jaakko Hintikka have taken primary interest in Kant's work, and makes it, a subject matter of contemporary philosophizing.8

So it can be said that Kant's philosophy has very much contemporary relevance in the field of philosophy, methodology of science and even certain fundamental questions regarding the cosmic origin of the world. So before starting the real investigation in Kant's critique of pure reason, it is necessary to have a look at the works of Kant, particularly those which are more relevant for this research work.

4.1.2 Kant's works:

Kant's entire life was mainly an academic life and so he produced many important philosophical texts during his academic and philosophic carrier. Generally his works are divided, chronologically into two parts. First part contains pre-critical
writing that is those works which are written before the publication of the first edition of ‘critique of pure reason’ in 1781. Actually Kant started his publication in 1755 and published many articles and texts between 1755 to 1770. This era is important as a background of Kant which contains many works which includes not only philosophy but also physical geography and astronomy. For our purpose, it is important for us to take note of his two pre-critical works.

1. Physical monadology
2. The only possible argument for the proof of the existence of God.

Moreover Kant had submitted his inaugural dissertation to Kosinberg university which was unpublished in Kant's life time but now it is published under the title 'Inaugural dissertation.' Between 1772 to 1781, Kant had published nothing. This decade is called the silent decade for Kant and it is assumed that during this period he devoted much of his time to the preparation of his critique of pure Reason. The first edition of critique of pure reason was published in 1781 and immediately became a hot subject of philosophical discussion even in Germany and abroad. It was generally remarked on the ‘critique’ that the book was very much difficult and so within two years of the publication of this book, Kant published his second work 'Prolegomena to any future metaphysics'. In 1786 Kant published his another important work with the title 'Metaphysical foundation of natural science' in which he complained that those portions of 'critique of pure Reason' which should be more explicit, has remained more obscure and there he declared that a second edition of 'critique of pure reason' is required which he published in 1787. The portion of the critique which was being referred by Kant as an obscure one was the transcendental deduction of categories which was completely re-written in second edition. With reference to the present research work our focus is mainly concentrated on Kant's critique of pure reason which is translated in English by Norman Kemp Smith and another recent translation by Paul Guyer. Though Paul Guyer's translation is the latest one, yet it is felt that on certain points Kemp Smith's translation is still better. Apart from critique, the another important work which is necessary for us is ‘The metaphysical foundations of natural science’. For this work we will make the use of the translation of Paul Guyer. Other works of Kant like ‘critique of practical reason,’ ‘critique of judgment,’ 'Metaphysical foundations of moral principles' and others are not related with the direct subject matter of this research work. So they will only be eventually consulted in this research work. So Kant's original works and the vast literature compel any one to be selected
and therefore the subsequent description and interpretation of Kant is done according to this line.

4.1.3 Kant's critique of Pure Reason:

Kant's 'critique of pure Reason' which was published twice, in 1781 and 1787, is considered one of the greatest publication in the entire history of western philosophy. According to Max Muller there are two books on which mankind can take pride, are 'critique of pure Reason' and 'Rgveda'. According to Strowsan 'critique of Pure Reason' is the most important philosophical work of modern western philosophy. The work is highly influential and Norman Kamp Smith give an example of a British general who always kept a copy of 'Critique of Pure Reason' with him during first world war, when Britain was at war with Germany. This book is written by providing a serial numbers to its paragraphs which was the method of philosophical publication in Kant's time. In this reference, first edition contains total 8551 paragraphs, while a second edition contains 883 paragraphs. Generally when both editions are published combindly, those portions which are not completely rewritten, these numerical citations of these paragraphs are made together.

It is necessary to give some introductory remarks about the articulation, division and structure of critique of pure reason.

The entire critique of pure reason is divided into two main parts. First part is transcendental doctrine of elements which contains almost all that is popularly known as the content of critique of pure reason. Second part is transcendental doctrine of method, which is less popular and less discussed part. Scholars and critiques of Kant generally do not put much attention. Particularly one of the main purposes of present research work is to explore, a possible logical relation between these two divisions and particularly to bring out a possibility of the logical derivation from first to second division.

The first division is divided into two parts. Part one contains transcendental aesthetics in two sections in that section first is on space and second section on time. This part contains much of the Kant's philosophy of mathematics, geometry and epistemological nature and scope of his theory of intuition in this part a possible answer is sought to the first question of Kant. 'How pure mathematics is possible?' Second part of first division is entitled as Transcendental logic. This part is divided into two divisions, in which division one is named as transcendental analytic which
mainly contains metaphysical and transcendental deduction of categories, discussion on the principles of experience like causality, Kant's refutation of idealism and his distinction between phenomena and noumena, the second division is entitled as 'Transcendental dialectic'. Actually present research work is focused particularly on this division. It covers Kant's theory of transcendental illusion and even the title of the critique of pure reason is lexicographically as well as philosophically justified in the discussion of this division. The main source of Kant's proof of the impossibility of metaphysics as a science lies in this division. Actually the three fold classical subjects of metaphysics are strongly criticized by Kant. That is the idea of self or soul, universe or world and God or ultimate reality are declared by Kant as transcendental illusions in this division, in the chapters on paralogism of pure reason, antinomies of pure reason and ideal of pure reason. According to Kant almost all classical metaphysics is the result of the employment of categories without the support of sensations. So when categories go beyond the field of possible experience then according to Kant, there is a type of dialectic in which transcendental illusion is a necessary outcome. So in the light of part one and division one of part two it is to be investigated that how these conclusions are logically derived from previous writings of the critique.

The second division of the book that is transcendental doctrine of method is divided into four chapters, chapter-1 is the discipline of pure reason, and Chapter-2 is the canon of pure reason. Chapter-3 is the architectonic of pure reason and Chapter-4 is the history of pure reason.¹²

Before these divisions, Kant's text contains prefaces to both editions in which preface of first edition contains paragraph A_VIII to A_XXII and in second edition preface contains paragraph B_VII to B_XIV. Both of these prefaces contains the exposition of many important ideas of Kant is philosophy including his problem and his Copernican revolution. And so it is important to take an appropriate note of both prefaces and particularly the enlarged preface of second edition.

With these general introductions about Kant's relevance his works and his critique of pure reason, now it is necessary to start a critical and comprehensive interpretation of Kant' critique of pure reason. This is done in the next subsection.

4.2 Kant's problem and Copernican revolutions:
In critique of pure reason Kant has introduced a definite problem with definite methodological terminology. And it is a significance of Kant's philosophy that his problem and its representative methodology are still in contemporary debate with same nominalistic reference. The problem is about the possibility of a knowledge regarding facts which are independent of experience and methodology is the division of judgments into two different types:

1) Which deals with fact and

2) Which say nothing about fact

Kant has given a definite linguistic name for these problems and before understanding their epistemological significance it is necessary to understand them as they are given in critique of pure reason. Kant's problem and his Copernican revolution in epistemology is framed in this terminology and so we start, as Kant did in his introduction, with the division of analytic and synthetic judgment. The definition is given by Kant in the following words in first edition.¹³

"In all judgments in which the relation of a subject to the predicate is thought (if I consider only affirmative judgments, since the application to negative once is easy), this relation is possible in two different ways. Either the predicate B belongs to the subject A as something that is (covertly) contained in this concept A; or B lies entirely outside the concept A, though to be sure it stands in connection with it. In the first case I call the judgment analytic, in the second synthetic."

In the introduction of the second edition Kant does not change anything in this definition and so it remains as it is in the first edition.¹⁴ Here it is true that the definition is being given for these propositions which contain subject and predicate. The sole criterion which is given by Kant is the relation of 'Belongs to'. Now it is necessary to investigate that what is the exact meaning of the statement that predicate-A belongs to subject-A. Here, according to Kant, in any proposition subject and predicate terms designate concepts. If a concept is contained in another concept then it is called as belonging to that concept. If we want to understand this terminology, and if we are slightly changing the designation of subject and predicate terms by class in the place of concept, then this "belongs to" relationship actually becomes the relationship of 'class-inclusion'.¹⁵ In the latter development of set theory this is called subset relationship and is defined as 'If each member of set-A is also the member of set-B then A is called subset of B and symbolized As A ∈ B.'¹⁶ In this terminology the
English word 'belongs to' is generally used for class membership and symbolically it is denoted as '$\in$' so in current terminology if we are leaving apart some lexicographical differences, it can be said that if the class or concept which is designated by predicate B is either included or is a subclass of subject A then the judgment relating these two classes or concepts is analytic.

The example of analytic judgment which is given by Kant is 'All bodies are extended'\(^{17}\), and the example and the discussion on it remain same in the second edition.\(^{18}\) Here Kant says that to find an extension "I do not need to go outside the concept."\(^{19}\) On the other hand, if the subject term of a categorical proposition does not contain its predicate term then the proposition is synthetic proposition. The example which is given for the case of synthetic proposition in both editions of 'critique of pure reason' is 'All bodies are heavy.'\(^{20}\) Kant says that the predicate 'heavy' is something entirely different from the concept of a body in general and therefore this is called a synthetic proposition.

Now it is necessary to think that what the epistemological reference of the division of this is proposition into analytic and synthetic proposition because, as it is quite clear, that this is not a formal division which has anything to do with the formal structure of a proposition. According to the classification of categorical proposition, according to Aristotelian logic, and these two propositions, 'all bodies are extended', 'All bodies are heavy' are universal affirmative or A-type of proposition. So pure logic does not make any differentiation in this definition though the definition may indicate that a formal differentiation is being made, it is so because there is no formal way to decide that whether a given predicate term actually belongs to the subject term of that proposition or not. And so there is a fixed epistemological reference, in the background in this division which is made clear by Kant in the following words.\(^{21}\)

"Now from this it is clear: I) that through analytic judgments our cognition is not amplified at all but rather the concept, which I already have, is set out and made intelligible to me; 2) that in synthetic judgment I must have in addition to the concept of the subject something else (x) on which the understanding depends in cognizing a predicate that does not lie in that concept as nevertheless belonging to it."

From this paragraph it is quite clear that according to Kant in any analytic judgment cognition is not amplified because the concept which is being designated by cognition is not finding anything new through the act of predication in the given
judgment. As it is clear from Kant's writing of the paragraph of B11 that in this type of judgments, the only epistemological process which occurs is that of clarification. So what is already there in the concept of subject term is only made intelligible through the act of predication in an analytic judgment.

