IV
Mean-Variance Efficient Portfolio Selection: Model Development
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The limited literature available in India in the area of portfolio selection compared to the efficient markets of the developed economies, such as United States of India (USA) and United Kingdom (UK) prompted us to conduct an in-depth study in this field. Although, the effect of various financial and accounting factors on security returns has been studied separately, yet no efforts have been made to integrate these factors for the benefit of an investor. The present quest tries to fills these voids. On the basis of knowledge gained from reviewing the research efforts of the past and the emerging issues in the Indian Capital markets, portfolio modelling has been attempted using the Quadratic programming approach.

Investors are faced with an array of factors while making the choice of securities for their portfolio. For making an optimal portfolio selection decision a serious consideration of investment objectives and limitations is required. Ambiguity on identifying the important factors from a given list of variables exists. Assimilation of the important factors already listed in literature, identification of new variables impacting portfolio selection decision and formulating a portfolio selection problem with multiple constraints requires attention.

The principle objective of this chapter is to develop a model to determine the Mean-Variance (EV) efficient sets in a constrained environment, for a class of portfolio selection problems. The main goal of an investor i.e. minimisation of risk/variance forms the objective function of the quadratic programming problem. The multiple objectives of today’s investor and their limitations are depicted through the formulation of linear constraints to this quadratic programming problem. An attempt to derive an efficient frontier for an investor with a single objective and multiple restrictions has been made. Theoretical contribution has been made by integrating and improving the existing quadratic programming portfolio modelling framework.
Multiple regression techniques and Granger causality tests which help in examining the explanatory power of variables and hence highlighting the more important amongst them have been discussed. Application of portfolio utility concept and the steps to compute it have been described in detail.

It is imperative to test the performance of the portfolios constructed using the model suggested vis-à-vis Markowitz’s efficient portfolio and the index. The measures for evaluation of portfolios as given by Sharpe (1966) and Treynor (1965) have been deliberated upon. The tests for equality of mean, variance and portfolio utility are also discussed in the end to compare the resultant portfolios.

IV.1 Research Design

The aim of an investor while selecting a portfolio is to minimise the risk for a given return subject to other constraints he/she faces. In this thesis an attempt has been made to aggregate a large number of constraints, to provide a solution which will be a trade-off amongst conflicting constraints without compromising in terms of Markowitz’s mean-variance efficiency. Mean-variance portfolio selection refers to the problem of finding an allowable investment policy satisfying all the constraints such that the risk measured by variance is minimised and expected returns are maximised.

The predictive power of variables explaining a cross section of returns such as beta, dividend, turnover, VaR, promoters holding, net profit, earnings per share, book to market ratio, Price to earnings ratio, portfolio utility, liquidity, conditional volatility etc have been investigated in isolation. This has limited the application of these studies simultaneously in investment management, for improving the efficiency of existing portfolio selection models. The resultant solution of standard portfolio selection models may be Pareto-optimal theoretically but may not perform well in the real world situations. The accumulation of important explanatory variables in a single portfolio selection model will lead to selection of portfolios which not only satisfy all identified constraints but also are efficient in the mean-variance sense.

IV.1.1 Mean-Variance Efficient Portfolio Selection: Theoretical Model

Following Markowitz, an investor has to choose from a set of n assets, \( A = \{a_1, \ldots, a_n\} \). Every asset \( a_i \) has a real valued expected return \( r_i \), and every pair
of assets \( \{a_i, a_j\} \) has a real valued covariance \( \sigma_{ij} \). The covariance matrix \( \sigma_{n \times n} \) is symmetric and its diagonal elements show the variance of assets \( a_i \). The values for return on securities \( r_i \), and their covariance \( \sigma_{ij} \) will be estimated from past data and are related to one fixed period of time. The portfolio to be constructed is a vector of real values \( X = \{x_1, \ldots, x_n\} \) where every \( x_i \) represents the fraction of funds invested in asset \( a_i \). According to Markowitz, the portfolio selection problem can be formulated as an optimisation problem over real-valued variables with a quadratic objective function and linear constraints. While Markowitz’s initial formulation was a bi-objective optimisation problem, this research work tackles the main objective of minimising overall variance in the quadratic objective function, ensuring the expected return \( E \). The other objectives of liquidity, earnings, dividend, utility, diversification etc have been formulated as linear constraints.

A Quadratic Programming Problem is framed where the objective function is to minimise variance (the most commonly used measure for risk). The problem has real financial market pressures and restraints formulated as linear constraints. This is a specific case of mathematical optimisation to determine a way to achieve the best outcome for a list of requirements (constraints) represented as linear relationships.

The academic work has focussed on the basic economic problem of assignment of weight to a security in a portfolio. The model will provide a solution which will give portfolio weights that will create portfolios which yield higher returns/ lower risk than either random buy and hold or existing portfolio selection models in the multiple constraint setting.

### IV.1.2 The Objective Function

The standard mean-variance portfolio selection model, standard analysis with upper bounds, Tobin-Sharpe-Lintner model, Black’s model have all tried to find maximum return-minimum risk portfolios. The recent day researchers Steinbach (2001), Zhao and Ziemba (2002), Lai, Wang, Xu, Zhu and Fang (2002), Goldfarb and Iyengar (2003) have developed models with minimising variance as the objective function. Mean-Variance efficient portfolios result in maximising the expected utility of the investor.
The objective function of our programming problem is to minimise the variance (the most widely used measure of risk) of the portfolio. This measure of risk is by its very nature quadratic.

**Objective Z: Minimise Variance**

**IV.1.3 Calculation of Risk/Variance of a Portfolio**

Variance of a portfolio is defined as

\[ V = \sum_{j=1}^{n} \sum_{i=1}^{n} x_i x_j \sigma_{ij} \]  

where,

\[ \sigma_{ij} = \text{E} [(r_i - \mu_i)(r_j - \mu_j)] \]  

is the covariance between \( r_i \) and \( r_j \). In particular

\[ \sigma_{ii} = \text{E} (r_i - \mu_i)^2 = V(r_i) \]

is the variance of \( r_i \).

**IV.1.4 Constraint Set**

The Quadratic Programming technique for optimisation of a quadratic objective function is subject to linear equality and linear inequality constraints. Constraints are conditions that a solution of an optimisation problem must satisfy. The set of solutions that satisfy all the equality and inequality constraints form the feasible set. A model is infeasible if no portfolio can meet its constraints.

The present day investor is multitudinous by his/her nature and has diverse objectives. The real world financial markets also impose additional restrictions/limitations on the process of portfolio selection. These objectives as well as limitations are incorporated in the model through the introduction of linear equality and inequality constraints.

