CHAPTER 3

PASSIVITY BASED GENERALIZED ROBUST CONTROLLER FOR BOOST RECTIFIER

3.1 INTRODUCTION

In recent years, single-phase switch-mode AC/DC power converters have been increasingly used in the industrial, commercial, residential, aerospace, military environment etc. To get constant output voltage and near-unity power factor, it is essential that the converter has to be controlled (Lin and Lu 2000). In recent power electronic researches, high power density, high power factor, high efficiency, low current distortion, and simple control scheme are strongly recommended for the industrial applications (Sanders et al 1991). This is due to the enforcement of strict harmonic regulations such as IEC 1000-3-2.

Voltage source converters provide excellent control over power flow in both forward and reverse directions. They can be operated as AC–DC converters to generate regulated DC voltage at high input power factor. The power flow can be easily reversed to operate the converter as a DC–AC converter (Lee et al 2000). This capability makes the system ideally suited to electric drives and line interactive UPS applications.

Conventional diode rectifiers or phase-controlled rectifiers have properties of simple structure and low cost. However, they have the inherent drawbacks that the power factor decreases when the firing angle increases and
the line current harmonics are relatively high. To overcome the above problems, many circuit configurations of the single-phase Switching Mode Rectifier (SMR) with low current distortion and unity power factor have been proposed in the past few years (Morici et al 1994). These circuit configurations are based on the full bridge diode rectifier followed by a boost, buck boost or cuk converter. Single-phase full bridge and half bridge SMR circuit configurations have capabilities of bidirectional power flow, reactive power control and high power factor.

Among these circuit configurations, single-phase unidirectional AC/DC converters with boost topology have been widely used as front-end power factor pre-regulator due to its good performance characteristics. The boost topology has properties of high power factor, low current distortion, high step-up voltage ratio and continuous input current. Boost rectifier topologies can be broadly classified as continuous mode and discontinuous mode conduction rectifiers (Escobar et al 2001). The single-phase two-level PWM continuous current mode rectifier with unidirectional power flow is presented here.

The Generalized State-Space Averaging method is a way to model the power converters as time independent systems, defined by unified set of differential equations and capable of representing circuit waveforms without discontinuities (Ortega et al 2002). Consequently, this approach is not suitable for modeling converters which have dominant oscillatory behavior such as the resonant type converters or large ripple PWM converters (Mahdavi et al 1997). Therefore, analysis of AC/DC power converters with ideal switches and parasitic components (capacitors of inductors) forming loops must be considered with more care. With the generalized state-space averaging method, the circuit state variables are approximated by a Fourier series expansion with time-dependent coefficients (Rosendo et al 1998). This
representation results in an unified time-invariant set of differential equations where the state variables are the coefficients of the corresponding Fourier series of the circuit variables.

3.2 SYSTEM ANALYSIS

The system dynamic behaviour of the full bridge boost rectifier is obtained by solving the system dynamics. Mathematical modeling also decides the details of the system that can possibly be studied by computer simulation. The starting point for modeling a converter, however, is by application of Kirchoff’s and Ohm’s law to the circuit, which provides first-order differential equations describing the state of current through inductor and voltage across capacitor.

The circuit of a single phase full bridge boost rectifier is shown in Figure 3.1. This circuit is composed in its main part by a complete (two legs) bidirectional bridge. The switches formed by diode in parallel to a transistor are controlled in its gate by switching signal which denotes the switch position function taking values in the finite set. The transistor works as a switch, which is turned on and off by the PWM control signal.

![Figure 3.1 Single phase full-bridge boost rectifier](image)
3.3 SLIDING MODE CONTROLLER DESIGN

This section introduces a robust motion control algorithm using partial state feedback for a class of non-linear systems in the presence of modeling uncertainties and external disturbances. Sliding Mode Control (SMC) is a well known technique due to its outstanding robustness properties against parametric uncertainties and external disturbances. Conventional SMC implementation utilizes the upper bound of each uncertainty to assure stability (Slotine and Li 1991). This procedure typically yields over conservative control gains which limits tracking accuracy (Grino et al 2001).

The effects of these uncertainties are first combined into a single quantity called perturbation. The major contribution of this work comes as the development and design of a robust observer for the state and perturbation which is integrated into a Variable Structure Controller (VSC).

3.3.1 Physical Model of the Full Bridge Boost Rectifier

The physical model of the boost rectifier (at its switching frequency) is derived by (Moura Roy and Olgac 1995).

\[
L \frac{dx_1}{dt} = -ux_2 - rx_1 + v_i \tag{3.1}
\]

\[
C \frac{dx_2}{dt} = ux_1 - \frac{1}{R} x_2 \tag{3.2}
\]

where, \( x_1 \) and \( x_2 \) are the input inductor current and the output capacitor voltage variables, 
\( v_i = E \sin (\omega t) \) is the ideal sinusoidal source that represents the AC-line source,
R - the DC-side resistive load,
\( r \) - the parasitic resistance of the inductor,
L - the inductance of the converter,
C - the capacitor of the converter,

the control variable \( u \) takes its values in the closed real interval \([-1,1]\) and represents the averaged value of the PWM (pulse-width-modulated) control signal injected into the real system.

In the actual implementation of the system it is assumed that the output voltage \( x_2 \), the input current \( x_1 \) and the source voltage \( v_s \) are available for the measurement. It is important to remark the system described in the above equations can be seen as the interconnection of the two subsystems with the different time constants and the particular subsystem is much slower than the other system. This fact has lead to the development of the classical control schemes for these systems consisting of two control loops the inner (fast) for shaping the inductor current, and the outer (slow) for regulating the output capacitor voltage. In this control architecture, the output of the outer control loop controller acts as the modulating signal in an AM modulator, with carrier \( v_s \), whose output is the reference for the inner loop (Damian Giaouris et al, 2007). The handicap of this control topology, caused by the outer voltage loop, is the need for big capacitors in the DC bus in order to prevent large over voltages in case of great load perturbations.