Kant's view regarding predication and his definition of analytic judgment has created much criticism and debate in the subsequent interpretation of Kant. Kant uses the term 'Does not go beyond the concept' and so he says that 'All bodies are extended' is declared as analytic proposition by Kant. Because according to him, there is no need to go beyond the concept of body for obtaining 'extension.' But how does Kant know this according to which process the analysis of the concept of body is to be made. We can arrive to extension through the analysis of the conception of body only if we are making this analysis according to the rationalistic epistemology of Descartes and Spinoza. Even if we take this course of analysis, according to Leibnitz, then no extension is found as belonging to the concept of body.22 So even in epistemological discourses, it is not clear that in which way Kant is stating that actual amplification of cognition is made in a synthetic judgment. Because Kant states that '7+5=12' or 'All judgments of mathematics are synthetic' and so they all are amplifying our cognition. It is very much difficult to derive a definite conclusion of in this regard; that what is the exact meaning of the amplification of cognition is to be understood by Kant in the division of analytic and synthetic judgments.

In the same epistemological reference, Kant has made a further division in classification of propositions. These divisions are apriori and aposteriori propositions. The meaning of apriori proposition is this that they must be independent of experience and all apostori propositions are depended on experience. It is generally true that all analytic propositions are apriori and all propositions which are related to experience are aposteriori. But if this dichotomy is to be accepted then one has to face either dogmatic or skeptical result of either empiricism or rationalism, But according to Kant there are certain propositions which are both synthetic apriori and this is an extended claim of Kant that if a study field or discipline, is to be taken as a theoretical science of reason then it must contain synthetic apriori proposition. So the fifth section of the introduction of second edition is entitled as 'synthetic apriori judgments' contain as principles of all theoretical sciences of reason.23 Here in 'Critique of pure reason' Kant claim of the presence of synthetic apriori judgment is
limited to mathematics and natural science (physics). For mathematics, Kant gives his argument in detail as follows.24

“To be sure one might initially think that the proposition "7+5=12" is a merely analytic proposition that follows from the concept of a sum of seven and five in accordance with the principle of contradiction. Yet if one considers it more closely, one finds that the concept of the sum of 7 and 5 contains nothing more than the unification of both numbers in a single one, through which it is not at all thought what this single number is which comprehends the two of them. The concept of twelve is by no means already thought merely by my thinking of that unification of seven and five, and no matter how long I analyze my concept of such a possible sum I will still not find twelve in it. One must go beyond these concepts, seeking assistance in the intuition that corresponds to one of the two, one's five fingers, say, or (as in Segner's arithmetic) five points, and one after another add the units of the five given in the intuition to the concept of seven, For I take first the number 7, and, as I take the fingers of my hand as an intuition for assistance with the concept of 5, to that image of mine I now add the units that I have previously taken together in order to constitute the number 5 one after another to the number 7 and thus see the number 12 arise, That 7 should be added to 5 I have, to be sure, thought in the concept of a sum = 7+5, but not that his sum is equal to the number 12. The arithmetical proposition is therefore always synthetic, one becomes all the more distinctly aware of that if one takes somewhat larger numbers, for it is then clear that, twist and turn our concepts as we will, without gelling help from intuition we could never find the sum by means of the mere analysis of our concepts."

The ideas which are being put here by Kant are almost universally unacceptable ideas in the further development of analytic and empiricistic philosophy. It is still surprising that even according to Kant a few propositions that the geometers presuppose are actually analytic. The example which is given by Kant is 'A=A' or 'A+B>A'.25 Because in modern mathematics for the expressions of mathematical propositions inequalities plays the role of same importance as that of equalities. Kant himself states the analyticity of these propositions in the following words.26

"To be sure, a few principles that the geometers presuppose are actually analytic and rest on the principle of contradiction; but they also only serve, as identical propositions, for the chain of method and not as principles, e.g.: a = a, the whole is equal to itself, or (a + b) > a, i.e., the whole is greater than its part. And yet even
these, although they are valid in accordance with mere concepts, are admitted in mathematics only because' they can be exhibited in intuition."

Here Kant is referring to the postulates of Euclidian geometry as analytical propositions. If they are analytic propositions then the theorems of Euclidian geometry, which are derived from them by formal deduction, must have the character of analytic propositions. But according to Kant most of the theorems of geometry including the theorem of Pythagoras are synthetic apriori propositions. It is a surprising element in Kant's methodology that inequalities are being considered as analytic and yet from them the derivation of synthetic proposition is possible. Actually in modern mathematics, there are many definitions of fundamental concepts which are expressed in the terms of inequalities.27 So Kant's emphasis on the division of analytic and synthetic proposition and its ground is inadequate as it is expressed in critique of pure reason.

The second important class of synthetic apriori propositions is the propositions of natural sciences according to Kant. Kant states that the principles or general laws of natural science which Kant terms as 'physica' are synthetic apriori propositions. Kant states his ground and justification in the following way.28 "Natural science (physica) contains within itself synthetic apriori judgments as principles. I will adduce only a couple of propositions as examples, such as the proposition that in all alterations of the corporeal world the quantity of matter remains unaltered, or that in all communication of motion effect and counter effect must always be equal. In both of these, not only the necessity, thus their a priori origin but also that they are synthetic propositions is clear. For in the concept of matter I do not think persistence, but only its presence in space through the filling of space. Thus I actually go beyond the concept of matter in order to add something to it a priori that I did not think in it. The proposition is thus not analytic, but synthetic, and nevertheless thought a priori, and likewise with the other propositions of the pure part of natural science."

Again this justification lacks the same rigor as it has been mentioned in the earlier discussion. According to Kant 'All bodies are extended' is an analytic proposition and the laws governing the motion of these bodies, which presuppose extension, are synthetic a priori propositions. So the question of the deduction is the same in the case of the propositions of natural sciences also. With all these considerations, it becomes quite clear that Kant is searching the ground for the
possibility of synthetic a priori judgments. If this problem is to be looked from historical point of view, then pre-Kantian western philosophical traditions would have responded to this problem in peculiar way. According to rationalism what can be called genuinely as knowledge can be obtained only through intellect and so according to rationalistic approach there is no fundamental or categorical difference between analytic and synthetic propositions. In principle, for them, all knowledge is apriori, that is independent on sense experience. On the other hand, for empiricism, there is no categorical difference between sense and reason and therefore all propositions are apostori according to empiricism. For empiricist philosophers a priori knowledge is not a genuine knowledge at all. Because according to Hume it expresses only relation among ideas. These extreme results are achieved on the ground of the maxims which are accepted in particular epistemological systems and these maxims are proceeding towards a particular type of dead ends in the close epistemological system. Kant wants to find out a way from this dilemma and so he re-articulates the basic epistemological problem in a particularly designed reconstructive sense.

If a combination or synthesis of empiricism and rationalism is required then it can be mentioned that it is a very much easy task, so far as its logical possibility is concerned. One has to simply state that for the origin of knowledge, the role of sense experience and intellect is necessary combinely required. And if we have to state this problem in Kantian terminology then the following situation arises:
1. According to rationalism all synthetic judgments are apriori
2. According to empiricism all synthetic judgments are apostori

If a logically combinatory way is to be found between these two extremes, then the only option which remains is this.

3. Some synthetic judgments are apriori. This is what Kant has mentioned. But this can be amounted only as a superficial synthesis. Even if there is only a single a priori proposition the question remains regarding its origin and its epistemological certainty. For example if we ask that how synthetic judgment apostori possible? Then in this question the pronoun 'How' refers to that particular mean, and mechanism of epistemological process, which makes this judgment possible and moreover also justifies it. And in this case, the answer is 'sense experience'. In the same way, if we inquire into the possibility of apriori judgments then the question 'How analytic judgment apriori is possible ?' is to be answered only on the ground of reason alone. But the question of the possibility of apriori judgment creates a problem for Kant and
for the solution of this problem, he makes a revolution in epistemology which is called Copernican revolution. So it is necessary here to understand the nature of this revolution and its role and function in the possible solution of Kant's problem.

In actual sense, the term 'Copernican revolution' belongs to astronomy. Copernicus was an astronomer who provided heliocentric modal in the place of classically accepted geocentric modal which was based on the astronomical notions of Aristotle and was fully worked out by Ptolemy. This geo-centric modal state that earth is the center of this universe and whole celestial sphere moves around it which is divided into seven different layers. In the place of this modal for finding a better explanation of the motions of certain celestial bodies. Copernicus postulated that 'Sun is in the center of the universe and whole celestial sphere moves around it'. Now from a philosophical point of view, both of these hypotheses are rather trivial and absurd. If universe is infinite than there cannot be nothing like a center of the universe and in a different point of view on the other hand, every point can be regarded as the center of the universe. But here for Kant, the question is not about any astronomical reference. Because in the time of Kant astronomy had made much progress and Kant himself propounded the nebula hypothesis which states the possibility of different island universes. And the whole of this is infinite. Here Kant wants to change the fundamental situation in epistemology. And the situation is this. Before Kant it was believed that in any epistemological process knower is neutral. This means that if any object is known to a particular knower then this knowing process does not make any change in the object which is being known and so any act of knowing or observation does not make any impact on the object which is being known or observed. Ontologically this seems a strange view because if there is a process between two entities x and y then how it can be possible that a particular entity y does not get any impact of that process and yet it is a part of that process. Kant wants to change this situation fundamentally. And he postulated that any knower is not simply receiving the known object which lies outside it but it constructs it own object of knowledge. For Kant the question is this that whether our cognition and our knowledge of object both are related in a particular way or simply our cognition should conform to the object of our knowledge. Kant says that if cognition can grasp an object then it must have its own structural properties or mechanism according to which it can grasp it. This is a fundamental change in the field of epistemology and Kant explains in detail in the following way.
"I should think that the examples of mathematics and natural science, which have become what they now are through a revolution brought about all at once, were remarkable enough that we might reflect on the essential element in the change in the ways of thinking that has been so advantageous to them, and, at least as an experiment, imitate it insofar as their analogy with metaphysics, as rational cognition, might permit. Up to now it has been assumed that all our cognition must conform to the objects; but all attempts to find out something about them apriori though concepts that would extend our cognition have, on this presupposition, come to nothing. Hence let us once try whether we do not get farther with the problems of metaphysics by assuming that the objects must conform to our cognition, which would agree better with the requested possibility of an a priori cognition of them, which is to establish something about objects before they are given to us. This would be just like the first thoughts of Copernicus, who when he did not make good progress in the explanation of the celestial motions if he assumed that the entire celestial host revolves around the observer, tried to see if he might not have greater success if he made the observer revolve and left the stars at rest. Now in metaphysics we can try in a similar way regarding the intuition of objects. If intuition has to conform to the constitution of the objects, then apriori; but if the object (as an object of the senses) conforms to the constitution of our faculty of intuition, then I can very well represent this possibility to myself. Yet because I cannot stop with these intuitions, if they are to become cognition, but must refer them as representations to something as their object and determine this object through them, I can assume either that the concepts through which I bring about this determination also conform to the objects, and then I am once again in the same difficulty about how I could know anything about them a priori, or else I assume that the objects, or what is the same thing, the experience in which alone they can be cognized (as given objects) conforms to those concepts, in which case I immediately seen an easier way out of the difficulty, since experience itself is a kind of cognition requiring the understanding, whose rule I have to presuppose in myself before any object is given to me, hence a priori, which rule is expressed in concepts a priori, to which all objects of experience must therefore necessarily conform, and with which they must agree. As for objects insofar as they are thought merely through reason, and necessarily at that, but that (at least as reason thinks them) cannot be given in experience at all the attempt to think them (for they must be capable of being thought) will provide a splendid touchstone of what we assume as the altered method
of our way of thinking, namely that we can cognize of things a priori only what we ourselves have put into them."