The set of constraints in this mean-variance efficient portfolio optimisation problem are listed below:

1. **Desired Expected Return**

   Used by Markowitz (1952, 1959) the expected mean of a portfolio ensures an aspired level of return to the investor.

   \[ E \geq \hat{E} \]  

   (4.4)
A positive $E$ represents the desired expected return of the portfolio and is defined as

$$E = \sum_{i=1}^{n} X_i \mu_i$$

(4.5)

where,

$$\mu_i = E (r_i)$$

(4.6)

$E$ is the expected value operator and refers to the expected value of return on a portfolio i.e. $E = E (R)$. And the return on the portfolio (R) is

$$R = \sum_{i=1}^{n} X_i r_i$$

(4.7)

where, $r_1, r_2, ..., r_n$ are jointly distributed random variables denoting the current period’s return on individual securities. The first differenced values of log of securities closing prices are calculated to depict the returns. Thus,

$$r_t = \ln (P_t) - \ln (P_{t-1})$$

(4.8)

$r_t$ is the returns on securities/index,

$\ln (P_t)$ is the natural log of closing price in time period $t$, and

$\ln (P_{t-1})$ is the natural log of closing prices in the previous time period.

$\hat{E}$ is the required rate of return on the portfolio.

2. Dividend Gains

To incorporate the current income requirement of an investor the dividend/dividend yield constraint has been introduced in the model.

$$D_i \geq Q_i$$

(4.9)

where,

$D_i$ = dividend income

$Q_i$ = quartile one of the distribution of dividend income

Dividends are declared at company’s annual general meeting by the management and given to its shareholders out of the company’s current or retained earnings. Dividends are usually given as cash (cash dividend), but they can also take the form of stock (stock dividend). Dividends provide an incentive to own stock in stable
companies even if they are not experiencing much growth. The companies that offer dividends are viewed as companies that have progressed beyond the growth phase, and no longer benefit sufficiently by reinvesting their profits, so they usually choose to pay them out to their shareholders.

The dividend constraint will restrict the companies in lowest twenty five percent of the distribution of dividend returns from entering the feasible solution.

3. Funds Exhaustion

As in the “standard” portfolio selection model an investor has to determine the percentage of funds that will be invested in various available securities. The selection of these fractions $X_1, X_2, \ldots, X_n$ invested in $n$ securities has to be such that the sum of all the fractions is 1. This constraint implies that hundred percent of the amount available has to be invested and no money can be kept idle.

$$\sum_{i=1}^{n} X_i = 1 \tag{4.10}$$

4. No Short Sales

The proportion of funds invested in each of the securities must be positive or zero. Negative fractions denoting short selling of securities do not form a part of the feasible set.

$$X_i \geq 0 \quad i = 1, \ldots, n \tag{4.11}$$

5. Upper Bounds

This constraint specifies the upper bounds required on investment in each security. If $U_i$ is greater than 1 for all $i$ then the obtainable set is unaffected. It may also be unaffected when some $U_i$ are less than 1 provided the corresponding $X_i$ in the solution set is less than $U_i$ at every point on the boundary. As the upper bounds $U_i$ are decreased the set of obtainable EV combinations will eventually shrink.

$$X_i \leq U_i \quad i = 1, \ldots, n \tag{4.12}$$

The boundary of feasible set even after introducing this constraint will consist of pieces of parabolas and horizontal and vertical line segments. Only parabolic segment can be contained within the efficient portion of the feasible EV set. In some cases the efficient portion may consist of just one segment or even a single point.
6. Lower Bounds

This buy-in thresholds constraint specifies a lower bound for securities to form a part of the portfolio. To reduce the transaction costs, this constraint is added to the portfolio selection problem. It prevents assets with small weights from being included in the portfolio. Securities below a lower bound \( l \), are not a part of portfolio. Hall and Tsay (1988) and Taksar, Klass and Assaf (1988) found the significance of transaction costs in forming portfolios with higher return.

\[ X_i \geq l \]  

\[ \text{(4.13)} \]

7. Beta

Sharpe (1964) related return on the security with return on the market portfolio. The measure of systematic risk component ‘\( \beta \)’ (beta) was calculated. It is the slope of Characteristic line which gives the relationship between market return and security return. \( \beta \) is a measure of sensitivity of a stock return to the return on the market index (Nifty 50 in this case). Taneja (2010); Mehta and Chander (2010) found beta to capture systematic risk and all betas to be statistically significant in the Indian stock markets. Beta has explanatory power to model returns and must be included in the portfolio selection model.

\[ \beta \geq \bar{\beta} \]  

\[ \text{(4.14)} \]

\[ \beta = \frac{\sigma_{im}}{\sigma_m^2} \cdot \rho_{im} \cdot \sigma_i \cdot \sigma_m \]  

\[ \text{(4.15)} \]

where,

\( \sigma_{im} \) = co-variance between security and market returns

\( \sigma_m^2 \) = market variance

\( \rho_{im} \) = correlation between security and market returns

\( \sigma_i \) = standard deviation in returns of a security

\( \sigma_m \) = market standard deviation in returns of market portfolio

\( \bar{\beta} \) = target beta

The variance of a portfolio is minimum when \( \rho = -1 \). The benefits of diversification are achieved by combining negatively correlated securities. For equal risk and return combination, the mean return of the portfolio is equal to the mean of
each of security in the portfolio, showing no variability over time. Any variance above or below the mean for set of securities in the portfolio is offset by returns of other assets in the portfolio. Risk is zero for such a portfolio. For different risk and return combinations, by combining negatively correlated securities one can minimise the standard deviation of the portfolio but cannot achieve a perfect zero as the standard deviation of securities are not equal. The variance of portfolio is less when \( \rho = 0 \), this consists of securities which are not related to each other. The portfolio risk will be highest if \( \rho \) among the securities is perfect positive i.e. \( \rho = +1 \).

8. Portfolio Value at Risk

\( VaR \) became a popular metric for risk measurement after the market crash of 1987. Further, the world wide adoption of Basel II norms since 1999 gave impetus to the use of \( VaR \). Portfolio value at risk (\( VaR \)) is a distribution-free statistical measure of possible portfolio loss. Market loss due to changes in value is for a holding period is measured through \( VaR \). While calculating \( VaR \) the portfolio composition is assumed to remain consistent throughout the holding period. It provides a realistic comparison of the risk profiles of portfolios with same holding period at the same confidence interval.

\[
VaR_n \leq \nu
\]  

(4.16)

where, \( \nu \) is the target value at risk, the maximum level of \( VaR \) of a portfolio to be included in the feasible set.

Harvey and Siddique (2000), Bekaert et al. (1998) and Das and Uppal (1999) advocate the need to incorporate the non-normalities of return distribution into the portfolio allocation decision. The downsized risk constraint in addition to standard deviation/variance must be incorporated in the portfolio allocation process. Morgan (1994), Campbell, Huisman and Koedijk (2001) through the use of \( VaR \) highlight the importance of non-normal characteristics of the expected return distribution and the length of investment time horizon on the optimal portfolio selection.

\[
VaR_n = \text{DailyVaR} \times \sqrt{n}
\]  

(4.17)

where, \( n \) is the holding period.