3.3.2 Steady-State Analysis

If the state vectors of the system is fixed assuming perfect control action, at the desired control action, at the desired values i.e \( x_{1d} = I_d \sin(\omega t) \), \( x_{2d} = V_d = \langle x_2 \rangle_{id} \) and neglecting higher order harmonics, an input-output balance is performed resulting in,
\[ P_i = \langle x_{id}v_s - rx_{id}^2 \rangle = \frac{1}{2}(EL_d - rL_d^2) \] (3.3)

\[ P_o = \frac{x_{2d}^2}{R} = \frac{v_d^2}{R} \] (3.4)

Since the input active power must be equal to the output active power \((P_i = P_o)\), then, \(\frac{1}{2}(EL_d - rL_d^2) = \frac{v_d^2}{R}\). This equation has two solutions

\[ I_d = \frac{E}{2r} \pm \sqrt{\left(\frac{E^2}{4r^2} - \frac{2v_d^2}{rR}\right)} \]

which are real if and only if \(\frac{v_d}{E} < \sqrt{\frac{R}{8r}}\) (Escober et al 2001). This condition is known as the boost condition of the power converter.

The smaller solution of this equation \(I_d = \frac{E}{2r} - \sqrt{\left(\frac{E^2}{4r^2} - \frac{2v_d^2}{rR}\right)}\) corresponds to a stable equilibrium (Grino et al 2001) and is the selected relation between the desired mean value of the DC capacitor \((V_d)\) and the amplitude of the desired inductor current \((x_{1d} = I_d \sin \omega t)\).

As it is known, the bidirectional boost rectifier has relative degree 1, regardless of the output \(x_1\) or \(x_2\). It is also known that if the output is \(x_2\), the system has a non-minimum phase behavior. For this reason, this system is usually controlled through current \(x_1\). In this case, the system has minimum phase behavior, i.e. its zero dynamics is stable. In order to verify this assertion, \(x_1\) is taken has a output of the system by fixing its value to \(x_{1d} = I_d \sin \omega t\) in equations (3.1) and (3.2) resulting in,

\[
\begin{align*}
\frac{dx}{dt} &= \frac{I_d^2 \sin(\omega t) \cos(\omega t)w_s}{C x_2} \frac{L_d^2 - I_d^2(\sin(\omega t))^2 r}{C x_2} + \frac{I_d^2 (\sin(\omega t))^2 E}{C x_2} - \frac{x_2}{RC} \\
\end{align*}
\] (3.5)
where $\bar{u}$ and $\bar{x}_c$ are the control variable and capacitor voltage respectively, in the zero-dynamics. Then the equation (3.5) describes the behavior of the zero-dynamics of the system.

To convert equation (3.5) into linear Ordinary Differential Equation (ODE), each side of this equation is multiplied by $x_2$ taking $\bar{z} = \frac{x_2}{2}$ and the linear ODE is obtained as in (3.6),

$$\frac{d\bar{z}}{dt} = \frac{I_d^2 \sin(\omega_d t) \cos(\omega_d t) \omega_d L}{C} - \frac{I_d^2 (\sin(\omega_d t))^2 r}{C} + \frac{I_d (\sin(\omega_d t))^2 E}{C} - \frac{2\bar{z}}{RC} \quad (3.6)$$

whose solution is $\bar{z}(t) = f(t) + p(t) + K$ where $f(t) = \frac{1}{2} C_1 \exp\left(-\frac{2t}{RC}\right)$ is the vanishing term ($\lim_{t \to \infty} f(t) = 0$) corresponding to the first order linear dynamics, $p(t) = A \sin(2\omega_d t) + B \cos(2\omega_d t)$ is the oscillating term and $K = \frac{V_d^2}{2}$ is the constant term. The DC value of $\bar{z}(t)$ in steady state results with the time period $T = \frac{\pi}{\omega_d}$ in (3.7) by averaging equation (3.6),

$$\frac{d\langle z \rangle_0}{dx} = \frac{1}{2C} (EI_d - rI_d^2) - \frac{2\langle z \rangle_0}{RC} = \frac{V_d^2}{RC} - \frac{2\langle z \rangle_0}{RC} \quad (3.7)$$

whose solution is $\langle z \rangle_0 = \frac{V_d^2}{2} + C_1 \exp\left(-\frac{2t}{RC}\right)$ \quad (3.8)

### 3.3.3 Controller Design and Operation

The controller block diagram is shown in Figure 3.2. Here the output variable $x_2$ is taken from main controller output which contains second
order harmonics is given to the linear controller which is outer control loop i.e. slower control loop. Then the inductor current is passed through sliding controller which the fast inner loop for shaping the inductor current and take a control vector from this loop is given to the basic rectifier block (Utkin 1977).

![Sliding mode control scheme block diagram](image)

**Figure 3.2** Sliding mode control scheme block diagram

### 3.3.4 Design of the Controller

This section is devoted to the design of both the control $u$ and $I_d$, since the latter operates as a control in a linear equation describing the dynamics of $\frac{x_2}{2}$. The control objectives can be written as $x_1(t) = I_d \sin (\omega t)$ and $z_0 = 0.5 v_d$. These requirements are demanded in steady-state, where $z = 0.5 v_d^2$.

As far as the first objective is concerned, sliding control is proposed since it is appropriate due to its very nature for switching converters, and it will provide a controlled system robust with respect to load variations. Thus
\( \sigma(x,t) = x_i - I_d \sin(\omega t) = 0 \) is considered as the switching surface and the relative degree of it is one. Following the standard procedure (Utkin 1978),

\[
u_{eq} = \frac{1}{x_2} \left( (E - r I_d) \sin(\omega r t) - \omega t L I_d \cos(\omega t) \right)
\]  

(3.9)

\[
u = \begin{cases} 
-1 & \text{if } \sigma(x,t) < 0 \\
+1 & \text{if } \sigma(x,t) > 0 
\end{cases}
\]

(3.10)

A necessary condition for sliding motion is \( x_2 \neq 0 \). The dot product of the x-gradient of \( \sigma(x,t) \) and controller vector is \( \frac{-x_2}{L} \) which in turn will be assumed negative. Furthermore, \(-1 \leq \nu_{eq} \leq +1\) defines the subset where the sliding motion occurs. The substitution of the zero dynamics in these inequalities results in necessary conditions to be held by the plant parameters.