This detailed expression of Copernican revolution states a fundamental change in epistemological principles in western philosophy. Kant has to make a synthesis but if this synthesis amount towards epistemological reference then a particular type of mechanism of knowledge is needed. And this mechanism should clarify the number, function and scope of epistemological faculties which provides the generation of knowledge. And this Copernican revolution state the role of the first faculty, that of sensibility which provides the provision of the representation of an outer object before knower, where cognition of that object is to be generated. There must be some pure forms of some formal grounds of this sensibility, and they are termed as 'initiation' by Kant. These pure forms are called space and time in Kant. And the discussion and consideration of these pure forms indicate the answer of the first question of Kant that how pure mathematics possible? For a discussion and interpretation of this matter it is necessary to examine the metaphysical and transcendental expositions of space and time which is done in the next sub section.

4.3  Kant's theory of space and time transcendental aesthetics:

Transcendental aesthetics is the name of the part of critique of pure reason which is in the first division of 'Transcendental doctrines of elements' The span of this discussion covers in critique of pure reason from $A_{22}$, $A_{49}$ and $B_{34}$ to $B_{75}$. This relatively small portion of critique of pure reason covers the great riches of Kantian epistemology which is criticized, evaluated and understood in greater details by Kant's scholars and critiques. Before understanding the exposition of space and time it is necessary to clarify the meaning of certain epistemological terms which are used by Kant in this particular reference.

In this regard, if it is started from the definition of sensibility then this definition is given by Kant in this way $^{33}$, "The effect of an object on the capacity of representation, in so far we are affected by it is sensation."

This definition of sensation is wide enough to encompass all those effects which can put any impact on the capacity of representation. Here representation is the most general epistemological term in Kant's epistemology which includes all that which is there, epistemologically in the mind. It is also necessary to note that for this type of effect there must be an object of that effect and this objectivity is carefully
articulated by Kant and therefore his philosophical position is not to be identified with subjective idealism like the philosophical position of Berkeley. If subjective idealism is to be refuted then there must be something in the side of knower which can be called as an element which supports the objective ground of cognition. And so there must be something which can be called pure representation, and pure form of representation according to Kant, a representation can be can be called 'pure' if it contains nothing which is to be encountered that belongs to sensations. And all principles which belongs to this apriori understanding of sensibility is called transcendental aesthetics. Kant defines transcendental aesthetics in the following way. "I call a science of all principles on a priori sensibility the transcendental aesthetic. There must therefore be such a science, which constitutes the first part of the transcendental doctrine of elements, in contrast to that which contains the principles of pure thinking, and is named transcendental logic."

It is necessary to take note of this position that Kant classifies this transcendental aesthetics in the realm of doctrines of elements. According to Kant this transcendental aesthetics is a science of all principles of apriori sensibility. If something is to be called as a science in Kant, then it must contain synthetic apriori judgments. The other thing is this that Kant here uses the term, 'apriori sensibility'. Now according to empiricism these two terms 'apriori' and 'sensations' cannot be put together because what belongs to the scope of sensation together with each sensation is apostori. This is the very definition of sensibility in general epistemological reference, but here Kant's Copernican revolution in epistemology takes a different type of position. Here aesthetics has two different types of dimensions. First dimension is empirical and second is transcendental. Therefore this term aesthetics which is the English translation of the German term 'Verhältnissen' is used by Kant in a particular reference with particular historical background from Baumgarten and other local German philosophers who had used this term in this way. The explanation of the use of the term 'aesthetic' in this reference has been provided by Paul Guyer in his translation in the following way. "The Germans are the only ones who now employ the word "aesthetics" to designate that which other call the critique of taste. The ground for this is a failed hope, held by the excellent analyst Baumgartn, of bringing the critical estimation of the beautiful under principles of reason, and elevating its rules to a science. But his effort is futile
for the putative rules or criteria are merely empirical as far as their sources are concerned, and can therefore never serve as a priori rules according to which our judgment of taste must be directed, rather the latter constitutes the genuine touchstone of the correctness of the former. For this reason it is advisable again to desist from the use of this term and to save it for that doctrine which is true science (whereby one would come closer to the language and the sense of the ancients, among whom the division of cognition into ἄισθητα καὶ υοητα was very well known).”

This use of the term aesthetics in this particular epistemological reference which makes it possible to put the adjective 'transcendental' before it, provides a justified claim of Kant regarding the possibility of synthetic apriori judgments in pure mathematics. There is a vast area of research in Kant's philosophy of mathematics which is currently being investigated by many well known Kantian scholars and philosophers. We will concentrate our attention Kant's theory or intuition as a background to his theory of transcendental exposition and metaphysical exposition of space and time. This is done in the following sub section.

### 4.3.1 Kant's view of intuition:

The term 'intuition' is used in the epistemology or metaphysics with different references. In idealistic and absolute idealistic reference, it is generally considered as a power of consciousness which can make a direct connection with absolute reality as it is actually there. Generally, this amounts to a supplementary device for a monistic ontology and so intuition in this reference is taken as an ontological status of reality rather than as an epistemological power of an empirical knower. But in Kant there is no such reference to the term intuition. Actually the German word which is used by Kant for intuition is 'Anschauung' and in 'Anschauung' there is no reference of this type of trans-empirical connotation of the word intuition. The second important thing is this that in the entire framework of Kantian epistemology, the term 'Anschauung' has been used only with reference to sensibility. But as Kant wants to represent some apriori condition of these intuitions, it is clearly understood in Kantian reference that he is definitely crossing the limits of sensations which are there in empiricistic epistemologies. For Kant, intuition is an epistemological faculty which can provide an outer object before representation of any cognition. It is an empirical fact that when any sensation is to be converted into a cognitively awarded knowledge, this particular knowledge of a particular object is always with the personal pronoun 'I', but having
the capacity of intuits something, this does not mean that there is a permanent and substantial 'I' and though this 'I' is common in every representation which the faculty of intuition provides it is not to be taken as permanent self or soul which is generally believed in classical metaphysics. Kant emphasized this point strongly that our intuition is not intellectual that is, it cannot create the manifold objects due to its own hidden metaphysical activities. But in the presence of an external object it can function only as a schema which can provide the objective and outer basis of cognition. Kant attempts to clarify the confusion between intellectual and sensible intuition in the following way.37

"But this persisting element cannot be an intuition in me for all the determining grounds of my existence that can be encountered in me are representations, and as such they themselves need something persisting distinct from them, in relation to which their change, and thus my existence in the time in which they change, can be determined. Against this proof one will perhaps say: I am immediately conscious to myself only of what is in me, i.e. of my representation of external things; consequently it still remains undecided whether there is something outside me corresponding to it or not. Yet I am conscious through inner experience of my existence in time (and consequently also of its determinability in time), and this is more than merely being conscious of my representation; yet it is identical with the empirical consciousness of my existence, which is only determinable through a relation to something that, while being bound up with my existence, is outside me. This consciousness of my existence in time is thus bound up identically with the consciousness of a relation to something outside me, and so it is experience and not fiction, sense and not imagination, that inseparably joins the outer with my inner sense; for outer sense is already in itself a relation of intuition to something actual outside me; and its reality, as distinct from imagination, rests only on the fact that it is inseparably bound up with inner experience itself, as the condition of its possibility, which happens here. If I could combine a determination of my existence through intellectual intuition simultaneously with the intellectual consciousness of my existence, in the representation I am, which accompanies all my judgments and actions of my understanding, then no consciousness of a relation to something outside me would necessarily belong to this, But now that intellectual consciousness does to be sure precede, but the inner intuition in which alone my existence can be determined is sensible, and is bound to a condition of time I however, this
determination and hence inner experience itself, depends on something permanent, which is not in me, and consequently must be outside me, and I must consider myself in relation to it, thus for an experience in general to be possible, the reality of outer sense is necessarily bound up with that of inner sense, i.e. I am just as certainly conscious that there are things outside me to which my sensibility relates as I am conscious that I myself exist determined in time. Now which given intuitions actually correspond to outer objects, which therefore belong to outer sense, to which they are to be ascribed rather than to the imagination that must be decided in each particular case according to the rules through which experience in general (even inner experience) is to be distinguished from imagination I which procedure is grounded always on the proposition that there actually is outer experience. To this the following remark can be added: The representation of something persisting in existence is not the same as a persisting representation; for that can be quite variable and changeable, as all our representations are, even the representations of matter, while still being related to something permanent, which must therefore be a thing distinct from all my representations and external, the existence of which is necessarily included in the determination of my own existence, which with it constitutes only a single experience, which could not take place even as inner if it were not simultaneously (in part) outer. The "How?" of this can be no more explained than we can explain further how we can think at all of what abides in time whose simultaneity with what changes is what produces the concept of alteration."

With reference to the main theme of present research work, Kant's view on intuition is fundamentally important. Kant is not putting any ontological limit on intuition as such but he remarks that, so far as the intuitions of human being are concerned, they cannot have intellectual intuition. Moreover Kant further assumes that not only for human beings but for any finite beings, this faculty of intuition cannot be intellectual and therefore accompanying personal pronoun 'I' with any intuitional cognition does not provide the proof of the existence of the substantial self. Any attempt to think or to derive the existence of a permanent self belongs to the paralogism of pure reason which is the first part of Kant's theory of transcendental illusion.

This faculty of intuition which belongs to a finite conscious being like human being has to work in the dimension of its pure form. These pure forms of intuition are space and time. In the epistemology of Kant space and time are not the objects of
perception they are formal precondition of the very existence of any knowledge which can be called as arised from perception. Therefore Kant provides two types of exposition for these forms of intuitions. First exposition is metaphysical and second exposition is transcendental. Following the text of critique of pure reason, both exposition of space have been evaluated in the next subsection.

4.3.2 Kant's view of exposition of space:

For Kant the term 'exposition' means a particular type of philosophical expression with due justification. Here exposition is not an empirical or scientific description for any given empirical object. But when exposition is to be made for space and time there are two factors which must be taken into account.