The variance co-variance approach uses the statistical parameter of portfolio standard deviation to capture the downside risk likely to be based in a specific period. It
measures the probability that the value of a portfolio will drop below a specified value in a particular time period.

\[ VaR_{\text{portfolio}} = \alpha \sigma_{\text{portfolio}} \] (4.18)

Introducing VaR into the model has the benefit of allowing the risk-return trade-off to be analysed for various associated confidence levels. Risk here becomes a function of investor’s risk aversion level, as the riskiness of an asset increases with choice of confidence level associated with the downside risk measures. VaR is a systematic way to segregate extreme events, which are studied qualitatively over long-term history and broad market events. VaR can also be analysed quantitatively using short-term data in specific markets.

9. Cut Off Rate

\[ C_i \] is the cut-off rate for percentage of investment in each of the securities in a portfolio with optimal \[ C^* \].

\[ C_i \geq C^* \] (4.19)

Elton, Gruber and Padberg (1978) recommended a methodology for deciding which securities should be selected to become part of an optimum portfolio. According to them, only those securities which have an excess return-to-beta ratio \( (T_n) \) greater than a unique cut of rate \( (C^*) \) should be selected from the universe of equities

\[ \text{Excess return-to-beta ratio } (T_n) = \frac{(R_i - R_f)}{\beta_i} \] (4.20)

\[ \text{Cut-off Rate } (C^*) = \frac{\sigma_m^2 \sum_{i=1}^{n} \frac{(R_i - R_f) \beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^{n} \frac{\beta_i^2}{\sigma_{ei}^2}} \] (4.21)

\[ T_n > C^* \] (4.22)

where,

\[ R_i = \text{expected return on stock } i \]

\[ R_f = \text{return on riskless asset} \]

\[ \beta_i = \text{expected change in the rate of return on stock } i \text{ associated with a 1 percent change in the market return} \]
\[
\sigma_m^2 = \text{variance in the market index}
\]
\[
\sigma_{ei}^2 = \text{variance of a stock’s movement that is not associated with the movement of the market index; this is the stock’s unsystematic risk}
\]

Sharpe’s single index model directly links the desirability of a stock to its excess return-to-beta ratio. In establishing a cut-off rate, all the securities whose excess return-to-risk ratio is above the cut-off rates are selected and whose ratios are below will be rejected.

10. Unsystematic Risk

It is desirable to separate the variations in rates of return into two components – one reflecting the portion of asset price movements caused by changes in the market and the other reflecting the portion of asset’s price movements caused by factors unique to the company and the industry. Here the unsystematic risk is related to factors such as labour strikes, inventions, research and development etc. which can be reduced through diversification. The statistical model separating total risk into its components is

\[
r_t = a + bm_t + e_t
\]  \hspace{1cm} (4.23)

where,
\[r_t = \text{rate of return of a security in time period } t\]
\[a = \text{intercept term}\]
\[b = \text{slope of the regression line}\]
\[m_t = \text{market rate of return in time period } t, \text{ and}\]
\[e_t = \text{random error about regression line in time period } t.\]

The statistic representing the random error \(e_t\) about the characteristic line is a measure of that portion of total risk affected by characteristics unique to the company or industry. However, in practice unsystematic risk is calculated as per the following formula.

\[
\text{Unsystematic risk} = \text{Total risk} - \text{Systematic risk}
\]

\[
e_t = \text{var}(r_t) - \beta^2 \text{var}(m_t)
\]  \hspace{1cm} (4.24)

where,
\[\beta = \text{measure of sensitivity of asset returns vis-a-vis market returns}\]
\[\text{var} (r_t) = \text{variance of the security returns}\]
\[\text{var} (m_t) = \text{variance of market returns}.\]
Kabito (2009) found unsystematic risk to be affecting returns significantly. Jeyachitra, Selvam and Gayathri (2010) found reduction in unsystematic risk with diversification yielding positive returns for Nifty stocks.

\[ e_t \leq Q_t \]  

(4.25)

where,

\( e_t \) = unsystematic risk

\( Q_t \) = quartile one of the distribution of unsystematic risk of security returns

11. Industry Diversification

To make the portfolios more stable industrial diversification becomes important. Hennessy and Lapan (2003); Jeyachitra, Selvam and Gayathri (2010); and others supported the concept of portfolio diversification. The constituent securities in the portfolio selected as per the model shall not be dominated by any specific industry. Like, the S&P CNX Nifty is a well diversified 50 stock index comprising for 24 sectors of the economy including Information technology, Automobiles, FMCG sector, Mining, Gas, Metals, Textiles, Banks, and Construction etc. On the contrary, Ivkovik, Sialm and Weisbenner (2008) warned against too much diversification leading to non performance of the portfolios.

The industry diversification constraint would set the maximum limit on the proportion of funds (weights) that can be invested in the stocks of a particular industry. This would break the dominance of an industry in the chosen portfolio.

\[ \sum_{z=a}^{j} I \leq \alpha \]  

(4.26)

where,

\[ \sum_{z=a}^{j} I = X_a + \ldots X_I \]  

(4.27)

\( X_a \ldots X_n \) are the proportion of funds invested in securities of a particular industry \( z \). \( \alpha \) = A fixed proportion of securities of an industry in the portfolio.

12. Company Diversification

According to clause 11 of the seventh schedule of Securities and Exchange Board of India (mutual fund) Regulations, 1996 on restriction of investments, a mutual fund scheme shall not invest more than five per cent of its NAV in the equity shares or...
equity related investments of a single company in case of an open ended company and ten per cent for close ended schemes.

The quantity of each security \( i \) (stock of a particular company) or related equity investments of a single company that is included in the portfolio is limited within a given interval. A minimum value \( \varepsilon_i \) and a maximum value \( \delta_i \) for each security \( i \) is specified such that

\[
x_i = \varepsilon_i \leq x_i \leq \delta_i
\]

where, \( i = (1, ..., n) \)

This constraint enables to consider the aspect of real-world finance as securities cannot be purchased in any quantity. Only minimum lot sizes can be bought and the amount of money to be invested in a single equity must be a multiple of a given minimum lot.

**13. Market Capitalisation**

Market Capitalisation is the total value of a company’s issued share capital as determined by its share price in the stock market. It is calculated as the number of ordinary shares in an issue multiplied by the previous day’s closing share price.

\[
\text{Market Capitalisation} = \frac{\text{share price} \times \text{shares in an issue}}{100}
\]

The market capitalisation constraint

\[
\Xi \geq Q_3
\]

where,

\( \Xi = \) market capitalisation of a company

\( Q_3 = \) quartile three of the distribution of market capitalisation

This constraint would ensure only the top twenty five percent of companies of the distribution of market capitalisation would be included in the feasible set.

**14. Free Float**

Free Float equities refers to stocks which are not held by the promoters and associated entities (where identifiable) of companies. Promoters’ holding, government holding in case of public sector undertaking, shares held by promoters through American Depository Receipts (ADR’s) or Global Depository Receipts (GDS’s), associate companies, employee welfare trusts, strategic stakes by corporate bodies,
investments under Foreign Direct Investment (FDI) category (where identifiable) and public lock-ins are subtracted to arrive at Free Float factor.