To achieve the second objective the variable \( \langle z \rangle_0 \) is regulated to \( \frac{v_z^2}{2} \) by applying classical linear control design to equation (3.7) with \( r = 0 \), where \( i_d \) acts as the control variable. This ODE describes the zero-dynamics, i.e. the ideal sliding dynamics. Taking the zero-dynamics as the dynamics of \( z = 0.5x_2^2 \) makes sense because the current loop is much faster than the voltage one, as already been pointed out. In addition, \( z(t) \) has a DC component and a fundamental harmonic at \( 2\omega_k \) which is removed through the linear notch filter.

### 3.3.5 Parameter Analysis

The plant parameters are important in the performance of the controlled converter. The load(R) and the input voltage can vary with time or
it can be affected by perturbations (Stankovic et al 2001). Inductance (L) and the capacitance (C) are design parameters and their values can be specified by the converter designer and can be assumed constant as long as the process takes. In this section, the influence of parameters L, C and R on the fulfillment of the control objectives is considered. Furthermore, a new parameter is taken into account namely an output voltage ripples lower or equal to 0.05 p.u.

First let it be assumed the steady-state for the input current, \( x_1 = x_{1d} = I_d \sin(\omega_d t) \). From equation (3.1) the steady state for the product \( ux_2 \) is given by,

\[
u_{sx}x_{2s} = [E \sin(\omega_d t) - \omega_d LI_d \cos(\omega_d t)]
\]

(3.11)

The change of variable \( z = 0.5 x_2^2 \) in equation (3.2) results in,

\[
c \frac{dz}{dt} = ux_2 x_1 - \frac{2z}{R}
\]

(3.12)

Thus from equations (3.11) and (3.12) the steady-state for the new variable \( z \) is,

\[
z_{ss}(t) = \frac{v_d^2}{2} \left[ 1 + \frac{2LCv_d^2 \omega_d^2 - E^2}{E^2(1 + \omega_d^2 R^2 C^2)} \cos(2\omega_d t) - \frac{\omega_d (E^2 R^2 C + 2LV_d^2)}{E^2 R(1 + \omega_d^2 R^2 C^2)} \sin(2\omega_d t) \right]
\]

(3.13)

By substitution of \( x_{2ss} = \sqrt{2z_{ss}} \) in equation (3.11) the steady state for the input variable is obtained, which is a quotient of periodic signals. The
numerator n(t) is pure sine of amplitude \( E\sqrt{1 + \frac{4V_d^4\omega_t^2L^2}{E^2R^2}} \) and the denominator,
\[
d(t) = \frac{1 + \frac{\omega_t^2V_d^4L^2}{E^2R^2}}{1 + \omega_t^2R^2C^2}.
\]

A necessary condition to avoid the saturation of the input variable, i.e.
\( 0 < u < 1 \), is \( \max\{n(t)\} < \min\{d(t)\} \) since both terms of the inequality are positive, it is equivalent to

\[
\min^2\{d(t)\} - \max^2\{n(t)\} = \left( V_d^2 - E^2 \left( 1 + \frac{4V_d^4\omega_t^2L^2}{E^4R^2} \right) - \sqrt{V_d^4 \left( 1 + \frac{4V_d^4\omega_t^2L^2}{E^4R^2} \right) \left( \frac{1}{1 + \omega_t^2R^2C^2} \right)} \right) > 0 \quad (3.14)
\]

The inequality in (3.14) implies \( V_d > E \), or equivalently, \( \langle x_{2d}\rangle_d > E \), recovering the boost character of this converter. The graph of \( \min^2\{d(t)\} - \max^2\{n(t)\} \) as a function of the variables \((L, C)\) in the range \( L \in [0.0005, 0.005], C \in [0.0005, 0.01] \) is depicted in Figure 3.3.

![Figure 3.3](image-url)  

**Figure 3.3** Min\(^2\) \( \{d(t)\} \) - Max\(^2\) \( \{n(t)\} \) as a function of the variables \((L, C)\) in the range \( L \in [0.0005, 0.005], C \in [0.0005, 0.01] \)
As for the second specification from the equation (3.13), the amplitude of voltage ripple is given in p.u by

\[
l_{pu} = \sqrt{1 + \left(\frac{E_4 R^2 + 4 \omega_i^2 \nu_d^4 L^2}{E_4 R^2 (1 + \alpha_i^2 R^2 C^2)}\right)} - 1
\]  

(3.15)

The graph of \(l_{pu}\) as a function of the variables (L, C) in the range \(L \in [0.0005, 0.005], C \in [0.0005, 0.01]\) is depicted in Figure 3.4.

![Graph](image)

**Figure 3.4** \(l_{pu}\) as a function of the variables (L, C) in the range \(L \in [0.0005, 0.005], C \in [0.0005, 0.01]\)

### 3.3.6 Simulation Results

The control design has been simulated in a single-phase active filter with the parameters shown in Table 3.1.
Table 3.1 Parameters of single phase full bridge boost rectifier

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\text{nom}}$</td>
<td>Nominal resistance</td>
<td>100</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$L$</td>
<td>Line Inductance</td>
<td>2.0</td>
<td>mH</td>
</tr>
<tr>
<td>$C$</td>
<td>Line Capacitance</td>
<td>1000</td>
<td>$\mu$F</td>
</tr>
<tr>
<td>$R$</td>
<td>Load Resistance</td>
<td>150</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$V_{\text{in}}$</td>
<td>Input Voltage</td>
<td>120</td>
<td>V</td>
</tr>
<tr>
<td>$V_o$</td>
<td>Desired Output DC Voltage</td>
<td>200 and 400</td>
<td>V</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
<td>50</td>
<td>Hz</td>
</tr>
<tr>
<td>$f_s$</td>
<td>PWM switching frequency</td>
<td>20</td>
<td>kHz</td>
</tr>
</tbody>
</table>

The simulink block diagram is shown in Figure 3.5 and the simulation results are shown in Figures 3.6 to 3.10.

![Simulink block diagram of full-bridge rectifier with sliding controller](image_url)
Figure 3.6 shows the response for output DC regulated voltage. The change in load disturbance from 60 Ω to 120 Ω is applied at time period T=1.2s. It is observed that the regulated output voltage exhibits a small deviation in magnitude.