1. According to Kant space and time are pure forms of intuition. Therefore it is to be justified that they are pure and simultaneously they are intuition. This task according to Kant, can be covered under the title of metaphysical exposition.

2. If something is metaphysically exposed, then Kant has to indicate its epistemological counterpart, that is, this pure forms of intuitions must be applied to a genuine discipline of knowledge. Here their truths can be ascertained. This exposition is the transcendental exposition of space and time. According to Kant Again following the order of Kant's text the metaphysical and transcendental exposition of space have been examined first in the next subsection.

4.3.2.1 Metaphysical exposition of space:

The metaphysical exposition of space is given in critique of pure reason in A23/ B38 to A25/ B41. There are two parts of this metaphysical exposition. In first part Kant proves that space is pure. That is, it is not something which is either based on experience or it is derived from experience. Kant provides his arguments for the first part in the following way.38

“Space is not an empirical concept that has been drawn from outer experiences. For in order for certain sensations to be related to something outside me (i.e., to something in another place in space from that in which I find myself), thus in order for me to represent them as outside one another, thus not merely as different but as in different place, the representation of space must already be their ground. Thus the representation of space cannot be obtained from the relations of outer appearance.
through experience, but this outer experience is itself first possible only through this representation."

Here what is emphasized by Kant is this that space is not an empirical concept which is either derived or drawn or originated from outer experience. Only in this case the metaphysical exposition is needed and possible. Otherwise for any concept or cognition which is either derived or based on outer sense experience only physical exposition is sufficient. But space is not this type of entity. According to Kant the reason is this that if I have to have an experience of anything which is outside me I must have a schema or a medium which can provide the justification of this particular use of the term 'outside me'. But this very representation of this 'Outside me' requires the representation of space. Now if space itself is something which is considered as given before me as outside me then even in this case the term 'outside me' will presuppose another epistemological schema which can put space as outside me and as an object of my knowledge. This schema must be spatial. And so we cannot accept his type of schema unless we accept the fallacy of infinite regress. Therefore in the first stage of metaphysical exposition space is pure and it is not derived or drawn from outer experience.

In the second part of the proof of his metaphysical exposition, Kant wants to prove that space is a pre-condition of all physical experience and yet it is not necessary for it to depend on these physical objects. This autonomous existence of space, which is called a priori in Kant's terminology, has been stated by Kant in the following way.39

"Space is a necessary representation a priori, which is the ground of all outer intuitions one can never represent that there is no space al-thought one can very well think that there are no objects to be encountered in it. It is therefore to be regarded as the condition of the possibility of appearances, not as a determination dependent on them, and is an a priori representation that, necessarily grounds outer appearances."

In this exposition Kant is particularly depended on Newtonian concept of absolute, flat and static space. Perhaps it is based on common sense that we cannot imagine material bodies without space but we can imagine space without any existence of physical objects. This one way dependency of material objects on space provides the grounds of its a priori character or pureness of space. But this view of space is not only counter philosophically notion which was there in Kant's time. These matters can be explicated and evaluated with the following arguments.
If we can imagine empty space then scientifically we are accepting Newtonian concept of absolute space with the application of the flat Euclidian geometry to it with the existence of positive definite metric and validity of Gallian transformations. But even in special theory of relativity, neither the metric is positive definite nor transformations are Gallian and so there are no absolute planes of simultaneity, in the language of Michael Friedmann, a well-known scholar of Kant. Special relativity provides the existence of Lorentz metric and demands the co-variances of Lorentz transformation and so this rules out any concept of 'absolute space and time' and in the theory of general relativity, space and time are not only mutually relative entities but they are also relative with mass and energy which is spatially extended existence. In this case, there is no fixed metric which can describe the geometrical properties of space. In other words it is the case that Euclidian geometry does not apply to space but for deriving the metric of a particular space-time geometry one has to solve the field equations of Albert Einstein and then they can get that particular metric which can determin the geometrical properties of space. So Kant's contention in the second point that space can exist without matter but matter cannot exist without space has become unfounded in the light of modern scientific discoveries. Moreover the view of space which has been taken in the rationalistic tradition, of Descartes and Spinoza is also opposite to that of Kant. According to this tradition space is an attribute of material substance and there can be no separate ontological existence of any attribute apart from substance. Yet from an exist epistemological point of view Kant wants to emphasis a priori character of space as the pure form of intuition and if space is to be taken in this particular reference then there is still much weight in Kant's arguments, that as a formal condition space, is a priori.

With these two points Kant established the pureness of space. Now in the second phase of metaphysical exposition Kant wants to prove that space is not a concept at all. Here Kant wants to emphasize that it is neither a discursive nor a relational concept. He states his grounds for this remarks in the following way. "Space is not a discursive or as is said, general concept of relations of things in general, but a pure intuition. For first, one can only represent a single space, and if one speaks of many spaces, one understands by that only parts cannot as it were precede the single all encompassing space as its components (from which its composition would be possible) but rather are only thought in it. It is essentially single the manifold' in it thus also the general concept of spaces in general rests
merely on limitations. From this it follows that in respect to it and a priori intuition (which is not empirical) grounds all concepts of them this also all geometrical principles, e.g., that in a triangle two sides together are always greater than the third are never derived from general concepts of time and triangle, but rather are derived from intuition and indeed derived a priori with apodictic certainty."

In this observation Kant makes an important claim about the theory of concept formation. If any concept is to be formed then it must have different separate existence under it. But so far as Kant has observed, one can have a concept of many spaces but again these many spaces are also to be considered within one single space and this is not the logical relationship between the concept and its essential example. So in the justification of this point of Kant the famous commentator of Kant H. J. Patton remarks that "A concept can have many instances under it but it cannot have many instances within it." So according to Kant space is not a concept at all. But again if it is not a concept at all then it cannot have a fixed set of characteristics which can determined its form or logical character.

In the last point, Kant wants to prove that as space is infinite in magnitude. It cannot be considered as a concept particularly in epistemological reference. Kant states his views in the following way. "Space is represented as a given infinite magnitude. A general concept of space (which is common to a foot as well as an ell) can determine nothing in respect to magnitude. If there were not boundlessness in the relations could bring with it a principle of their infinity."

Here Kant is claiming that space can be given as a representation of infinite magnitude. Now it is very difficult to understand that how infinity can be given in a representation. Another point which is claimed by Kant is this that in a general concept of space nothing is to be thought about its magnitude. These both views are surprisingly novel. If space is infinite and there is only one space, then due to infinity just like due to its unity, no process of concept formation is possible. But the matter is controversial if we look to many concepts which are there in modern mathematics. These concepts include infinity in themselves and yet there is no harm in the process of concept formation. For example, we may take the concept of any infinite set either of natural numbers or real numbers which are infinite and yet the concepts of those sets are formed in mathematics. But Kant is emphasizing here that space is given as an infinite magnitude and this is very much difficult to understand either
epistemologically or geometrically. But there are some points in Kant's argument that if space is infinite then just like its singularity it makes difficult the process of concept formation.

With this exposition, according to Kant the metaphysical exposition of space becomes complete. Here Kant claims that space is a pure form of intuition and now what remains is this that there must be a discipline or a field of knowledge where the synthetic a priori judgment about space are to be obtained. This has been stated by Kant in the transcendental exposition of space which is seen and evaluated in the next sub-section.

4.3.2.2 Transcendental exposition of Space:

After providing metaphysical exposition of space what remains is its transcendental exposition. The term transcendental has various reference, and generally, in metaphysics it is used for any state of reality which is beyond space, time and causality. But in Kant's critical philosophy the term 'transcendental' has been used in a very specific sense. In epistemological reference it is used for the possibility of a ground of a priori knowledge. So if a transcendental exposition of space is to be made, then it is to be shown that for this pure form of intuition, there is possibility of the origin of synthetic a priori knowledge or in other words there must be some synthetic a priori judgments which are to be considered as belonging to the structure of space as such. Naturally there are many empirical statements regarding the given part of the space for example, the area of a given rectangle or triangle or the volume of a given sphere or cone a matter of empirical investigation and therefore these propositions about space or more generally about the given part of the space are synthetic a posteriori but there must be a discipline or science of space which can provide a universally certain proposition about space. In Kantian terminology he demands the apodictic certainty of the proposition regarding the space and he thinks that the classical geometrical propositions can provide this type of example. Kant states this exposition as follows. 46

“The apodictic certainty of all geometrical principles and the possibility of their a priori construction are grounded in this a priori necessity. For if this representation of space were a concept acquired a posteriori which was drawn out of general outer experience the first principles of mathematical determination would be nothing but perception. They would therefore have all the contingency of perception and it would
not even be necessary that only on straight line lie between two points but experience would merely always teach that. What is borrowed from experience always has only comparative universality, namely through induction. One would therefore only be able to say that as far as has been observed to date, no space has been found that has more than three dimensions.”

According to Kant all the theorems of Euclidian geometry are synthetic a priori propositions and so they all are having the character of apodictically certain propositions. Kant further explains that if space would have been an empirical concept, abstracted from outer experience, then this type of apodictic certainty cannot be achieved. It is remarkable that other general propositions about geometry are found by Kant as having comparative universality. Here Kant takes an important example about the dimensionality of space and he remarks that it is only an observed fact up to the date that no space has been found in more than three dimensions. In his commentary to the Kant’s ‘critique of pure reason’, N. K. Smith remarks that according to Kant “space is three dimensional is neither an analytic proposition nor a synthetic a priori judgment” he further remarks that the three dimensionality of our space can be derived in some way from the inverse square law of gravitation. In this way, in the transcendental exposition of space, Kant attempts to prove that what is ascertain in the metaphysical exposition of space is epistemologically established with apodictic certainty.

After metaphysical and transcendental exposition of space these both expositions are to be made for time also. Kant makes these two expositions almost with the similar arguments which will be dealt in the next subsection.

4.4 Kant’s view of exposition of time:

The problem of Time as remained a very much puzzling problem since the beginning of western philosophy. In modern western philosophy, Descartes and Spinoza are almost silent on this problem of time and Leibnitz rejects the reality of time. For Kant time is also a pure form of intuition. But even if it belongs to sensibility it has to play the role of inner sense which will be clarified in the discussion of transcendental logic. But here Kant is anxious to prove that like space, time is also a pure intuition and so a metaphysical and a transcendental exposition of time is also necessary. The meaning and function of both type of exposition are just
like that of space. Therefore the text of critique we will briefly account and evaluate the metaphysical exposition of time in the following subsection.