The companies eligible for inclusion of the model should have free float of at least fifteen per cent which is higher than the free float required for inclusion in S&P CNX Nifty (ten per cent). Free float factor (Investible Weight Factor–IWF) for each company in the index is determined based on the public shareholding of the companies as disclosed in the shareholding pattern submitted to the stock exchanges by the companies on a quarterly basis.

\[ \lambda \geq 0.15 \]  

(4.31)

where, \( \lambda \) = free float factor.

15. Promoter’s Holding

Promoter’s holding, government holding in case of public sector undertaking, shares held by promoters through American Depository Receipts or Global Depository Receipts form the promoters holding. It should not be very low so as to disinterest the promoters in the well being of the organisation. It should not be extremely high either that the interests of the shareholders are always sub-ordinated. Contrary to the above float adjusted market capitalisation constraint, here the shareholdings of the promoters are specified.

A constraint for minimum promoter’s holding has been included in the portfolio selection model.

\[ \xi \geq 0.10 \]  

(4.32)

where, \( \xi \) = Securities held by the promoters of a company.

16. Institutional Holding

Similar to the above mentioned promoter’s holding constraint, the Institutional holding constraint specifies the minimum shareholding with associate companies, employee welfare trusts, strategic stakes by corporate bodies, mutual funds, financial institutions, foreign institutional investments (FII’s), investments under Foreign Direct Investment (FDI) category (where identifiable) and public lock-ins. The amount of securities with the above mentioned institutions should not be less than ten percent. Adding this constraint in the model would assure our investor that stake of other institutional parties is also present in the companies included.

\[ \zeta \geq 0.10 \]  

(4.33)
\[ \zeta = Y_c + Y_{ew} + Y_{cb} + Y_{mf} + Y_{FII} + Y_{FDI} + Y_o \]  
(4.34)

where,  
\( Y_c \) = amount of securities held by associate companies  
\( Y_{ew} \) = amount of securities with employee welfare trusts  
\( Y_{cb} \) = strategic stakes by corporate bodies  
\( Y_{mf} \) = securities with mutual funds  
\( Y_{FI} \) = securities held by financial institutions  
\( Y_{FII} \) = investments under foreign institutional investors  
\( Y_{FDI} \) = investments under Foreign Direct Investments  
\( Y_o \) = any other shares held by institutions

17. Sales

The sales constraint would result in inclusion of only the top fifty per cent of securities of the distribution of securities as per sales criteria.

\[ S \geq Q_2 \]  
(4.35)

where,

\( S \) = net standalone sales of the company  
\( Q_2 \) = quartile two of the distribution of sales

18. Net Profit

Ballestero (1998) emphasised on preferences for profitability and safety of an investor. Net profit is a measure of the company’s profitability after accounting for all the costs. Often referred as the bottom line it is calculated by subtracting company’s total costs from the total revenue.

\[ \pi \geq Q_3 \]  
(4.36)

where,

\( \pi \) = net profit  
\( Q_3 \) = quartile three of the distribution of net profit

The net profit constraint has been set at Quartile three, which implies only the top twenty five per cent of securities in the distribution of net profit are included in the selected portfolio.
19. Earnings Per Share

Earnings per share are the portion of a company’s profit allocated to each outstanding share of common stock. EPS serves as an indicator of a company’s profitability per share.

\[ eps = \frac{(\text{net income} - \text{preference dividend})}{\text{average number of shares}} \]  

(4.37)

The EPS constraint,

\[ eps \geq Q_2 \]  

(4.38)

where,

\( eps \) = earnings per share

\( Q_2 \) = quartile two of the distribution of earnings per share

will lead to inclusion of only the top fifty percent securities of distribution according to security’s EPS.

20. Book to Market Ratio Constraint

Fama and French classified the stocks into three groups of portfolios; one of low book-to-market equity (B/M) ratio, one of medium B/M ratio and the last being of high B/M ratio. Three classes of book equity-to-market equity (B/M) value (low B/M, medium B/M and high B/M) are created. The stocks are divided into three book-to-market groups (high, medium and low) for the top 30 percent, middle 40 percent and bottom 30 percent of the book-to-market values.

The book-equity value of the stocks is the respective book value of common shareholder’s equity plus the balance sheet deferred tax (if any) and minus the book value of preferred stocks and the book-to-market equity ratio is constructed by dividing their book-equity value with their market-equity value.

Post and Levy (2005), Taneja (2010), Mehta and Chander (2010), Saleh (2010) emphasised on the superiority of Fama and French’s size and book to market ratio in explaining the market returns in global as well as Indian context.

The constraint here would limit the stocks included to only the top twenty five percent (high group) book-to-market values.

\[ B/M \geq Q_3 \]  

(4.39)

where,

\( B/M \) = book to market ratio

\( Q_3 \) = quartile three of the distribution of book to market values
21. Price-to-earnings (P/E) Ratio

P/E Ratio is a valuation ratio of a company’s current share price compared to its per-share earnings.

\[ P / E \text{ Ratio} = \frac{\text{Market Value per share}}{\text{EPS}} \] (4.40)

A high P/E suggests that investors are expecting higher earnings growth in the future compared to companies with a lower P/E. It’s useful for the investors to compare the P/E ratios of one company to other companies in the same industry, to the market in general or against the company’s own historical P/E. An important problem that arises with the P/E measure is that the denominator (earnings) is based on an accounting measure of earnings that is susceptible to forms of manipulation, making the quality of the P/E only as good as the quality of the underlying earnings number.

\[ Q_1 \leq P / E \text{ Ratio} \leq Q_3 \] (4.41)

where, \( Q_1 \) and \( Q_3 \) are quartile one and three respectively of the distribution of P/E ratio.

A P/E ratio neither too high nor too low is desirable as a company with high P/E represents overvalued stock vis-à-vis it’s earning potential and a low P/E ratio represents lack of demand for a script despite good earnings. Hence, top twenty five and bottom twenty five percent of the distributions are excluded from the portfolio. Due to the presence of this constraint in the portfolio selection problem the feasible set would be reduced stocks of companies in the middle fifty percent of distribution as per P/E Ratios.

22. Number of Mutual Funds which Invested in the Security during the Financial Year

This constraint would exhibit the popularity of the stock among the mutual fund managers. The expertise, knowledge and experience of mutual fund managers can be in some way incorporated in our portfolio selection process through this constraint.

\[ \eta \geq Q_3 \] (4.42)

where,

\[ \eta = \text{number of mutual funds invested in the security} \]

\[ Q_3 = \text{quartile three of the distribution of number of mutual funds that invested in securities} \]
The constraint would ensure that scripts which are most popular among the mutual fund managers (top twenty five percent of the distribution) would enter the portfolio.