![Graph showing output voltage response with R = 60Ω and R = 120Ω](image)

**Figure 3.6 Response output voltage of the given full-bridge rectifier**

Figure 3.7 shows the responses of the input inductor current and supply voltage under change in load (100%). It shows that the input inductor current is almost in phase with the input supply voltage. During the increase in load, the power factor has considerable lag in maintaining near unity value as the addition of a large disturbance creates a ripple in magnitude and hence the sliding surface enters in to chattering. However at lower values of load change, the controller is robust enough not only in regulating the output voltage but also in maintaining the power factor at near unity.
Figure 3.7  Responses of both the input inductor current and input supply voltage

Figure 3.8 presents the response of the input inductor current. From the figure it is obviously inferred that the requirements of the AC-DC converter are fulfilled only when it is operated with the power factor closer to one and the output capacitor voltage \( \langle x_2 \rangle_0 \) should be driven to the constant reference value \( \langle x_2 \rangle_{0d} \), where \( \langle x_2 \rangle_{0d} > E \) in order to have boost behavior.

Figure 3.8  Response of input inductor current
The responses of supply voltage, input inductor current and output capacitor voltage are shown in Figure 3.9. The responses of input inductor current \( (x_1) \) and output capacitor voltage \( (x_2) \) are obtained for different load changes (Figure 3.10) from \( R = 160 \Omega \) to \( R = 10 \Omega \) at time \( t = 1.6 \text{sec} \). In this simulation, a load change of \( R = 40 \ \Omega \) is given as an intermediate load between the two previous loads at \( t = 1.6 \) sec. From the responses it is observed that the SM controller performs good in maintaining the regulated output voltage at a constant value. However, the in-phase nature of inductor current and supply voltage is not so better than that of the proposed IDA-PB controller wherein the power factor is near optimal to unity.

![Figure 3.9](image-url)

**Figure 3.9** Responses for the input inductor current \( (x_1) \), and output voltage \( (x_2) \) of the rectifier
Figure 3.10 Responses for different load settings

3.4 PCHS AND GSSA

Port Controlled Hamiltonian Systems (PCHS), with or without dissipation, generalize the Hamiltonian formation of classical mechanics to physical systems connected in a power-preserving way (Galaz et al 2001). The significant mathematical object of the formulation is what is called a ‘Dirac’ structure, which contains the information about the interconnecting network. A main feature of the formation is that the interconnection of Hamiltonian subsystems using a Dirac structure yields again a Hamiltonian system (Escobar et al 1999). A PCH model encodes the detailed energy transfer and storage in the system and is thus suitable for control schemes based on, and easily interpretable in terms of the physical dynamics of the system (Li 1999).

PCHS are passive in a natural way, and several methods stabilize them at a desired fixed point have been devised (Ortega 2002). VSS, especially in power electronic applications, can be used to produce a given periodic power signal to feed, for instance, an electric drive or any other
power component. In order to use the regulation techniques developed for PCHS, a method is necessary to reduce a signal generation or tracking problem. One powerful way to do this is the Generalized State Space Averaging (GSSA) technique (Mahdavi et al 1997). In this method, the state and control variables are expanded in a Fourier-like series with time-dependent coefficients. For periodic behavior, the coefficients will evolve to constants. In many practical applications physical consideration of the task is to solve finite dimensional reduced system to which standard techniques can be applied (Garcia et al 2001).

In this work PCHS techniques to a GSSA model of a boost-like full-bridge rectifier are applied. In many applications, such as the control of doubly-fed induction machines (Ortega et al 2000), power can flow in both directions through the back-to-back (rectifier + inverter) converter connected to the machine. Since the aim of the control scheme is to keep the intermediate DC-bus to a constant voltage, i.e. the rectifier’s load current can have any sign (although it can be supposed to be, approximately, piecewise-constant in time). Hence the need for a different solution than that found in (Petrovic et al 2001) arises. It should be noticed that standard procedures to solve the bidirectional case cannot be applied, since, no matter which output is chosen, either DC bus voltage or AC-current, the zero dynamics is unstable for one of the modes of operation.

3.5 REVIEW OF GSSA METHOD

The GSSA method was introduced in (Sanders et al 1991) and expanded later to highlight the accuracy of this modeling technique applied to describe the dynamic behavior of DC/DC converters in (Mahdavi et al 1997) and (Caliscan et al 1999).
The method is based on the fact that a signal $x(\tau)$ on the interval $\tau \in [t - T, t]$ can be represented by the Fourier series.

$$x(\tau) = \sum_{i=-\infty}^{\infty} (x)_i(t) e^{i\omega_0 \tau}$$

where $\omega_0 = 2(\pi/T)$ and $(x)_k(t)$ are the time-dependent complex Fourier coefficients given by,

$$\langle x \rangle_k(t) = \frac{1}{T} \int_{\tau}^{\tau'} x(\tau) e^{-jk\omega_0 \tau} d\tau \quad (3.16)$$

To reconstruct $x(\tau)$ from its Fourier coefficients, (3.16) can be reformulated as,

$$x(\tau) = \langle x \rangle_0 + 2 \sum_{j=1}^{\infty} (R(x), \cos(j\omega_0 \tau) - I(x), \sin(j\omega_0 \tau)) \quad (3.17)$$

where the time argument $t$ of $\langle x \rangle_k$ has been dropped to simplify the notation, and $R(\langle x \rangle_k)$ and $I(\langle x \rangle_k)$ are the real and imaginary parts of $\langle x \rangle_k$. In order to use this representation for the $x(\tau)$ in a state-space model of a system, two useful facts concerning differentiation with respect to time and computation of the average of a product are,

$$\frac{d \langle x \rangle_k(t)}{dt} = \left[ \frac{dx}{dt} \right]_k(t) - jk \omega_k \langle x \rangle_k(t) \quad (3.18)$$

$$\langle qx \rangle_k = \sum_{i=-\infty}^{\infty} \langle q \rangle_{k-i} \langle x \rangle_i \quad (3.19)$$
3.6 PCHS DYNAMICS OF SINGLE PHASE BOOST RECTIFIER