### 4.4.1 Metaphysical exposition of time:

The metaphysical exposition of time is briefly summarized by Kant. The plan of Kant is almost same so far as the metaphysical exposition of time is concerned. Kant proves that time is an a priori intuition and also that time is a concept but not an intuition. Kant starts the proof of the first point in the following way.\(^{48}\)

“Time is not an empirical concept that is somehow drawn from an experience. For simultaneity or succession would not themselves come into perception if the representation of time did not ground them a priori. Only under its presupposition can one represent that several things exist at one and the same time (simultaneously) or in different times (successively).”

In this proof Kant takes a similar stands which he has taken in the case of space. For space it was argued that if some representation refers to the object which is outside me then for the formal ground of the use of this ‘outside’ term will require the pure form of intuition in the form of space as a priori condition. Here Kant is referring the another important characteristic which belongs to all representations. If a representation is denoting an object which is ‘outside me’ then this type of representations must have the character of either simultaneity or succession but these concepts of simultaneity and the concept of time as there a priori formal condition and so Kant says that time is not an empirical concept which is drawn or derived from outer experience. For further proof of a priori nature of time Kant provide his arguments as follows.\(^{49}\)

“Time is a necessary representation that grounds all intuitions. In regard to appearances in general one cannot remove time though one can very well take the appearances away from time. Time is therefore given a priori. In it alone is all actuality of appearances possible. The later could all disappear. But time itself, as the universal condition of their possibility cannot be removed”

In the second point what Kant is emphasizing is this that we can imagine or think time without appearances but we cannot think or take appearances without time. Here it seems little bit strange, it may be taken as a psychological possibility that one can imagine an empty space in the form of pure darkness without any material objects residing in it. But it is even psychologically very much difficult that how an empty
time, from which in the words of Kant, if the all appearances are taken away how this time can be imagined. Other observations which we have made in the previous subsection for the case of space stands valid for the case of time also that is with the arrival of special and general theory of relativity, the absolute character of time has been removed from physics and therefore Kant's view which is basically presupposing the Newtonian concept of absolute time bears a character of a counter scientific notion.

In this way it can be concluded that in the metaphysical exposition of time, Kant attempts to prove that time is a pure intuition, it is empirically real yet it is transcendentally ideal. In the realm of empirical reality, the Newtonian concept of absolute time is almost uncritically accepted by Kant. This metaphysical exposition of time is to be followed by transcendental exposition of time which provides the ground of the possibility of synthetic a priori judgments which is dealt in the following subsection.

### 4.4.2 Transcendental exposition of time:

The meaning and scope of the transcendental exposition of time is almost similar to the transcendental exposition of space in Kant. Its aim is to provide the apodictic certainty of mathematical propositions with reference to time. In the case of space the concerning mathematical discipline is taken by Kant as Euclidian geometry but the situation for the applied discipline for time is not clearly stated by Kant in his 'critique of pure reason.' Kant states his transcendental exposition of time in the following words.50

"This is a priori necessity also grounds the possibility of apodictic principles of the relations of time, or axioms of time in general. It has only one dimension different times are not simultaneous, but successive (just as different spaces are not successive, but simultaneous). These principles could not be drawn from experience, for this would yield neither strict universality nor apodictic certainty. We would only be able to say: This is what common perception teaches, but not: This is how matters must Stand. These principles are valid as rules under which experiences are possible at all, and instruct us prior to them, not through it."

In this observation Kant makes an important remark about the nature of time. It is clear that Kant is not here adopting the concept of time as a relational concept, as it is accepted by Leibnitz. Kant states that time has only one dimension and its
different parts or moments lacks the characteristic of co-existence as it has been ascribed to the concept of space. In this claim of non-simultaneous character of time Kant is adopting the Newtonian concept of absolute time. Now Newton states his concept in his 'Principia Mathematica' as,\(^{51}\)

"Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequal) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year."

In this famous definition of absolute time Isaac Newton is observing the objectively real status of time together with its flow. This absolute character is stated in Galilean transformation, as it has been mentioned earlier in this work, by the equation \(t = t'\). But it is also clear that these equations do not hold in modern physics particularly with reference to the Lorentz transformations. But here Kant is putting emphasis on one dimensionality of time and non-coexistence of its different moments. Now it is a question of current debate on Kantian scholars, that which branch of mathematics can provide the desired synthetic apriori judgments which can give the apodictic certainty to temporal pure intuition. The majority of scholars including Norman Kemp smith, believe that, according to Kant, arithmetic is the branch of mathematics which can provide synthetic apriori judgments with reference to metaphysical exposition of time. It may be surprising that what can be the possible connection between time and arithmetic. It is altogether intuitively clear that geometry is the science of space or by geometrical propositions we can derive and describe actual objective characteristics of space. But in which sense the propositions and theorems of arithmetic are supposed to describe the properties of time. The only possible explanation which can be given to this question is this. As it is supposed that time has one dimension and there is a definite order among its moments, we must have a similar mathematical structure which is a totally ordered set.\(^{52}\) This means that for any two moments of time, for example \(x\) and \(y\), if \(x\) occurs before \(y\) than this occurrence is unchangeable. This property can be observed in real number system and therefore it may be possible that according to Kant arithmetic is the science of time but here there are certain mathematical considerations which make these assumptions incorrect.
Arithmetic is a mathematical structure and as there are different possible geometries there can be different possible arithmetical systems. The question will be this that which arithmetical system can provide the ground of the apodictic certainty to temporal propositions? Perhaps the answer will be: It is arithmetic based on the axioms of Peano, together with its development up to real number system due to Dedikand and Caushy. But it is also true that all the theorems of real number system cannot be applied apriori to the properties of time. Moreover in pure mathematics the number theory is extended to complex number system which can be represented, geometrically, on a plane and not on line. So it can be concluded that according to Kant, in the transcendental exposition of time, there is no specific indication of the mention of any particular discipline, yet a certain part of arithmetic and number system may provide an arena where the possibility of synthetic apriori propositions can be justified.

With the metaphysical and transcendental exposition of time Kant has provided the proof of space and time as pure intuitions as well as the indication of concerning disciplines for synthetic apriori judgments in the wider perview of Kantian epistemology. It is necessary to undertake a brief and yet compact apprehension on the overall status of space and time in Kantian philosophy. This is also a natural order of the following of the composition of the text of critique of pure reason and so in the next section, a general estimation of Kant's position of space and time has been done.

4.5 Kant's overall view of space and time:

In the entire history of western philosophy the epistemological and ontological status of space and time are generally not taken together with due appropriate seriousness. The absolute reality of space is generally accepted in materialistic ontology and a world view which contains infinitely extended absolute space together with some discretely distributed location of material objects are accepted as a description of nature only in Greek atomism of Democritus. In the subsequent development of western philosophy, no philosophical system has accepted this absolute character of space at ontological level. In the perview of modern western philosophy for Spinoza, and for Descartes, space is something real yet not an independent entity. While for Leibnitz it is completely unreal. In the case of empiricism, space is a subjectively imposed entity which cannot be considered as real. For time there is not much serious consideration which has been made in western
philosophy apart from the Leibnitz’s view of unreality of time. And in medieval philosophical assertions which deny the objectivity and reality of time only on theistic grounds. So the historical position, which was open before Kant, for a serious consideration of space and time was the mechanistic modal of Newton which he laid down in his 'Principia mathematica'. For Newton space and time are absolutely real. But this position cannot be accepted in Kant, because in that case Kantian philosophy must have turned in to materialistic ontology in which for epistemological considerations much room cannot be provided. So before Kant, there are two fold tasks. In the first option, Kant cannot deny the entire objective character of space and time because in that case his philosophy cannot be differentiated from Berkeley’s subjective idealism. At the same time, he cannot accept absolute objectivity of space and time because it mixes his position as materialistic ontology. So, according to Kant, space and time are empirically real yet transcendentally ideal. Kant states this view clearly for time in the following words.  

"Against this theory, which concedes empirical reality to time but disputes its absolute and transcendental reality insightful men have so unanimously proposed one objection that I conclude that it must naturally occur to every reader who is not accustomed to these considerations."  

This observation of Kant states his famous statement which stands correct for space also, that for his epistemology, the empirical reality and transcendentality of space and time are inevitable necessary ingredients. Here it is to be exemplified that what is the actual meaning of Kantian statement that space and time are empirically real and yet transcendentally ideal. Because in the historical development of Kantian scholarship some idealist scholars and interpreters of Kant, like Adward Caird had, already mentioned that Kant's position is much nearer to absolute idealism than it is generally supposed. So it is necessary to bring out the essential meaning of the empirical reality of space and time according to Kant.

In order to clarify the meaning of the term empirically real it is necessary to understand the exact meaning and significance of the term 'empirical' according to Kant. For Kant there is a fundamental limit of knowing process and knowing power and space and time both plays their role in the establishment of this limit. Any object which can be the subject of a possible experience must be put; as a representation, 'before the knower through the outer sense whose pure form is space. While whatever has been put before knower through outer sense must be intuited through the inner
sense, or empirical apperception in the terminology of 'transcendental analytic.' In every case this is a uniformly adopted epistemological position in Kant that without this combined role of space and time, no empirically possible cognitive phenomenon can be generated and therefore like subjective idealism space and time cannot be considered as mere subjectively generated or interpreted cognitions from the side of knower. Because they are preconditions of the generation of any cognition. If these were not the case, then there could be no room for the possibility of synthetic apriori proposition in mathematics. The empirical reality of space and time is estimated and justified on the evidence of the possibility of mathematics, and as mathematics is objectively real and empirically valid, space and time are also having empirical reality.

With this empirical reality of space and time Kant does not believe that they are ultimately, or in the language of Kant, transcendently real. Before considering this matter further, it is necessary to know what is ultimately or transcendently real in Kant or what does Kant actually mean by the term 'transcendently real'? According to Kant ultimate reality is that which can or does exist without any reference to any cognition of any knower. For this type of reality Kant uses the term 'thing-in-itself' or 'Noumena'. According to Kant, this noumena or thing-in-itself are totally unknowable for any finite cognitive power. This is a problematic concept and so here, in transcendental aesthetics, when Kant denies the transcendental reality of space and time, he simply means that space and time are neither noumena nor any transcendental characteristic of noumena. And so space and time do not possess any transcendental reality.