23. Risk Penalty

Risk Penalty is investor specific and depends upon the risk tolerance of an investor. It is calculated by relating the total risk of a portfolio to the risk tolerance level of an investor.

\[
Risk\ Penalty = \frac{\text{Risk squared}}{\text{Risk tolerance}}
\]  

(4.43)

Risk squared is the variance of the portfolio return. Risk tolerance is a percentage point ranging from 0 to 100 denoting investor’s preference of a risk-free asset over a risky equity portfolio. 90% and above of risk tolerance can be regarded as a very high risk tolerance whereas a tolerance of less than 20% could be regarded as low.

\[
\rho \leq Q_2
\]  

(4.44)

where,

\[
\rho = \text{risk penalty}
\]

\[
Q_2 = \text{quartile two of the distribution of risk penalty}
\]

24. Utility

Markowitz (1952) stated a portfolio selection rule based on risk penalty and portfolio utility. A portfolio in the efficient frontier shall have a high Portfolio utility/risk adjusted rate of return compared to the other available portfolios.

\[
\nu \geq Q_1
\]  

(4.45)

where,

\[
\nu = \text{Portfolio Utility}
\]

\[
Q_1 = \text{Quartile deviation 1 of the distribution of Portfolio Utility}
\]

\[
\text{Portfolio Utility/Risk-adjusted return} = \text{Portfolio expected return} - \text{Risk penalty}
\]  

(4.46)

With the help of this constraint one can include those securities which have a high portfolio utility by constraining the portfolio to stocks only in the first quartile of the distribution of stocks as per their portfolio utility.
25. Liquidity

Griffin, Nardari and Stulz (2007) signified the importance of relationship between past returns and liquidity in the markets for the individual investor for deciding his portfolio. Bekaert, Harvey and Lundblad (2007) studied the impact of market liquidity on expected returns in nineteen emerging markets. Liquidity of the securities was found to be an important factor in explaining return on a portfolio globally. Agarwalla and Pandey (2010) found that information and liquidity effect of block trades at NSE have a permanent price impact.

Liquidity in the context of stock markets implies a market where large orders can be executed without incurring a high transaction cost. The transaction not in terms of the fixed costs typically incurred like brokerage, transaction charges, depository charges etc. but refers to the cost attributable to lack of market liquidity. Liquidity comes from the buyers and sellers in the market, who are constantly on the lookout for buying and selling opportunities. Lack of liquidity translates into a high cost for buyers and sellers.

The electronic limit order book (ELOB) available on NSE is an ideal provider of market liquidity. This allows all investors in the market to execute orders against the best available counter orders. The market thus possesses liquidity in terms of outstanding orders lying on the buy and sell side of the order book, which represent the intention to buy or sell. When a buyer or seller approaches the market with an intention to buy a particular stock, he can execute his buy order in the stock against such sell orders, which are already lying in the order book, and vice versa.

Market impact cost is one of the most appropriate measures of the liquidity of a stock. It accurately reflects the costs faced when actually trading an index. For inclusion in the index, the security should have traded at an average impact cost of 0.50% or less during the last six months for 90% of the observations, for the basket size of Rs. 20 Million.

Impact cost is cost of executing a transaction in a security in proportion to the weight age of its market capitalization as against the index market capitalization at any point of time. This is the percentage mark up suffered while buying / selling the desired quantity of a security compared to its ideal price = (best buy + best sell) /2. It is a practical and realistic measure of market liquidity, closer to the true cost of execution faced by a trader in comparison to the bid-ask spread. It is computed separately for buy
and sell and varies as per transaction sizes and outstanding orders at any given point of time. When a stock is not sufficiently liquid, a penal impact cost is applied. In mathematical terms, it is the percentage mark up observed while buying / selling the desired quantity of a stock with reference to its ideal price (best buy + best sell) / 2.

As it is for inclusion of a stock in Nifty, the impact cost of a stock should not be more than 0.50 for being a part of the portfolio. This would imply presence of only the most liquid securities in the portfolio.

\[ i \leq 0.50 \] (4.47)

where, \( i \) = Impact cost of the stock.

### 26. Conditional Volatility


The conditional market volatility is estimated using GARCH (1,1) as a model of stock returns as suggested by Bollerslev, Chou and Kroner (1992). The GARCH specification for stocks listed at NSE returns is as follows:

\[
R_m = \mu + \epsilon_m + \theta \epsilon_{m-1}
\] (4.48)

\[
\sigma^2_m = \alpha_m + \beta_m \epsilon^2_{m-1} + \gamma_m \sigma^2_{m-1}
\] (4.49)

where,

\( R_m \) = the return of the stock

\( \sigma^2_m \) = the conditional variance of the Nifty stock return.

\( (\epsilon^2_{m-1}) \) = past squared residual \( \epsilon^2_{m-1} \)

\( (\sigma^2_{m-1}) \) = past conditional variance

\( \beta \) = ARCH coefficient

\( \gamma \) = GARCH coefficient

\( \epsilon_m \) is distributed \( N(0, \sigma^2_m) \)
Equation 4.48 is the conditional mean equation of the return, it accounts for the first-order serial correlation in market returns partly induced by non-synchronous trading. Equation 4.49 specifies the conditional variance as a linear function of past squared residual ($\varepsilon_{m-1}^2$) and past conditional variance ($\sigma_{m-1}^2$).

From the conditional variance equation, the ARCH coefficient, $\beta$, can be viewed as the news coefficient, whereas the GARCH coefficient, $\gamma$, reflects the impact of old news on volatility. The sum of ($\beta + \gamma$) is the measure of volatility persistence. If $\beta + \gamma$ is close to 1, then a shock in a given period $t$, will persist for many periods into the future.

The conditional volatility constraint is as follows:

\[
\beta + \gamma \leq Q_1
\]

(4.50)

where,

$\beta$ = ARCH coefficient

$\gamma$ = GARCH coefficient

$Q_1$ = quartile one of the distribution of conditional volatility coefficients

The constraint would enable inclusion of the securities with the lowest twenty five percent conditional volatility securities in the portfolio.

27. Volume of the Security (in absolute numbers) and its Turnover (in Rupees)

The liquidity of an investment is a prime concern of an investor. Higher the volume and turnover of a stock, the more liquid it would be regarded. Griffin, Nardari and Stulz (2007) found a positive relationship of returns and turnover. This relationship was found to be statistically and economically significant. Hvidkjaer (2008) also studied the impact of small trades on the cross-section of stock returns. Tsuchida, Zhou, and Rachev (2012) analysed the changes in portfolio performance under weight and turnover constraints. Sun (2011) studied the complex relationship between trading volume and securities price. DeMiguel, Plyakha and Uppal (2011) found Sharpe ratios of the portfolio and certainty-equivalent return is accompanied by a higher turnover. Many insights into the portfolio theory are gained by analysing the volume and turnover of securities.
Adding the following two constraints to the problem of portfolio selection would result in inclusion of top twenty five percent stocks with the highest volume and turnover.

\[ v_s \geq Q_3 \]  \hspace{1cm} (4.51)

where,

- \( v_s \) = volume of security (in absolute numbers)
- \( Q_3 \) = quartile three of the distribution of volume

\[ t_s \geq Q_3 \]  \hspace{1cm} (4.52)

where,

- \( t_s \) = turnover of security (in Rupees)
- \( Q_3 \) = quartile three of the distribution of turnover