The model describing the averaged dynamic behavior of the circuit is given in equation (3.20) by (Astolfi et al 2002),

\[
\begin{align*}
\frac{d\phi(t)}{dt} &= -\frac{u(t)}{C} q(t) - \frac{r}{L} \phi(t) + V_i(t) \\
\frac{dq(t)}{dt} &= \frac{u(t)}{L} \phi(t) - i(t)
\end{align*}
\tag{3.20}
\]

where, \(\phi(t)\) \(- magnetic flux through inductor (Wb)\)

\(q(t)\) \(- electrical charge in capacitor (Coulombs)\)

\(r\) \(- resistance modeling the parasitic resistive effect of the inductor and the switches (\(\Omega\))\)

\(u(t)\) \(- the position of the switches taking values in the discrete set\)

\(i(t)\) \(- is the load current (A)\)

\(V_i(t)\) \(- E \sin(\omega_0 t) is the AC voltage source of amplitude and angular frequency (V)\)

\(\omega_0\) \(- 2\pi f_0, f_0 being the fundamental frequency in Hertz.\)

The load is assumed to be resistive, then \(i(t) = q(t)/RC.\)

The control objectives for this rectifier are,

i) The DC value of the output voltage \(q(t)/C\), should be equal to a desired constant value \(V_d > E.\)

ii) The power factor of the converter should be equal to one. This means that, in steady state, the inductor current \(\phi(t)/L\) should follow the sinusoidal signal with the same frequency and phase as the AC-line voltage source.
i.e., \( \phi^*(t) = LI_d \sin (\omega_0 t) \)

where, \( I_d \) is the appropriate constant value fulfilling the aforementioned objective. The second control objective does not correspond to a tracking problem because amplitude \( I_d \) depends on variable \( i_1(t) \).

A useful variable transformation, which simplifies (3.20), is obtained through \( v(t) = -u(t)q(t) \) and \( z = [z_1 \ z_2] = [\phi(t), (1/2)q(t)^2] \). The system in the new variables is,

\[
\frac{dz_1(t)}{dt} = -\frac{r z_1(t)}{L} + \frac{v(t)}{C} + v_i(t) \\
\frac{dz_2(t)}{dt} = -\frac{v(t) z_1(t)}{L} - i_i(t) \sqrt{2z_2(t)}
\]

The energy in the storing elements and of this system can be described by,

\[
H_z(t) = \frac{z_1(t)^2}{2L} + \frac{z_2(t)}{C}
\]

Equations (3.21) and (3.22) can be rewritten as,

\[
\begin{bmatrix}
\frac{dz_1(t)}{dt} \\
\frac{dz_2(t)}{dt}
\end{bmatrix} =
\begin{bmatrix}
0 & v \\
-v & 0
\end{bmatrix}
\begin{bmatrix}
\partial H_z \\
\partial z
\end{bmatrix} -
\begin{bmatrix}
r & 0 \\
0 & Ci_i \sqrt{2z_2}
\end{bmatrix}
\begin{bmatrix}
\partial H_z \\
\partial z
\end{bmatrix} +
\begin{bmatrix}
v_i \\
0
\end{bmatrix}
\]

(3.24)
This corresponds to a PCHS system of the form,

\[
\dot{z} = [J_T(v) - R_T(x)] \frac{\partial H_T}{\partial z}(z) + g_T
\]  

(3.25)

where \( J_T = -J_T^T \), \( R(x) = R^T(x) \geq 0 \) are the matrices describing the interconnection structure and damping, respectively. The last inequality results from and because the load voltage is non negative. The input voltage is considered as an external disturbance modelled by vector (Jianping Xu et al 1997). In order to obtain the simplest coherent GSSA model the harmonic content of the states and the input in steady state are determined.

### 3.6.1 Steady State Analysis

In order to obtain the steady-state zero dynamics, the assumption in equations (3.24) and (3.25) is taken into account and then, \( v \) and \( z_2 \) are solved where \( z_i^*(t) = LI_d \sin(\omega_i t) \) is the desired dynamics and \( i_i(t) \) is the load current assuming a resistive load. The steady-state response yields,

\[
z_2^*(t) = \alpha_{z2} + \beta_{z2} \sin(2\omega_2 t + \theta_{z2})
\]

(3.26)

\[
v^*(t) = C(E - rI_d) \sin(\omega_0 t) + I_d \omega_0 LC \cos(\omega_0 t)
\]

(3.27)

where,

\[
\alpha_{z2} = \frac{I_d RC^2}{4} (E - rI_d),
\]

\[
\beta_{z2} = \frac{I_d RC^2}{4} \sqrt{\frac{(E - rI_d)^2 + (I_d \omega_0 L)^2}{1 + (\omega_0 RC)^2}}
\]

and

\[
\tan(\theta_{z2}) = \frac{(E - rI_d) - \omega_0 RC(I_d \omega_0 L)}{\omega_0 RC((E - rI_d) + (I_d \omega_0 L))}.
\]
\( I_d \) can be obtained through power balance.

\[
I_d = \frac{E}{2r} m \left( \frac{E}{2r} \right)^2 - \frac{2V_d^2}{rR}
\]  

(3.28)

The negative sign has been chosen since it yields a stable equilibrium point with lower power consumption. The total stored energy in steady-state is,

\[
H_r(t) = \alpha_H + \beta_H \sin(2\omega_t t + \theta_H)
\]  

(3.29)

where,

\[
\alpha_H = \frac{I_d CR}{4} (E - rI_d) + \frac{LI_d^2}{4},
\]

\[
\beta_H = \frac{\alpha_H}{\sqrt{1 + (\alpha_H RC)^2}}
\]

and

\[
\tan(\theta_H) = \frac{1}{\omega_H RC}.
\]