Here, at this point some further consideration is being needed. The entire Kantian position on noumena together with his agnostic stage is a highly disputable philosophical position in Kant's framework. It is criticized by all types of scholars of Kant including Hegel. At present our intension is not to make a complete critical estimation of the problematic concept of noumena and Kant's agnostic position. But it can be asserted that when Kant denies the transcendental reality of space and time in the above mentioned sense then situation becomes somehow obscure. It may be assumed that things-in-themselves are lacking geometrical properties or they are supposed to be transcend the linear or ordinal properties of Arithematic; then spatial structure. And so is it also to be taken as an impossibility that no arithmetical
characteristics can be assigned to things in themselves in other words if we assert a proposition that

Seven (thing-in-itself) + five (thing-in-itself) = twelve (thing-in-itself) whether this expression asserts a proposition or it is simply a meaningless expression. If it is meaningful then whether it is true or false. If true then is it providing an example of a synthetic apriori judgment. All these questions are almost unanswerable in Kantian philosophy. Another point is this that, in Kant the transcendental, ideality is related with a particular type of epistemological conviction rather than an ontological status. So when we state that space and time are transcendentally ideal, we have to admit that they are the noumenal conditions of finite consciousness. But Kant does not accept this situation and therefore he takes space and time as empirically real and transcendentally ideal.

Space and time are pure form of intuition and therefore they provide the ground of synthetic apriori judgments. Kant says that in a finite consciousness when the process of judgment is to be occurred for any cognition and so it is to be assumed that in any finite consciousness, intuition cannot be intellectual and therefore for synthetic apriori judgments we must take space and time are empirically real. Kant expresses his view in detail in the following way.⁶⁰

“Thus, if it were to be supposed that space and time are in themselves objective and conditions of the possibility of things in themselves, then it would be shown, first, that there is a large number of a priori apodictic and synthetic propositions about both, but especially about space, which we will therefore here investigate as our primary example. Since the propositions of geometry are cognized synthetically a priori and with apodictic certainty, I ask: Whence do you take such propositions, and on what does our understanding rely in attaining to such absolutely necessary and universally valid truths? There is no other way than through concepts or through intuitions, both of which, however, are given, as such, either a priori or a postori. The latter, namely empirical concepts, together with that on which they are grounded, empirical intuition, cannot yield any synthetic proposition except one that is also merely empirical, i.e. a proposition of experience; thus it can never contain necessity and absolute universality of the sort that is nevertheless characteristic of all propositions of geometry. Concerning the first and only means for attaining to such cognitions, however, namely though mere concepts or a priori intuitions, it is clear that from mere concepts no synthetic cognition but only merely analytic cognition can be attained.”
Here Kant remarks about the general theme of his epistemology and its justification in the realm of sensibility. The concept of knowledge either in rationalism or in Kant, contains the factors of universality and certainty in itself. It is true that no proposition containing empirical concept or any particular cognition, can have this type of characteristic. So in the perview of empiricism, it is generally believed that universality and necessity can be assigned only to analytic propositions. But for Kant universality and necessity should also be assigned to factual knowledge and synthetic proposition. This becomes possible because space and time are pure form of intuition and so they provide the ground of synthetic apriori judgments which contain the characters of universality and necessity. Methodologically, this amounts to Kant’s view of the status of mathematic itself or in other words his philosophy of mathematics. This is briefly discussed and estimated in next section.

4.5.1 Kant's philosophy of mathematics:

Philosophy of mathematics is an important branch in contemporary analytic philosophy and its roots are found in the beginning of western philosophy, particularly in the philosophy of Plato, mathematics has remained always an ideal structure or modal for systematic thinking and it provides a genuine example of the application of deductive method and universal as well as necessary knowledge. Kant's views on philosophy of mathematics are not generally accepted in analytic tradition yet they have profound influence on it. Before considering Kant's philosophy of mathematics in the light of currently interpreted situation, it is useful to make a preliminary survey of different trends of philosophy of mathematics.

Among analytic school of philosophers. Where are these different trends or schools of philosophy of mathematics.

1) Logicism
2) Formalism
3) Intuitionalism

Each of their principle is explained and stated briefly as follow.

1) Logicism:
This school of philosophy of mathematics is originated in the latter half of 19th century particularly from the philosophy of Frege. In logicism there is a complete identification between mathematical and logical truth and it is claimed that all mathematics can be derived, in principle, from logic. Deduction is the only adopted
methodology and mathematical truths are theorems of logic. This view is held among analytic philosophers like Russell and Wittgenstein and logical positivist. Particularly Carnap adheres this view and attempts to reduce in the form of the syntax of language. The key idea as the ground of logicism is this that whole mathematics can be derived from some basic postulates of logic, for practical purpose even the first order predicate calculus is sufficient. So from Kant's view of logic, either formal or transcendental, it can be asserted that logicism does not constitute Kant's view of mathematics. Because Kant's formal or transcendental logic does not even permit the use of nested quantification. Therefore Kant's philosophy of mathematics is not consistent with logicism of 20th century.

2) Formalism:

Formalism is a philosophy of mathematics which provides a separate status to mathematical derivability. Mathematical deduction, which pass from mathematical forms to other forms state a position in which formal deduction and truth are to be identified. Actually, in principle, formalism is a grand programme which states that by a sufficiently enriched computing devices or deriving agencies, whole mathematics can be proved, or in other words, each mathematical truth can be established as a mathematical theorem. This view is led particularly by Hilbert and is generally called as Hilbert’s programme. This view is also criticized when Gödel’s first incompleteness theorem established that this was an impossible task and a complete axiomatic formalization of mathematics is impossible. No doubt, Kant supports Euclidian geometrical systems as an ideal mathematical formalization, but for Kant the essence of mathematical truth does not solely lie in its deductibility Because, as we have seen, in the transcendental exposition of space, that geometrical truths are not simply ascertained from their deductibility form the axioms of Euclidian geometry but apriori character of this truth is based on their applicability to space. So formalism is not a view of philosophy of mathematics which could be accepted by Kant.

3) Intuitionism:

Intuitionism is a philosophy of mathematics which accepts a separate ontological existence of mathematical concepts and entities. Actually this philosophy of mathematics has been propounded by Pythagoras and supported in part by Plato. According to this school neither the existence of mathematical objects are purely symbolic nor form or mathematical procedures for the derivation of theorem is purely mechanical. So according to this school which is led in contemporary
philosophy by Brower, and supported by the thinkers of mathematical philosophy like Lucas and Penrose, held that mathematical objects are ontologically there and comprehension of each mathematical truth is not a computational and algorithmic procedure. Even Kurt Gödel does not accept either formalist or logicistic view. In an unpublished article which was prepared by him for the contribution of the volume of the philosophy of Rudolf Carnap, he claims that mathematics is not just the syntax of language. Therefore it can be said that with a viewpoint of Kant, the existence of synthetic apriori propositions confirms partly a form of intuitionism. This is so because like Plato, Kant does not presuppose a separate ontological realm for the justification of the objectivity of mathematical concepts. But according to Kant there is, somehow, one-one correspondence between mathematical truths and actually existing facts. And therefore mathematical truths can provide, in Kant, a ground for the possibility of synthetic apriori judgments.

4.5.2 Kant's view and absolute space and time of Newton: 

As we have pointed out in the preceding subsection regarding Kant's overall view of space and time, the general Kantian theme can be summarized in the statement that space and time are empirically real but transcendentally ideal. This empirical reality of space and time provides the ground theme of Kant's philosophy of mathematics in the subsequent development of the text of "critique of pure reason". This saves Kantian epistemology from the charge of subjective idealism which Kant undertakes in the first book of transcendental dialectic under the title of Refutation of idealism. The same objectivity of space and time at the level of empirical reality can be seen in antinomies of pure reason where Kant proves contradictory statement about universe as a whole. All this consideration suggest that Kant is not supporting any subjective idealistic view about the nature of space and time with reference to the evaluation of the central focus point of present research work where Kant's doctrine of transcendental illusion is to be examined in an overall view, It is necessary to take a general note regarding Kant's view and Newton's absolute space and time.

Newton's space and time are fixed ontological realities. Einstein, in his relativity theory, states that Newton's space can be thought either at rest or at least without any acceleration. Newtonian concepts of space and time provides a picture of universe in which space is flat Euclidian and extended up to infinity. Space is three dimensional and geometrically the metric which is defined on this space is positive
definite metric and which is invariant under Galilean transformations. This type of conception of space, which is absolute and this type of absolute time are to be generally considered as a type of *causa sui*. If Kant accepts that Newton's view of space and time are empirically valid, then this type of conception of space and time cannot be used for the derivation of antinomies of pure reason. But it can be remarked that with the Newtonian view point of space and time it is inconsistent to derive at least first antinomy of pure reason particularly the thesis of that antinomy.

### 4.5.3 Kant's concept of geometry:

Before the construction of formal axiomatic systems in logic and mathematics in 20th century, Euclidian geometry was the only example of an axiomatic system. Kant takes geometry in the realm of pure discipline as a piece of truth and yet for the apodictic certainty of geometrical propositions. Kant takes the one-one correspondence between the geometrical properties of actually existing three dimensional space and formal geometrical theorems of Euclidian geometry. All possible geometrical theorems are somehow actualized in actually existing three dimensional space. This correspondence is the very ground of calling geometrical propositions synthetic.

In 20th century this position of Kant has been generally criticized in analytic tradition with a stand that after the construction of non-Euclidian geometries in 19th century the very ground of the synthetic character of geometrical proposition does not remain the safe. But it is necessary to think that what is actually lost for Kant by the arrival of non-Euclidian geometries. What Kant has actually said for the relationship between geometry and the property of space does not refer to any particular truths of geometry. Kant states this in the following words:

"Geometry is a science that determines the properties of space synthetically and yet a priori. What then must the representation of space be for such cognition of it to be possible? It must originally be intuition; for from a mere concept no propositions can be drawn that go beyond the concept, which, however, happens in geometry."

Here Kant does not indicate the acceptance of any particular concept or theorem of Euclidian geometry. Moreover, it is still more interesting that, for Kant even, 'space is three dimension' is not an analytic proposition at all. Kant states, in his pre-critique work, in ‘theories of heaven’ that –‘space is three dimensional’ is a
synthetic proposition. And the three dimensionality of space can be derived from inverse square law of gravitation.\textsuperscript{73}

Taking all this matter together it can be said that like a professional thinker or philosopher of philosophy of mathematics Kant states only a position of apodictic certainty of geometrical propositions. Even if Riemannian geometry may applied to curved space as it has been shown by Einstein in his general theory of relativity then the philosophical question remains the same. Why it is so that there is one-one correspondence between a geometrical formalism and an actually existing space of a universe and in that realm the position of Kant is still relevant as it becomes clear from the following concluding remark of Kant.\textsuperscript{74}

"Thus our explanation alone makes the possibility of geometry as a synthetic a priori cognition comprehensible. Any kind of explanation that does not accomplish this, even if it appears to have some similarity with it, can most surely be distinguished from it by means of this characteristic."