28. Volume and Turnover of the Futures on a Security

The popularity of a stock and its liquidity is further enhanced through the increasing volume and turnover of its stock futures. The lead-lag relationship between the spot market and its futures has been tested all across the globe. The future markets have been found to be leading the spot markets, processing the information faster and transmitting it to the cash market (Stoll and Whaley 1990; Abhyankar 1995, 1998; Thenmozhi 2002; Raju and Karande 2003). Additional insights into security returns can thus be gained by introducing a constraint on the futures segment of the market. Through the introduction of this constraint, perspective of the derivatives segment of the market is gained. The future prospects of the stock can also be evaluated based on volume and turnover of the futures on a security.

\[ v_f \geq Q_3 \]  \hspace{1cm} (4.53)

where,

- \( v_f \) = volume of futures of a security
- \( Q_3 \) = quartile three of the distribution of volume of futures of security

\[ t_f \geq Q_3 \]  \hspace{1cm} (4.54)

where,

- \( t_f \) = turnover of futures of a security
- \( Q_3 \) = quartile three of the distribution of turnover of futures of security
29. Open Interest

Copeland and Galai (1983), Easley et al. (1998) etc have suggested that derivatives should not be considered redundant in a market with information related frictions. Non-price measures of activity such as open interest contain information about future performance of the underlying asset. Bhuyan and Chaudhary (2001) investigated the effect of options open interest information on trading decisions. Trading strategies based on distribution of option’s open interest were found to generate better returns. Due to the lead-lag relationship experienced in Indian derivatives and securities market, the open interest effect on portfolio optimisation has been incorporated through this constraint.

The total number of options and futures contract that are not closed or delivered on a particular day are referred to as open interest.

\[ O \geq Q_3 \]  

where,  

- \( O \) = open interest
- \( Q_3 \) = quartile three of the distribution of open interest

The addition of this constraint would limit the feasible set to stocks of companies having the highest open interest (twenty five percent) in derivatives segment.

30. Cardinality

An investor may wish to specify the number of assets in his/her portfolio for the purpose of monitoring and control. Jobst, Horniman, Lucas and Mitra (2001) examined the effects of cardinality constraints and transaction round lot restriction on the portfolio selection problem. The constraints though of high practical importance make the efficient frontier discontinuous. Approaches for computation of this frontier were suggested. Chang, Meade, Beasley and Shariha (2000) calculated the efficient frontier for cardinality constrained portfolio optimisation problem. Fernandez and Gomez (2007) applied a heuristic method to trace out the efficient frontier for portfolio selection problem including cardinality and bounding constraints. Soleimani, Golmakani and Salimi (2009) solved using genetic algorithm portfolio selection problem with cardinality constraint. The cardinality constraint is an important variable in determining the portfolio composition of an investor. Hence, it is a part of this model.
The number of securities that can compose the portfolio is bounded i.e. we have two values $k_{\text{min}}$ and $k_{\text{max}}$.

$$1 \leq k_{\text{min}} \leq k_{\text{max}} \leq n,$$  \hspace{1cm} (4.56)

$$k_{\text{min}} \leq \sum_{i=1}^{n} z_i \leq k_{\text{max}},$$  \hspace{1cm} (4.57)

where,

$\begin{align*}
    k_{\text{min}} &= \text{minimum number of securities in the portfolio} \\
    k_{\text{max}} &= \text{maximum number of securities in the portfolio} \\
    z_i &= \text{number of securities in the portfolio}
\end{align*}$

For testing of the model, portfolios are being created with stocks at Nifty, hence $k_{\text{max}}$ would be fifty whereas we fix $k_{\text{min}}$ to be ten.

**IV.2 Multivariate Regression: Model Formulation**

It is important to find out the financial variables that explain the cross section of returns on a security. Certain variables having significantly higher explanatory power of returns and must be included in the portfolio selection model. The factors which do not have a predictive power may be considered less important and thus could be excluded from the portfolio selection model. To find out the variables which can significantly explain returns, this multivariate regression analysis is conducted. The multiple regression analysis undertaken regresses the returns on a security and excess returns to standard deviation ratio on a number of financial factors. Two multiple regression equations were estimated.

**Multiple Regression for Returns**

Regression equation with return on the security as the dependant variable and earnings per share, dividend, free float, impact cost, institutional holding, market capitalisation, net profit, price to book value ratio, price-earnings ratio, promoters shareholding, sales, turnover, unsystematic risk and volume as independent variables was formulated.

$$R_i = \alpha + \beta_1 \text{eps} + \beta_2 D_i + \beta_3 \lambda + \beta_4 d + \beta_5 \xi + \beta_6 c + \beta_7 \pi + \beta_8 \frac{p}{b} + \beta_9 \frac{p}{e} + \beta_{10} \varepsilon + \beta_{11} S + \beta_{12} t + \beta_{13} e + \beta_{14} v \tag{4.58}$$
where,

\[ R_i = \text{return on } i^{th} \text{ security} \]
\[ \alpha = \text{constant term} \]
\[ \text{eps} = \text{earnings per share} \]
\[ D_i = \text{dividend declared} \]
\[ \lambda = \text{free float} \]
\[ i = \text{impact cost} \]
\[ \zeta = \text{institutional holding} \]
\[ \mathbb{C} = \text{market capitalisation} \]
\[ \pi = \text{net profit} \]
\[ \frac{p}{b} = \text{price-book value ratio} \]
\[ \frac{p}{e} = \text{price-earnings ratio} \]
\[ \xi = \text{promoters holding} \]
\[ S = \text{sales} \]
\[ t = \text{turnover} \]
\[ \varepsilon = \text{unsystematic Risk} \]
\[ v = \text{volume} \]
\[ \beta_1, \beta_2, \ldots, \beta_{14} = \text{Regression coefficients} \]

**Multiple Regression for Excess Returns to Standard Deviation Ratio**

Regression equation with excess return to standard deviation on the security as the dependant variable and earnings per share, dividend, free float, impact cost, institutional holding, market capitalisation, net profit, price to book value ratio, price-earnings ratio, promoters shareholding, sales, turnover, unsystematic risk and volume as independent variables was formulated.

\[
S_i = \alpha + \beta_{\text{eps}} \text{eps} + \beta_{D_i} D_i + \beta_\lambda \lambda + \beta_i i + \beta_\zeta \zeta + \beta_\mathbb{C} \mathbb{C} + \beta_\pi \pi + \beta_\frac{p}{b} \frac{p}{b} + \beta_\frac{p}{e} \frac{p}{e} + \beta_\xi \xi + \beta_1 S + \beta_{1,t} t + \beta_1 \varepsilon + \beta_1 v
\]

(4.59)

where,

\[
S_i = \frac{(R_i - R_f)}{\sigma_i}
\]

(4.60)
\[ R_f = \text{return on the risk-free asset} \]
\[ \sigma_i = \text{standard deviation of the } i^{\text{th}} \text{ security} \]

All other terms are the same as in multiple regression equation 1 explained above.