Equations (3.20), (3.26), and (3.27) show that a suitable GSSA model of the system, useful for controller design purposes, should contemplate the first harmonic Fourier components for \( z_1(t) \), the zero and second harmonic Fourier components for \( z_2(t) \) and the first harmonic Fourier components for \( v(t) \). As for the Hamiltonian \( H_T(t) \), from equation (3.29), the DC component and second harmonic should be considered. If, in addition, \( C \) is chosen to obtain a low voltage ripple in the capacitor, then \( \beta_{z_2} \) and \( \beta_H \) are negligible with respect to \( \alpha_{z_2} \) and \( \alpha_H \), respectively. Hence, the second harmonic Fourier components of \( z_2(t) \) and \( H_T(t) \) are not considered for further steps (Petrovic et al 2001).
3.6.2 Rectifier as PCHS in GSSA Parameters

Although the most general GSSA model of a system has infinite dimension, the harmonic contents of signals in steady state can be used to find accurately enough finite dimensional GSSA models. To this aim, using equation (3.19) and taking into account the Fourier components, the bilinear product \( v(t)z_1(t) \) in (3.19) can be approximated as,

\[
\langle v_z \rangle_0 = \sum_{k=-\infty}^{\infty} \langle v \rangle_k \langle z \rangle_k \approx 2 \left( \langle v \rangle_1^R \langle z \rangle_1^R + \langle v \rangle_1^I \langle z \rangle_1^I \right) \quad (3.30)
\]

Furthermore,

\[
\langle i_q \rangle_0 \approx 2 \left( \langle i \rangle_1^R \langle q \rangle_1^R + \langle i \rangle_1^I \langle q \rangle_1^I \right) + \langle i \rangle_0^R \langle q \rangle_0^R \quad (3.31)
\]

As it has been assumed \( q(t) \) has predominantly DC harmonic components, the complex coefficients of order one in (3.31) are discarded. Hence, using (3.19), (3.30) and (3.31), the GSSA model of the system defined by (3.22) becomes,

\[
\frac{d\langle z \rangle_0}{dt} = -\langle i \rangle_0 \sqrt{2\langle z \rangle_0} - \frac{2}{L} \langle v \rangle_1^R \langle z \rangle_1^R - \frac{2}{L} \langle v \rangle_1^I \langle z \rangle_1^I
\]

\[
\frac{d\langle z \rangle_1^R}{dt} = -\frac{r}{L} \langle z \rangle_1^R + \frac{1}{C} \langle v \rangle_1^I + \omega_b \langle z \rangle_1^I
\]

\[
\frac{d\langle z \rangle_1^I}{dt} = -\frac{r}{L} \langle z \rangle_1^I + \frac{1}{C} \langle v \rangle_1^I + \omega_b \langle z \rangle_1^R - \frac{E}{2}. \quad (3.32)
\]

Let, \( x = [\langle z \rangle_0, \langle z \rangle_1^R, \langle z \rangle_1^I] \) be the state vector and \( u = [\langle v \rangle_1^R, \langle v \rangle_1^I] \) control vector and
\[
x^* = \left[ \frac{C^2 V_d^2}{2}, 0, -\frac{LI_d}{2} \right]
\] (3.33)

be the desired equilibrium.

The original control problem has become a regulation problem in the GSSA domain. For simplicity, let us denote the load current DC component by \( I_o = \langle i_i \rangle_o \). Then, the system in (3.32) can be written as the PCH system

\[
\begin{align*}
\frac{dx_1(t)}{dt} &= 0 - u_1 - u_2 \\
\frac{dx_2(t)}{dt} &= u_1 0 \frac{\alpha_0 L}{2} \frac{\partial H}{dx} - 0 \frac{r}{2} 0 \frac{\partial H}{dx} + \left[ C I_o \sqrt{2x_1} 0 0 \right] \\
\frac{dx_3(t)}{dt} &= u_2 - \frac{\alpha_0 L}{2} 0 \frac{r}{2} \left[ 0 0 \frac{r}{2} \right]
\end{align*}
\] (3.34)

otherwise in a more compact form,

\[
\dot{x} = \left[ J(u) - R(x) \right] \frac{\partial H}{dx} + g
\] (3.35)

where, \( J(u) \) and \( R(x) \) are the interconnection and damping matrices, respectively, and \( g \) vector is an external disturbance. \( H(x) \) is the DC component of the Hamiltonian in \( H_T(z) \) in equation (3.23) i.e.

\[
H(x) = \frac{1}{C} \langle z_2 \rangle_0 + \frac{1}{L} \langle \phi \rangle_{r_2}^2 + \frac{1}{L} \langle \phi \rangle_{i_2}^2
\] (3.36)

\[
H(x) = \frac{1}{C} x_1 + \frac{1}{L} x_2^2 + \frac{1}{L} x_3^2
\] (3.37)
The GSSA system in equation (3.34) preserves the PCH structure of the system in equation (3.24), with the remarkable advantage of a regulation control objective. The control law depends on the output voltage DC component and requires measuring the DC output current to guarantee robustness with respect to load variations.

3.7 IDA-PB CONTROLLER DESIGN

The objective of the IDA-PBC approach (Rosendo et al 1998) is to design a feedback control \( u = \beta(x) \), such that the closed-loop dynamics is the PCH reference system.

\[
\dot{x} = [J_d(x) - R_d(x)] \frac{\partial H_d}{dx}(x)
\]  

(3.38)

where, \( J_d(x) = -J_d^T(x) \) and \( J_d(x) = R_d^T(x) + 0 \) are targeted interconnection-and-damping matrices, and the new energy function \( H_d(x) = H(x) + H_d(x) \) has a strict local minimum at the desired equilibrium.

3.7.1 Conditions for Stable Equilibrium

Following (Rosendo et al 1998), the controller design is proceeded in the standard manner.

(i) Structure preservation

Given \( J_d(x) \) and \( R_d(x) \), let \( J_a(x) \) and \( R_a(x) \) be defined by,

\[
J_d(x) = J(x, \beta(x)) + J_a(x) = [J(x, \beta(x)) + J_a(x)]^T
\]

(3.39)

\[
R_d(x) = R(x) + R_a(x) = [R(x) + R_a(x)]^T
\]

(3.40)
The desired dynamics is achieved if it is possible to find functions $\beta(x)$ and $k(x) = \partial H_a(x)/\partial x$ satisfying,

$$[J(x, \beta(x)) + J_d(x) - (R(x) + R_d(x))] k(x) = - [J_d(x) - R_d(x)] \partial H_a(x)/\partial x + g$$

(3.41)

(ii) Integrability

$K(x)$ is the gradient of a scalar function.

