2.1 FUNDAMENTALS OF FLOW SEPARATION

In 1904, Prandtl showed that flows with low friction in the vicinity of bodies can be subdivided into two regions: a thin layer close to the body, the so-called boundary layer originally called friction layer due to the predominance of friction, and the remaining flow, the potential flow where friction effects can be neglected. In the boundary layer itself, the flow at the wall must follow the no-slip condition. Hence, the boundary layer is decelerated by the wall, but accelerated by the outer flow. The static pressure, constant across the boundary layer, is governed by the main flow.

2.1.1 Flow Separation as a Boundary Layer Phenomenon

A study has been made of some contributions to the problem of shock induced boundary layer separation by Love (1955) and additional analytical and experimental results were also presented. The probable ranges within which the pressure rises and flow deflections associated with separation may be expected to be were shown. Consideration is given to the effects of Mach number, adverse pressure gradient and Reynolds number for laminar boundary layers and to the effects of Mach number, Reynolds number, and ratio of specific heats for turbulent boundary layers.
It is well know that on a flat plate at zero angle of incidence with no pressure gradient, the thickness of the boundary layer increases indefinitely along the length of the plate. However, if a pressure gradient exists, growth of the boundary layer is more complex. A pressure gradient on the boundary layer is termed as favourable if the pressure decreases in the downstream direction, and unfavorable or adverse if the pressure increases in the downstream direction.

If the pressure gradient on a boundary layer is favourable as may be expected in a converging channel, the fluid elements within the boundary layer are accelerated by it. This partly counteracts the retarding action of the viscous drag induced at the solid boundary. Therefore, in the presence of a favourable pressure gradient, separation of the boundary layer does not take place and the boundary layer is said to be held by the favourable pressure gradient.

Consider the growth of a boundary layer under an adverse pressure gradient, say on the wall of a diverging channel as shown in Figure 2.1. In such a channel the streamlines are divergent and an adverse pressure gradient is developed in the channel.

Referring to Figure 2.1, let A be a point on the channel wall just upstream from the diverging part of the channel where boundary layer has normal velocity distribution. In the diverging part of the channel beyond A, where an adverse pressure gradient is developed, the fluid elements close to the wall have to overcome not only the viscous drag due to wall shearing stress, but also a retardation due to the adverse pressure gradient. In contrast, the fluid elements at the outer edge of the boundary layer have to overcome mainly the retardation due to the adverse pressure ingredient, the viscous drag
there being much less as compared to that near the wall. As a result, fluid
elements close to the wall are decelerated more rapidly compared to those at
the outer edge of the boundary layer, thus producing a distorted velocity
distribution diagram as shown at point B of Figure 2.1.

This process continues at successive downstream sections until the
slope of the velocity distribution diagram at the wall \( \partial u / \partial y = 0 \), as shown at
point C in the Figure 2.1. Further downstream, the fluid elements close to the
wall flow in the reverse direction, as shown at D. This reverse flow pushes the
incoming downstream flow away from the wall thus causing separation of the
boundary layer from the wall at the point C, as shown in the Figure 2.1.

![Figure 2.1 Separation of a Boundary Layer from a Wall](image)

### 2.1.2 Separation of Laminar and Turbulent Boundary Layer

Although the mechanics of separation of laminar and turbulent
boundary layers are basically the same, the positions of points of separation
are different for the two types of boundary layer. In a laminar boundary layer
the exchange of momentum between the faster moving external flow and the slower moving boundary layer flow takes place through viscous shear which is a relatively slow process. In contrast, in a turbulent boundary layer, because of random fluctuations of the transverse velocity components, rapid exchange of momentum takes place between the faster moving external flow and slow moving boundary layer flow. Therefore against an adverse pressure gradient, the fluid elements of a turbulent boundary layer are kept in motion over a longer distance as compared to the fluid elements in a similarly placed laminar boundary layer. As a result of this, under identical other conditions a turbulent boundary layer separates at a more downstream location compared to a laminar boundary layer.

In flows with favourable or zero wall pressure gradient, the boundary layer is attached to the wall. This can be different in the case of an adverse wall pressure gradient. If the wall pressure increases in the main flow direction, kinetic energy of the fluid particles is transformed into potential energy. However, fluid particles close to the wall only have a small kinetic energy because of their lower velocity. Therefore they are stopped by the pressure rise, and may be even forced to flow in the reverse direction. In this case the boundary layer is separated from the wall, and the recirculation region is developed in the vicinity of the wall.

Flow separation requires the existence of both friction and an adverse wall pressure gradient in a flow along a body. If one of these two conditions is suppressed, flow separation can be prevented. Prandtl (1952) proved this with different experiments, e.g. with a flow around rotating cylinders or with a diffuser with boundary layer suction. Also, flow separation might not occur if the adverse pressure gradient is weak. In this case, the normal exchange of momentum inside the boundary layer can be sufficient to
transport momentum from the mean flow to the wall; consequently, the kinetic energy of the particles close to the wall can be high enough to withstand the pressure rise without separation. Turbulent boundary layers with their characteristic high lateral exchange of momentum therefore separate much later than laminar boundary layers, where the momentum transport only consists of molecular movements. At the separation point of two-dimensional boundary layers, planar or axisymmetric, the wall shear stress becomes zero,

\[ \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_w = 0 \] (2.1)

From this equation, and the velocity profile, the behaviour of the derivatives of \( u \) in wall-normal direction can be estimated.