As it is clear from the observation of Kant that when a formal structure like geometry is considered in a co-relation with actual three-dimensional space that its proposition acquire the apodictic certainty in Kantian sense. It is an altogether different question that which geometrical structure can describe the actual space and if we look at the currently investigated concept of multiverse then there is a possibility that every mathematical structure is related to its factual counterpart.\textsuperscript{75} And so Kantian theory of geometry is still relevant because it takes the question at a deeper level where the central focus is the relation between formalism and realistic. The cosmological ideas of Kant and antinomies of pure reason will be considered in the subsequent development of this work, and particularly in 8\textsuperscript{th} chapter at present it can be concluded that the inventions of non-Euclidean geometries do not refute Kant's position completely. Therefore, in the 20\textsuperscript{th} century when analytic philosophy refutes Kant's concept of synthetic apriori judgments in mathematics then it is not so simple matter. And there is an urgent need of a reconsideration of the relation of formalism and reality. Leaving the detailed examination for a subsequent consideration, currently Kant's view of intuition together with its relation with space and time is going to be elaborated in the next sub section.

\section*{4.5.4 Kant's View of intuition and space-time:}
The terms intuition has many references and in metaphysical or spiritualistic discourses, it is generally taken as a third order of knowledge which can make a direct connection with ultimate reality. Actually space and time are pure forms of intuitions and according to Kant they apply to all conscious beings that are finite. By stating that space and time as the forms of intuitions, Kant thinks that empirical reality can be imparted on it. Kant states clearly in the following words.

"If I say: in space and time intuition represents both outer objects as well as the self-intuition of the mind as each affects our senses, i.e., as it appears, that is not to say that these objects would be a mere illusion. For in the appearance the objects, indeed even properties that we attribute to them are always regarded as something really given, only insofar as this property depends only on the kind of intuition of the subject in the relation of the given object to it then this object as appearance is to be distinguished from itself as object in itself. Thus I do not say that bodies merely seem to exist outside me or that my soul only seems to be given if I assert that the quality of space and time in accordance with which, as condition of their existence, I posit both of these lies in my kind of intuition and not in these objects in themselves. It would be my own fault if I made that which I should count as appearance into mere illusion."

This form of sensibility which is stated by Kant as one of the two fundamental powers of mind, is not to be taken as illusion. The most general term which is used by Kant is 'representation', and this intuition is also a representation whose task is to put empirically external objects before mind for the generation of cognition. In a well-known observation Kant states intuition as a source in the mind for knowledge in the following words.

"Our cognition arises from two fundamental sources in the mind, the first of which is the reception of representations (the receptivity of impressions), the second the faculty for cognizing an object by means of these representations (spontaneity of concepts); Through the former an object is given to us, through the latter it is thought in relation to that representation (as a mere determination of the mind). Intuition and concepts therefore constitute the elements of all our cognition, so that neither concepts without intuition corresponding to them in some way nor intuition without concepts can yield cognition. Both are either pure or empirical. Empirical, if sensation (which presupposes the actual presence of the object) is contained therein; but pure if no sensation is mixed into the representation. One can call the latter the matter of sensible cognition. Thus pure intuition contains merely the form under which
something is intuited and pure concept only the form of thinking of an object in
general. Only pure intuitions or concepts alone are possible a priori, empirical ones
only a posteriori."

Kant's observation clearly states that Human power of sense perception is
directly related or controlled by the spatiotemporal form. If space and time are
empirically real then the objects which are being presented before knower are not
either illusory or mere subjective state of the cognition of knower as it is the case in
the philosophy of Berkeley. But if space is empirically real, and it is a form of
intuition and it can be conceived as infinite given magnitude, then what is the
epistemological ground for the derivation of antinomies about the finite and infinite
nature of space. This question will be dealt with a greater emphasis in the subsequent
chapter. But at present it can be said that Kant's theory of intuition is closely related
with his theory of space and time and therefore in the consideration of spatiotemporal
antinomies it must be taken into account.

4.6 Conclusive evaluation:

Kant’s theory of space and time provides an epistemological justification of
the possibilities of synthetic apriori judgments in mathematics moreover it provides
the answers to Kantian question that how is pure mathematics possible? But even
though Kant’s answer to this question may not be convincing from a modern point of
view, it must be taken seriously regarding the ontological question about the
relationship between formalism and reality. The central theme of present research
work is concern with the third question of Kant in the form ‘how is metaphysics as a
natural pre-disposition possible?’ In the answer of this question Kant puts his theory
of transcendental illusion, which will be the central point of consideration of this
research work. Before considering that it is necessary to have a close look of Kant’s
theory of understandings and natural principles which is given in the second part of
the transcendental doctrines of elements. This will be studied and evaluated in the
next chapter.
Notes and references.

1. Kant’s theory of space and time is having much contemporary relevance. Many well-known commentators and philosophers have commented on Kant’s view of transcendental aesthetics. All the standard critics of Kant, including H. J. Patton and N. K. Smith, have devoted much place to transcendental aesthetics and Kant’s views on space, time and geometry. The contemporary philosopher like Hintikka has also paid much attention on Kant’s theory of space and time. The famous German critique Hans Vaihinger has devoted to volumes to Kant’s transcendental aesthetics in his work commenter *Kommentar zur kritik der reinen vernunft (1881 1892 stuttgart)*


3. Big bang cosmology is the name of that standard cosmological modal, according to which, the origin of the universe together with space and time is considered before 13.7 billion years ago. cf. Wall R.M. (1984) ‘general relativity’ Chicago University press, Wineberg Stephen (1972) ‘Gravitation and cosmology’, Wily publications, New York. With the modification of inflationary universe scenario, big bang cosmology is currently considered as the standard version of cosmological modals after the discovery of cosmic microwave background radiation in 1965. This particularly amounts the significance of Kant’s first antinomy

4. These three companions are,
	Hintikka has also contributed many important research articles regarding Kant’s philosophy of mathematic and its relation with transcendental aesthetics of critique of pure reason. Relevant theme of these works to the present research work will be undertaken in this chapter during a critical evaluation of Kant’s theory of space and time and its relation with foundation of mathematics
9. In his work theory of heavens Kant made his famous nebula hypothesis which develops the Kant’s conception of island universe. Actually Kant was the first person who anticipated that our galaxy is not the only stellar system which is there in the universe. He postulated that there are many different island type stellar systems which are contained in the universe. This hypothesis, in part, was confirmed after 1920 by astronomical discoveries, particularly due to Hubble. It can be said that this astronomical consideration has played some role in Kant’s derivation of antinomies of pure reason.
11. Even recently published the Cambridge companion to Kant critique of pure reason contains only one article written by A. W. Moore with the title ‘The transcendental doctrine of method’ p-p.310-326
12. This type of division and its presentation in the form of a table is given by Kant only in the first edition of critique of pure reason. The paragraph number is given as A_{XXIV}. In the second edition of critique of pure reason, this table of content is dropped. Yet almost all the divisions and parts of critique of pure reason remain same in both editions.


   Here the translation of Paul Guyer makes a note that ‘Kant’s own copy of the first tradition here adds ‘I exist’ is an analytic judgment’, ‘a body exists is a synthetic one’.


   In the second edition of Critique of pure reason, Kant has omitted this paragraph and simply stated in B11 that analytic judgments are judgments of clarification and synthetic judgments are judgments of amplification. cf, B11, P. 141

22. Actually even Newton’s physics and his ‘Principia Mathematica’ do not accept this type of inherent relationship between the concept of body and the concept of extension. Historically this inherent relationship is in a certain sense is accepted in the general theory of relativity where Einstein refers to Descartes in the following words. “Descartes argued somewhat on these lines ‘space is identical with extension and extension is connected with bodies hence there is no space without bodies and therefore there cannot be any empty space’.” cf. Einstein Albert (2001) ‘Relativity the special and the general theory.’ Dover publication, London, Appendix-5, ‘Relativity and the problem of space.’


   Kant’s claim is not limited only for theoretical disciplines like epistemology and logic. According to him even ethics must contain synthetic apriori propositions if it is to be treated as a theoretical science of reason. cf. Kant Immanuel (2003) ‘Critique of practical reason’, translated by Paul Guyer, Cambridge University press, Cambridge


   The introduction of the first edition of critique of pure reason does not contain this paragraph.

Actually these propositions are axioms of Euclidian geometry which is given in ‘Elements by Euclid’


27. Particularly the concept of limit is expressed in the terminology of inequalities and yet there is an equivalent expression of the same concept in equality. These matters run in this way.

‘Let \( f \) be a real valued function and ‘\( L \)’ is a real number, \( f \) is defined in the punctured delta neighbourhood of \( a \), if

\[
\forall \varepsilon > 0, \exists \delta > 0 \quad 0 \leq |x - a| < \delta \implies 0 < |f(x) - L| < \varepsilon
\]

Then the limit of the function \( f \) is \( L \) as it tends to \( a \) and it is presented symbolically as,

\[
\lim_{x \to a} f(x) = L
\]


In this paragraph Kant states two examples for the justification of the presence of synthetic apriori proposition in physics. These examples are the law of the conservation of momentum and Newton’s third law of motion. It is well known in the realm of physics that after the arrival of Einstein’s special theory of relativity these laws are to be taken as the approximation of wider universal laws. The law of the conservation of momentum has become the law of the conservation of energy and momentum and Newton’s third law is not correct as nothing can travel faster than light. cf. Einstein Albert (1998) ‘The meaning of relativity’ Dover publication, London

29. This modal was successful in the explanation of certain motion of celestial bodies. There were many correct predictions including the predictions of solar and lunar eclipse which were made on account of this modal. But in due course of astronomical developments it was found that all celestial motions are not explained on the ground of this modal. So according to Kant, in a similar analogy, it is found that the traditionally accepted concept of knower and known and the views about their mutual relationship cannot explain the nature and possibility of synthetic apriori judgments and so Kant attempted to make a fundamental revolution in the field of epistemology.

30. This is a fundamentally important question; not only for ontology but also in a scientific discourse but unfortunately before the arrival of quantum mechanics no one has questioned this in physics. In quantum mechanics it is believed that any act of
observation makes a difference or distortion in the system which is being observed and so this amounts towards a fundamental theoretical limit in the accuracy of observation. This has challenged the classical deterministic view as well as the traditional concept of causality in physics. cf. Dirac P.A.M. (1967) ‘The principles of quantum mechanics’, Oxford university press, Oxford. In the first introductory chapter of this work Dirac provides the arguments that any act of observation cannot leave that system undisturbed which is being observed and philosophically this puts some serious questions before the extremely realistic world view which adheres the belief that the universe is just out that and we are simply observing it as it is there.