That is, $\frac{\partial k_i}{\partial x_j}(x) = \frac{\partial k_j}{\partial x_i}(x)$

(3.42)

(iii) Equilibrium condition

$$\frac{\partial H_d}{\partial x}(x^*) = 0$$

(3.43)

If conditions (i) – (iii) hold, then $x^*$ is a (locally) stable equilibrium point of the closed-loop system.

The aforementioned procedure is particularised for the full bridge boost rectifier controller defining $J_d(x) = J(x, \beta(x))$ and $R_d(x) = R(x)$, i.e., $J_d(x) = 0$ and $R_d(x) = 0$.

(i) Structure preservation

Equation (3.41) yields,

$$0 = -I_a C \sqrt{2x_k} k_1 - u_1 k_2 - u_2 k_3$$

(3.44)

$$0 = u_1 k_1 - \frac{r}{2} + \frac{\omega L}{2} k_3$$

(3.45)

$$0 = u_2 k_1 - \frac{\omega L}{2} k_2 + \frac{E}{2}$$

(3.46)
Then, from (3.45) – (3.46),

\[ u_1 = \frac{-r k_2 + \omega L k_1}{2k_1} \]

\[ u_2 = \frac{\omega L k_2 + R k_3 - e}{2k_1} \quad (3.47) \]

(ii) **Integrability**

Replacing (3.47) in (3.44) and taking into account that
\[ k(x) = \frac{\partial H_a}{\partial x}, \]
the following partial differential equation is obtained.

\[ 2I_0 C \sqrt{2x_1} \left( \frac{\partial H_a}{\partial x_1} \right)^2 = -r \left( \frac{\partial H_a}{\partial x_2} \right)^2 - \left( r \frac{\partial H_a}{\partial x_3} - E \right) \frac{\partial^2 H_a}{\partial x_3} \quad (3.48) \]

As control inputs \( u_1 \) and \( u_2 \) only depend on the output voltage dc component, \( k_2 = k_2(x_1) \) and \( k_3 = k_3(x_1) \).

Then, by the integrability condition, \( \frac{\partial k_i}{\partial x_i} = \frac{\partial^2 H_a}{\partial x_i \partial x_i} = \frac{\partial^2 H_a}{\partial x_i \partial x_i} = 0 \).

For \( i = 2, 3 \) and \( k_2 = a_2 \) and \( k_3 = a_3 \) are indeed constant. Thus, the Partial Differential Equation (PDE) is actually an Ordinary Differential Equation, whose solution is given by,

\[ H_a(x) = \frac{2}{3} \sqrt{\frac{2x}{I_0 C}} x_1(a_2^2 r + a_3^2 r - a_3 E) + a_2 x_2 + a_3 x_3 \quad (3.49) \]
(iii) Equilibrium Assignment

From equation (3.49) and the definition of $H_a (H_a = H_d - H)$, the following conditions on $a_2$, $a_3$ and $I_d$ so that $x^*$, from equation (3.15), is a singular point of $H_d$ are derived:

\[
\frac{1 + \sqrt{-2 I_o C \sqrt{C^2 V^2_d (a_2^2 r + a_3^2 r - a_3 E)}}}{3 I_o C \sqrt{C^2 V^2_d}} = \frac{\sqrt{2 (a_2^2 r + a_3^2 r - a_3 E)}}{6 \sqrt{-I_o C \sqrt{C^2 V^2_d}} ((a_2^2 r + a_3^2 r - a_3 E)}
\]

$a_2 = 0$ and $a_3 - I_d = 0$. \hspace{1cm} (3.50)

This equations system has two solutions,

\[
\begin{cases}
  a_2 = 0, I_d = \frac{E + \sqrt{E^2 - 8 I_o V_d r}}{2 r}, a_3 = I_d \\
  a_2 = 0, I_d = \frac{E - \sqrt{E^2 - 8 I_o V_d r}}{2 r}, a_3 = I_d
\end{cases}
\]

and \hspace{1cm} (3.51)

Then, taking the latter solution, $k_1$ and the control inputs derived in (3.47) are,

\[
k_1 = -\frac{\sqrt{2} \sqrt{I_o^3 V_d C \sqrt{2 x_1}}}{2 I_o C \sqrt{x_1}}
\]

\hspace{1cm} (3.52)

\[
u_1 = -\frac{\omega L (-E + \sqrt{E^2 - 8 I_o V_d r C \sqrt{V_0 V_d}})}{4 r V_d}
\]

\[
u_2 = -\frac{E + \sqrt{E^2 - 8 I_o V_d r C \sqrt{V_0 V_d}}}{4 V_d}
\]

\hspace{1cm} (3.53)

where, $V_0$ denotes the output voltage DC component $\langle v_0 \rangle_0$ and $\sqrt{2 x_1} = \langle q \rangle_0 = C \langle v_0 \rangle_0$
3.8 SIMULATION RESULTS OF PASSIVITY CONTROLLER

3.8.1 Linear Load

The entire system behaviour is simulated as a discrete control system using MatLab and Simulink. The continuous IDA-PB controller is discretized and implemented through a PWM so that the simulations are closer to the actual system (Rodriguez et al 2001). The closed-loop diagram is outlined in Figure 3.11. The actual rectifier is shown at the top of the figure with pulses as the input signal and the couple $i(t)$ and $v_\alpha(t)$ as the output measured variables. The input voltage $v_i(t)$ is given as input in the controller and IDFT blocks. The real-time discrete Fourier transform is carried out in the RDFT block. It is based on a recursive discrete Fourier transform technique (Rosendo et al 1998) which allows the right Fourier coefficients to be obtained at sampling times. The controller block computes the suitable averaged Fourier components for the control signal $u(t)$ while the inverse discrete Fourier transform is performed in the IDFT block to obtain the discrete $v(kT)$ control signal. Finally, the fixed frequency pulses are provided by the PWM block by pulse-width modulating the real valued $u(t)$ control signal. The parameters of single phase boost rectifier are listed in Table 3.1.