**IV.3 Granger Causality Tests**

The Granger (1969) approach examines the question whether \( x \) causes \( y \) to find how much of current \( y \) can be explained by past values of \( y \) and then find whether adding lagged values of \( x \) improves the explanation or not. If \( x \) helps in prediction of \( y \) then \( y \) is said to be granger caused by \( x \). This would happen when coefficients of lagged values of \( x \) are statistically significant. Granger causality measures precedence and information content and does not imply that \( y \) is the effect or result of \( x \). Regressions run are of the form:

\[
y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \ldots + \alpha_l y_{t-l} + \beta_1 x_{t-1} + \ldots + \beta_l x_{t-l} + \epsilon_t
\]  

(4.61)

where,

\( y_t = \text{returns on security} \)
\( x_t = \text{the causation factors such as eps, dividend, net profit, promotional and institutional holding, impact cost, unsystematic risk, sales, volume, turnover etc.} \)
\( \alpha_t = \text{coefficient of lagged values of returns} \)
\( \beta_t = \text{coefficient of lagged values of various causation factors.} \)

F-statistics are reported for each such equation. These are the Wald statistics for the joint hypothesis:

\[
\beta_1 = \beta_2 = \ldots = \beta_l = 0 .
\]  

(4.62)

**IV.4 A Utility Approach**

Attitude of an individual comprises of a spoken component comprising of person’s belief and an unspoken component reflecting his feelings and emotions. Financial risk tolerance captures both these aspects. Financial risk tolerance may be inferred by the portfolio allocations already made by the individuals (Schooley and Worden, 1996), interpreting the responses to alternate investment choices (Hey, 1999) and subjective questions (Hanna, Gutter & Fan, 1998).
An investor is said to be risk averse if he prefers less risk to more risk, all else being equal. Such an investor will require compensation for taking additional risk. When the additional expected return for undertaking equal additional risk is increasing, it refers to a ‘Diversifier’ who is treated as a rational investor. When the additional return for equal additional risk is expected to fall progressively then such an investor is known as a ‘plunger’. The opposite of risk aversion is risk seeking (sometimes called risk loving). A risk seeking investor prefers more risk for little return, all else being equal. Financial theories generally assume investors are not risk seeking. However, risk seeking behaviour is observable in actual life. People who play lotteries or gamble at casinos accept a negative expected return in exchange for the thrill of financial risk.

Between risk aversion and risk seeking is a state called risk neutrality. An investor is risk neutral if he is indifferent to risk. He will neither pay to avoid it nor to take it. In a nutshell, risk does not affect his decisions. Financial theories generally assume investors are not risk neutral. All individuals keep their investments in the form of a portfolio. An individual would not keep the savings in only one form say cash. An investor tends to allocate the savings in a manner that it gives rise to a portfolio.

A large number of individuals follow the principle of safety first and try to reduce as far as possible the chance of a catastrophe. The possibility of a disaster directly affects utility function of an investor. The utility function if interpreted in terms of minimizations of the chance of disaster, assumes only two values, e.g., one if disaster does not occur, and zero if it does. The level of disaster may change depending on the changes in return and loss.

Markowitz extended the work on portfolios to include utility analysis (Kindly see figure IV.1). “The utility function has three inflection points. The middle inflection point is defined to be at customary level of wealth. Except in cases of recent windfall gains and losses, customary wealth equals present wealth. The first inflection point is below; the third inflection point is above, customary wealth. The distance between the inflection points is a non decreasing function of wealth. The curve is monotonically increasing but bounded; it is first concave, then convex then concave and finally convex.”
An investor would try to maximize the utility. Expected value of utility is given by

\[ U = U(C_1, C_2, \ldots, C_t, W_T) \]  \hspace{1cm} (4.63)

where,

- \( C_t \) = real value of consumption in period \( t \)
- \( T \) = time of death
- \( W_T \) = bequest
- \( U \) = utility of the investor’s lifetime consumption pattern.

The problem of portfolio arises when the investor does not wish to consume in one period rather desires to carry it over to the next period. This portfolio problem then requires the selection of that combination of investments which will yield him maximum utility. In the classical framework, an investor faces infinite possible combinations of risk free asset and the market portfolio but the ultimate allocation depends on the investors’ utility function.

Figure IV.2 plots the efficient combination of portfolios and indifference curves of a normal investor (diversifier) in the capital markets. Y axis represents the expected returns and X axis represents the expected risk. The indifference curves A1 to A6 represent the utility to an investor in risk-return space. The utility map has not been restricted only to the positive quadrant. The intercept ‘a’ of the concave quadratic utility function could be negative depicting the minimum risk investor would need to take with the risk-return combinations of assets available in the capital markets. A positive intercept would depict the risk-free rate of return.
RFR represents the risk free rate of return. The efficient frontier proposed by Markowitz consists of all risky assets. All points below this efficient frontier are attainable but undesirable. All points above this efficient frontier are desirable but unattainable with a combination of only risky assets. The Capital Market Line (CML) represents the risk-return combinations of available assets. The point of tangency of the efficient frontier and Capital market line represents the market portfolio M. Market portfolios is that portfolio which consists of all risk assets namely equities, bonds, derivatives, real estate, antiques, art and all other risky assets required for a completely diversified portfolio. The combination of risky assets in market portfolio results in elimination of the unsystematic risk. The ray joining RFR and B is dominant to all points on the Markowitz's efficient frontier as they have a combination of risky and riskless assets. With the inclusion of a risk less asset, one can increase the return of the portfolio without increase in the risk of that portfolio. Points to the left of M represents lending portfolios (say L). In lending portfolios, investors construct a portfolio having a combination of Market portfolio and riskless assets i.e. one can lend money at risk free rate. Points to the right of M represents borrowing portfolios (say B). In borrowing portfolios, investors construct a portfolio with partly own funds and partly by borrowing funds at the risk free rate. Points L and B depend upon the indifference curves of an investor and were originally plotted by Sharpe (1964). For a diversifier, the utility curves will shift to the left of M i.e. on ray M-RFR. Risk aversion in investors often results in investors allocating their funds (W) on the ray RFR-M having a combination of riskless asset and Market Portfolio (M). As the wealth increases and a desire to earn more wealth is often resulting in investors moving from ray RFR-M to M-B thereby creating leverage portfolios.

A diversifier would invariably prefer minimum risk portfolio thereby investing in either Market portfolio or lending portfolios thereby minimising the unsystematic risk. A condition of equilibrium would exist when the slope of utility curves becomes equal to the slope of the capital market.
To undertake the utility analysis risk Penalty is calculated using the formula,

\[ \text{Risk Penalty} = \frac{\text{Risk Squared}}{\text{Risk Tolerance}} \]  

Utility is calculated by subtracting the value of Risk Penalty from the value of expected return.

\[ \text{Utility} = \text{Expected Return} - \text{Risk Penalty} \]

Risk squared is the variance of return of the portfolio. Risk Tolerance is a number from zero to hundred. Equation 4.64 and 4.65 are used for quantitative utility analysis. The amount of risk tolerance shows the investor’s willingness to bear more risk for more return. Low (High) tolerance indicates Low (High) willingness. Risk Penalty is less as risk tolerance is increased.