This detailed observation of Kant which has been made by him in the preface of the second edition of his Critique of pure reason is fundamentally important for his entire epistemological framework. There are divergent opinions of the scholars of Kant which are derived from this. Some critiques derive idealistic elements from it. cf. Caird A.M. (1952) ‘The critical philosophy of Kant’ vol-I and II, Cambridge University press, Cambridge. On the other hand some commentators wants to incorporate this position as a fundamentally important factor in Kant’s philosophical stand. cf. Paton H.J. (1963) ‘Kant’s metaphysics of experience’ vol-I and II. While some commentators of Kant like N.K. Smith finds this as an obscure position in Kant. Here we simply observe that this is a fundamental change in epistemology whose implications are not properly drowned in the subsequent development of western philosophy.

33. Kant Immanuel (2000) ibid, A_{20},B_{34}, P.135
34. Kant Immanuel (2000) ibid, A_{21},B_{35}, P.136
37. Kant Immanuel (2000) ibid, B_{XL},B_{XLI}, p-p.121-122

It is important to note that in the translation of N. K. Smith ‘derived’ in the place of the term ‘drawn’ is used. The meaning is same up to a certain a certain extent but if the meaning of the both is to be taken together then it can be said that space has no empirical counterpart in the content of any outer experience.

40. A positive definite metric is expressed by the following equation,
\[ ds^2 = dx^2 + dy^2 + dz^2 \]
And this metric is co-variant under Gelelilon transformation which is the fundamental transformation in Newtonian physics.
\[ x = x' - vt \]
\[ y = y' \]
\[ z = z' \]
\[ t = t' \]
this notion of space provides a view of space which is absolute, eternal, unchangeable and untransformable but this view is not an agreement with the current view of science.

41. A negative Lorentz metric is defined as
\[ ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \]
Where ‘c’ is the velocity of light and the Lorentz transformations are expressed as following
\[ \frac{x' - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \]
\[ y = y' \]
\[ z = z' \]
\[ t = \frac{t - \frac{vx}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \]
This Lorentz metric provides the existence of light cone on every particular events of this universe and so the existence of absolute plane of simultaneity has been ruled out. The light cone can be represented in the following way.
42. The general metric tensor in general theory of relativity is defined as
\[ ds^2 = g_{ik}dx^i dx^k. \] This is the general formula which serves as the schema of distance function. For obtaining a particular type of formula for the measurement of distance in general relativity, one has to solve the field equations of Albert Einstein under specified conditions. Historically for the surrounding geometry of a non-rotating spherical star this field equations were solved by Schwarzschild in 1916 and with the subsequent exact solutions for rotating spherical star. Exact solution has been found by Kerr in 1962. cf. Hawking S.W. and Eliies G.E.R.F (1973) ‘The large scale structure of space, time’ chapter-5, ‘exact solutions’. In this chapter Hawking surveys different metrics including Robertson Walker metric which is used on the cosmic scale. Here the point is this that contrary to Kant’s belief there are no fixed geometrical properties at all. As Einstein remark’. The geometrical properties of space are not independent but they are determined by matter. cf. Einstein Albert (1952) ‘Relativity the special and general theory’ Dover publication, Appendix-I

43. Kant Immanuel (2000) op.cit. P.158
44. Paton H.J. (1962) ‘Kant’s metaphysics of experience’ vol-I
45. Kant Immanuel (2000) op.cit., P.159
46. Kant Immanuel (2000) ibid P.158
Here N.K. Smith puts great emphasis on this contention of Kant. He quotes Kant with the effect that a geometrical science of all possible spaces of different dimension would have been of the greatest important but this task is very much difficult for human being. No doubt it was difficult, or rather impossible in the age of Kant, but currently in theoretical physics and cosmology, there is a possibility of the physical existence of spaces which have more than three dimensions. In superstring theory, Bosnic strings requires 26 dimensions and heteroictic superstrings require 10 dimensions. Moreover Kant’s contention regarding the relationship between three dimensionality of space and inverse square law is also prophetically visionary. Because it is now proved that if the physical space is to be taken in four dimensions then the law of gravitation will take the nature of inverse cube law in the place of inverse square law in three dimensions. cf. Hawking S.W. (2010) ‘The grand design’ Bantam press, London. And Polechinski Joe (2004) ‘String theory’ vol-I and II, Cambridge University press, Cambridge.

49. Kant Immanuel (2000) ibid P.162
50. Kant Immanuel (2000)ibid P.162
52. Partial and total orders on a given set are purely mathematical concept which plays an important role in the development of number theory and set theory. In the present context the meaning is simply this that for any two real numbers x and y only one of the following statements can be true
   X<Y
   X=Y
   X>Y
53. In the development of number system the negative integers are defined generally as the ordered pairs of natural numbers and rational numbers are defined as the ordered pairs of integers. Real numbers are defined either as the Dedikind cut on Q, ie the set of rationals or a Caushy sequence on Q. in neither definition a real number is simply an isolated element or number so it is very difficult to think that in which sense it can represent a particularly isolated moment of time. cf. Walter Rudin (1977) ‘The principle of mathematical analysis’ P. H. I. New Delhi
54. The complex numbers are defined as the ordered pairs of R or the set of complex numbers becomes a structure on R x R. among complex numbers the property of orders is not to be held. For example, for any two complex number a + ib and c + id where \( i = \sqrt{-1} \) only equality relation between them can be defined as if \( a = c \) and \( b = d \) then \( a + ib = c + id \). But there is no definition of inequality which can be given between two complex numbers therefore it can be said that any number system cannot apriori describe the properties of time and so far as the empirical applications of a mathematical structure is concerned it can be said that in quantum mechanics the mathematical structure which is given is a complex Hilbert Space. cf. Dirac P.A.M. (1964) ‘The principles of quantum mechanics’, Oxford University press, Oxford, London


In the second section of doctrine of elements Kant uses the term ‘empirical apperception’ almost similar to the term inner sense in the first section of transcendental aesthetics. The point is this that the row material which is being supplied through outer sense must be intuited through inner sense.

58. Kant Immanuel (2000) ibid, A\textsuperscript{235}/B\textsuperscript{294} to A\textsuperscript{269}/B\textsuperscript{315}

In this agnostic position of Kant it is believed that there are things in themselves which must be supposed as the ground or cause of the phenomena of possible experience. But there are no conditions of knowledge which can be assigned for the knowledge of noumena or things-in-themselves. p-p.354-365


In this work Kant is criticised by Hegel on the basis that agnostic epistemology is self contradictory

60. Kant Immanuel (2000) op. cit. p-p.187-188
   In this work, Russell adheres to the logicism of Frege which is fully developed in his famous work ‘Principia Mathematica’ which is written together with A.N. Whitehead. Wittgenstein also presents his view with supports logicism in his ‘Tractatus’. cf. Wittgenstein L (1961) ‘Tractatus logico philosohicus’ R. K. P. London.

62. First order predicate calculus is that in which only individual variables are quantified. As this calculus as an axiomatic system can be shown as consistent and deductively complete which have been shown by Gödel in 1930. So this is not the subject of Gödel’s incompleteness theorems and so most of analytic philosophers and working mathematicians generally do not bother for the second order formulation of a mathematical theory. cf. Hinttika Jacckko (1998) ‘The principles of mathematic revisited’ the Cambridge University press, Cambridge. Yet the current research in this realm of mathematical logic indicates a re-evaluation of higher order logic. cf. Stewart Sperio (2001) ‘Foundation without foundationalism a case for second order logic’, Cambridge university press, Cambridge. Looking all these things it can be summarized that if higher order logic is to be taken as a background of mathematical formalization than the basic contention of logicism collapse. Because entire deductive process becomes a subject of Godel’s first incompleteness theorem regarding deductive completeness.


66. Kant Immanuel (2000) ibid B_{275}, P.326. Here Kant states the theorem for the refutation of idealism as “The mere, but empirically determined, consciousness of my own existence proves the existence of objects in space outside me” B_{276}, P.327

67. Kant Immanuel (2000) ibid A_{428}/B_{456} to A_{432}/B_{460}, P.470-472

69. It is necessary for any equation of physics which expresses a law of nature should be invariant under a parameter group of transformation. In Newtonian physics all general laws are invariant under Galilean transformation. But Maxwell equations of electromagnetism are not invariant under Galilean transformation.

70. Kant Immanuel (2000) op. cit. A_{426}/B_{454}, P.470. The thesis of the first antinomy is ‘The world has a beginning in time, and in space it is also enclosed in boundaries.’

71. There are many types of non-Euclidian geometries which are based on different assumptions than Euclid’s postulates. In Euclidian geometry it is postulated that in a plane if there is a point which is not on a given line-1 then there one and only one line which can pass through that point which is parallel to l. In the geometries of Riemann and Lobewski this postulate has been dropped. According to Riemann there are no parallel lines and according to Lobewski all lines are parallel. cf. Kobiyazi and Nomezu (1962) ‘The foundations of differential geometry’ vol-I and II John Wily and sons, New York


73. Kemp Smith N.K. (1961) ‘A commentary to Kant critique of pure reason’ chap-3. Here Kemp Smith states that according to Kant there is a co-relation between the number of the dimensions of space and the form of the law of gravitation. Here it is also observed by Kant that there may be a possibility of more than three-dimensional spaces and the science of all these possible spaces would be of extremely important but at the same time it would probably be beyond the human reach. It is also to be noted that the recent discoveries of string theory and particularly M-theory, Kant’s observations are found relevant up to a certain extent. For example, in our three dimensional space the law of gravitation can be stated as,

\[ f = G \frac{m_1 \cdot m_2}{r^2} \]

But if our space is four dimensional then the law of gravitation would be

\[ f = G \frac{m_1 \cdot m_2}{r^3} \]

Or more generally in a d-dimensional space the law of gravitation will take the form of
\[ f = G \frac{m_1 m_2}{r^{d-1}} \]


75. Carter Bertrand (2007) ‘Universe or multiverse, Cambridge university press, Cambridge. In the discussion of multiverse hierarchy Tag mark discusses the question of the level of multiverses and at fourth level he contends that there may be a factual universe for every possible mathematical structure which includes all types of possible geometries. This multiverse concept estimates the number of universes as \(10^{10^{10^{10^7}}} \) or \(10^{10^{10^{10^7}}}\)

So the basic question which is currently being asked by Stephen Hawking and many other scientist that ‘why mathematics is magically being applied to the world?’ Finds some answer in multiverse theory. cf. Hawking S.W. (2010) ‘The grand design’ Bantam press, New York

76. For example, Spinoza in his work on the improvement of human understanding states that there is a third order of knowledge which encompasses and grasps reality itself. cf. Spinoza B. (1961) ‘On the improvement of Human understanding in collected works of Spinoza’ vol.-i Dover publication, London.

77. Kant Immanuel (2000) op. cit. B_{69}, P.-190

78. Kant Immanuel (2000) ibid, B_{75/A_{51}}, P.193

79. Kant Immanuel (2000) ibid, B_{22}, P.147