3.8.2 Nonlinear Load

Using averaged sensed variables instead of actual ones lets this controller to deal with nonlinear loads. To this end, simulation of this kind of load has been considered by means of resistance values composed by a mean value plus a triangular ripple (20% amplitude) at the fundamental frequency. The results are obtained for output DC voltage regulation performance and the power factor correction performance for the case of a nonlinear load whose mean values change from 20 $\Omega$ to 500 $\Omega$. 
Figure 3.11 The structure of the IDA-PB control approach

3.8.3 Control Objectives

The desired regulated DC output voltage and the power factor should be 200V and near unity, respectively. It is important to note that the IGBT switches are oversized for this particular application resulting in undesirable power losses and harmonic distortion. Theses power losses has been taken into account increasing the series resistance r by a switch loss resistance ($r_{sw}$) is assumed to be 0.3 $\Omega$. The system performance could be improved by replacing these switches with low power ones.
3.8.4 Dynamic Response of Uncontrolled and Controlled Rectifier

The response of the uncontrolled rectifier is shown in Figure 3.12. As seen from the Figure 3.12, the voltage output of the rectifier is less than the desired voltage and also the line current is not exhibiting the sinusoidal waveform.

![Figure 3.12 Response of the uncontrolled rectifier](image)

The responses of the IDA-PB controlled rectifier for output voltage and line current waveform in front of load changes are shown in Figure 3.13.
Figure 3.13 Responses for output voltage and line current

3.8.5 Optimal Responses of Rectifier with IDA-PBC

Figure 3.14 exhibits the simulink model of the proposed control scheme. Subsequently, Figures 3.15 – 3.16 show the dynamic responses of the system under load variation. Figure 3.16 shows the power factor correction performance for the case of nonlinear load whose mean value changes from 50 Ω to 500 Ω.
Figure 3.14 Simulink model of the passivity control scheme

Figure 3.15 Unity power factor and voltage regulation responses
Figure 3.16 Responses for $V_s$, $I$, and $V_o$ which exhibit robustness under load parameter variations (50 $\Omega$ to 500 $\Omega$)

The obtained output voltage for the three values of the load, i.e. $R = R_{\text{nom}} = 20\Omega$ for $0 \leq t < 0.3$, $R = 500\ \Omega$ for $0.3 \leq t < 0.5$ second, and $R = 100\ \Omega$ for $0.5 \leq t < 0.8$ second, is depicted in Figure 3.16. The inductor current $i(t)$ and the scaled input voltage $v_i(t)$ are also depicted for each of the selected loads. These figures show that the specifications are fulfilled. The desired regulated DC output voltage and the power factor is $V_o = 200V$ and near-unity respectively.

3.9 RESULTS AND DISCUSSION

As seen from the Figure 3.12 the line current of ordinary uncontrolled bridge rectifier is not sinusoidal, also there is no boost up of output voltage. However system itself is not robust and also the system power factor is also not unity (Figure 3.12). From Figure 3.15, it’s observed that, when compared to SMC performance, it is possible to get a pure sinusoidal
line current using the proposed bridge rectifier model and boost up of output voltage can also be realized with respect to a wider range of load variations too.

3.9.1 DC Bus Voltage Regulation

Initially the DC bus voltage rests at the diode rectifier level with a resistive load of $R = 20 \, \Omega$. Then the control action is applied keeping the load resistance and the output voltage increases to the desired DC value. Afterwards, the load changes from $R = 20 \, \Omega$ to $R = 500 \, \Omega$ and from $R = 500 \, \Omega$ to a no-load condition were applied at $t = 0.3 \, s$ and at $t = 0.5 \, s$, respectively. Figure 3.16 shows the shape of the DC bus output voltage. As it can be seen, the controlled system is robust with respect to load variation.

3.9.2 Line Current and Power Factor Behavior

Figures 3.15 and 3.16 show that the line current shape is pure and power factor value for two load conditions, namely $R = 50 \, \Omega$ and $R= 500 \, \Omega$ is unity as both the line current and voltage are in phase. As it can be seen, the higher the power managed, the power factor is closer to unity. It is important to remark that the $\cos \varphi$ is equal to one in both cases.

3.10 ADVANTAGES OF THE PASSIVITY CONTROL TECHNIQUE

The passivity-based controller design methodology is based on the average models of the boost rectifier derived from Euler-Lagrange dynamics considerations. The derived equations describing the average behavior of the AC/DC converter coincide with the generalized state space average GSSA models. The controller is designed based on the well known “energy shaping
plus damping injection” ideas of the passivity-based approach. The control method proposed here has been designed for a non linear GSSA model with an inherited Port-Controlled Hamiltonian (PCH) structure. The design takes the advantage of this structure to solve the aforementioned GSSA regulation problem and the closed-loop system robust to load variations compared to the Sliding Mode Controller.

Besides the stability theorems proposed in literatures, the advantage of this setting is that the assignment of parallel damping does in general not involve the use of current sensors but only needs the measurements of the voltages. A major advantage of parallel damping in comparison with series damping injection has been that it makes the closed-loop system robust, in the sense that it does not require adaptive extensions in case the load is unknown or varying. The closed-loop system is robust to wide load variations achieving unity power factor in the AC mains and load voltage Regulation.

3.11 CONCLUSION

In this chapter, the passivity-based controller (PBC) design procedure for Euler-Lagrange EL systems in the context of power converters is presented in terms of the damping injection principle. The dynamic performance analysis of single phase full bridge boost-type rectifier is analyzed for regulated output voltage and unity power factor at the AC mains. In the case considered here, a nonstandard tracking control problem for a full-bridge boost rectifier results in a regulation one because of GSSA expansion for phasor coefficients. An IDA-PB control has been designed measuring the load current and the load voltage, and presuming the input voltage is known.
The design procedure for a dynamic SM control scheme for the single phase AC/DC rectifier has also been presented which suggests a sliding surface dynamically defined in order to fulfill two specifications in a single input system, namely unity power factor and output voltage regulation. A converter parameter design procedure that can be used to minimize the capacitor has also been proposed. The theoretical predictions have been validated by means of simulation results.