Utility analysis is undertaken using both quantitative method and graphical analysis. For the graphical analysis, the portfolio return and variance of all the constructed portfolios are plotted over the indifference curves of various types of investors (diversifier, plunger, risk neutral and risk lover). The indifference curves
exhibit the level of utility derived by a particular type of investor from a specific risk-return trade-off. By superimposing the risk-return combination of portfolios on the utility/indifference curves, the choice of portfolio by the investor is determined graphically.

IV.5 Performance Measures for Portfolios

Sharpe (1966) gave a summary measure of portfolio performance. The measure adjusts performance for risk. It measures risk premiums of the portfolio (the excess return required by investors for the assumption of risk) relative to the total amount of risk in the portfolio. His index is given by

\[ S_i = \frac{(\bar{r}_i - r^*)}{\sigma_i} \]  

(4.66)

where, \( S_i \) = Sharpe Index
\( \bar{r}_i \) = average return on portfolio i
\( r^* \) = riskless rate of interest
\( \sigma_i \) = standard deviation of the returns of portfolio t.

The index summarises risk and return of a portfolio in a single measure, categorising the performance of a portfolio on a risk adjusted basis. A larger value denotes better performance of the portfolio. The Sharpe ratios have been computed for the eight portfolios modelled, Markowitz’s portfolio and index portfolio Nifty 50 for comparison of performance.

Treynor (1965) provided a measure for portfolio performance based on the concept of characteristic line. This is a linear representation of an otherwise curvilinear relationship between market rate of return and portfolio rate of return. The ideal portfolio lies to the left of the imaginary 45 degree line starting at the origin. The slope of the characteristic line is the beta coefficient. The steeper this line more volatility the portfolio possesses. The concept has been incorporated into a single index to measure portfolio performance accurately.

\[ T_n = \frac{(r_n - r^*)}{\beta_n} \]  

(4.67)
where,

\[ T_n = \text{Treynor Index} \]
\[ \bar{r}_n = \text{average return on portfolio n} \]
\[ r^* = \text{riskless rate of interest} \]
\[ \beta_n = \text{beta coefficient of portfolio n} \]

The risk and return are summed up in a single number categorising the performance of the portfolio. The Treynor ratio for all the portfolios have also been calculated to rank the portfolios.

### IV.6 Tests for Equality

This tests the null hypothesis that all series in the group have the same mean, median (distribution), or variance.

#### 1. Mean Equality Test

This test is based on a single-factor, between-subjects, analysis of variance (ANOVA). The basic idea is that if the subgroups have the same mean, then the variability between the sample means (between groups) should be the same as the variability within any subgroup (within group).

The between and within sums of squares are defined as:

\[
SS_B = \sum_{g=1}^{G} n_g (\bar{x}_g - \bar{x})^2
\]  
(4.68)

\[
SS_W = \sum_{g=1}^{G} \sum_{i=1}^{n_g} (x_{ig} - \bar{x}_g)^2
\]  
(4.69)

where,

\[ x_{ig} = i^{th} \text{ observation in group } g \]
\[ I = 1, \ldots, n_g \text{ for groups } g = 1, \ldots, G \]
\[ \bar{x}_g = \text{Sample mean within group } g, \]
\[ \bar{x} = \text{Overall sample mean.} \]

The F statistic for the equality of means is computed as:
\[ F = \frac{SS_g / (G - 1)}{SS_p / (N - G)} \]  

(4.70)

where, \( N \) = total number of observations.

The F-statistic has an F-distribution with \( G - 1 \) numerator degrees of freedom and \( N - G \) denominator degrees of freedom under the null hypothesis of independent and identical normal distribution, with equal means and variances in each subgroup.

2. Variance Equality Tests

Variance equality tests evaluate the null hypothesis that the variances in all subgroups are equal against the alternative that at least one subgroup has a different variance.

F-test

It computes the variance for each subgroup and denotes the subgroup with the larger variance as L and the subgroup with the smaller variance as S. Then the F-statistic is given by:

\[ F = \frac{s_L^2}{s_S^2} \]  

(4.71)

where, \( s_g^2 \) = the variance in subgroup \( g \)

This F-statistic has an F-distribution with \( n_L - 1 \) numerator degrees of freedom and \( n_S - 1 \) denominator degrees of freedom under the null hypothesis of equal variance and independent normal samples.

IV.7 Summary and Conclusions

In this chapter, an attempt has been made to provide the justification for the research design. A mean-variance efficient portfolio selection model is developed theoretically with thirty constraints. The main objective of an investor is minimisation of risk (variance) from the portfolio as this leads to maximisation of his utility. In addition to the classical constraints of funds exhaustion, no short sales and upper bounds many new constraints have been added for accommodating the needs and limitations of a present day investor. The investor’s desire for capital returns and dividend gains are modelled as constraints. Liquidity measures such as volume, turnover and impact cost (a
new variable available from National Stock Exchange’s official website) are introduced to tackle investor’s desire for a liquid portfolio. Diversification has been incorporated through industrial diversification and company diversification constraints.

To discourage very small fractions of shares in the portfolio leading to high transaction costs, lower bounds have been introduced. Beta factor which has been found to significantly explain returns is added as a constraint in the problem. The unsystematic risk of a company and its conditional volatility is also included. Accounting figures and ratios such as net profit, sales, earnings per share, price-earnings ratio and book to market ratio are included to accommodate for the earnings from a portfolio. Investors are also interested in securities where other stakeholder’s interest is substantive. This desire has been considered through promoter’s shareholding and institutional shareholding constraint. The lead-lag relationship between the spot market and derivatives market and its effect on the equity portfolio selection is assimilated through the open interest constraint, volume and turnover of futures as the security constraint.

The Value at Risk, cut off rate, market capitalisation and free float factor are also considered. The number of mutual funds investing in the security depicts the popularity of a script among the technically competent mutual fund managers. This number has been added as a constraint. Risk penalty and Portfolio utility measures as given by Markowitz are also set as constraints in the portfolio selection problem. A cardinality constraint to limit the size of the portfolio has also been added.

Multivariate regression technique was discussed to identify the variables which have the explanatory power to estimate returns. Two multiple regression equations are estimated one with returns as the dependent variable and the other with excess return to standard deviation ratio as the dependent variable. The list of independent variables included all the portfolio constraints. The methodology for Granger causality tests to find out relationship of causation between returns on a security and the variables set as constraints in the programming problem has been also discussed.
The Portfolio Utility concept of Markowitz has been discussed for its application to the research work. The methodology followed for calculating the utility and portfolio evaluation ratios has been elaborated. The performance evaluation measures as proposed by Sharpe (1966) and Treynor (1965) have been explained. The tests for equality of returns, variances and portfolio utilities among the alternate portfolio selection model formulations have been deliberated upon.