In order to get a closer understanding of the separation processes, the momentum equation in wall-parallel direction is considered. The chosen non-conservative formulation is valid for a Newtonian fluid in a Cartesian coordinate system, neglecting volumetric forces:

\[
\begin{align*}
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \\
- \frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left[ 2 \mu \left( \frac{\partial u}{\partial x} - \frac{\text{div} \, \vec{v}}{3} \right) \right] + \frac{\partial}{\partial y} \left[ \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial z} \left[ \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \right]
\end{align*}
\] (2.2)

If an arbitrary point at the wall is considered, the non-slip condition yields \( u = v = w = 0 \) for all velocity components as well as for their derivatives with respect to time and to the wall-parallel directions \( x \) and \( z \). Substituting \( \text{div} \, \vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \), and by assuming a constant viscosity across the boundary layer, the expression is simplified as follows:
\[ 0 = -\frac{\partial P}{\partial x} - \frac{2}{3} \mu \frac{\partial}{\partial x} \left( \frac{\partial v}{\partial y} \right) + \mu \frac{\partial^2 u}{\partial y^2} \] (2.3)

Because \( \partial / \partial x (\partial v / \partial y) = \partial / \partial y (\partial v / \partial x) \), the second term on the right side of Equation (2.3) becomes zero. Consequently, the following formula is valid for an arbitrary location at the wall:

\[ \mu \left( \frac{\partial^2 u}{\partial y^2} \right)_w = \frac{\partial P_w}{\partial x} \] or \[ \frac{\partial \tau_w}{\partial y} = \frac{\partial P_w}{\partial x} \] (2.4)

In this context, Equation (2.4), was derived directly from the momentum equation, Equation (2.2), only assuming a constant viscosity in the boundary layer, and is therefore valid for any point at the wall, including separation and recirculation zones. Schlichting (1997) derived Equation (2.4) from the classic boundary layer equations, which only represent an approximation of the flow. The aforementioned derivation from the momentum equation shows that Equation (2.4) is not only an approximation, but also an exact solution for the flow at the wall.

Equation (2.1) can be used to show that \( \partial^2 u / \partial y^2 > 0 \) at the separation point. Since the dynamic viscosity is always positive, Equation (2.4) yields that in order to have separation the wall pressure gradient must be adverse:

\[ \left( \frac{\partial P_w}{\partial x} \right)_{x, \text{separation}} > 0 \] (2.5)

### 2.1.3 Shock-Wave Boundary Layer Interactions

The above expressions are valid for subsonic as well supersonic flows. In the following we will however only discuss the case with turbulent
supersonic flows, having an adverse pressure gradient of sufficient strength to cause the boundary layer to separate. When a supersonic flow is exposed to an adverse pressure gradient it adapts to the higher-pressure level by means of a shock wave system. Basically, separation occurs when the turbulent boundary layer cannot negotiate the adverse gradient imposed upon it by the inviscid outer flow. Thus, flow separation in any supersonic flow is a process involving complex shock wave boundary layer interactions.

2.1.4 The Basic Interactions

The shock wave boundary layer interaction has been extensively studied in the last fifty years with the help of basic experiments, see e.g. Donaldson (1952). The three basic configurations involving interaction between a shock wave and a boundary layer in supersonic flows are schematically represented in Figure 2.2. In all of these cases, the incoming outer flow is uniform flow streaming along a flat plate. The first and conceptually most simple configuration is the wedge (or ramp) flow. Here, a discontinuity in the wall direction is the origin of a shock wave through which the supersonic flow undergoes a deflection equal to the ramp angle \( \alpha \), Figure 2.2 (a).

The second type of flow is associated with the impingement on the wall of an incident oblique shock, which cause a deflection of the incoming flow, see Figure 2.2 (b). The necessity for the downstream flow to be parallel again to the wall causes the formation of a reflecting shock issuing from the impingement point.
Figure 2.2  Basic shock/boundary layer interactions in supersonic flow
  a) Ramp flow. b) Shock reflection. c) Step induced separation, adopted from Ostlund (2002)
The third flow is induced by a step of height ‘h’ facing the incoming flow, see Figure 2.2 (c). Such an obstacle provokes separation of the flow at point S. The rapid pressure rise accompanying separation gives rise to a shock wave emanating from a place very close to the separation point S, and a separated zone develops between the separation point S and the step.

A review of experiments concerned with the description of the steady flow field produced by the separation of a turbulence boundary layer ahead of a forward facing step is presented by Zukoski (1967). The dependence of the induced pressure field at the wall on Reynolds number, Mach number, and step height is considered. It was shown that the pressure rise in the separation region expressed in normalized form is independent of Mach number and Reynolds number and that the scale for the separation phenomena is the boundary layer thickness. In addition, it was found that the plateau pressure rise is independent of Reynolds number for the turbulent regime and that the induced side force increases linearly with Mach number.

It has been shown in many experiments (Chapman et al 1958), that the major part of the shock / boundary layer interaction properties are nearly independent of the cause having induced the separation, whether being either a solid obstacle or an incident shock wave. In fact the features of the static wall pressure for the above different experimental configurations are the same, see Figure 2.3. The wall pressure has a steep rise shortly after the beginning of the interaction at I. The flow separates from the wall at S, located a distance \( L_s \) from I. The wall pressure then gradually approaches a plateau with almost constant pressure, labeled the plateau pressure \( P_p \). The extent of this plateau reflects the size of the closed recirculation bubble and \( P_p \) thus corresponds to the wall pressure in the bubble. A second pressure rise can be observed as the reattachment point at \( R \) is approached. These
characteristics are independent on the downstream geometry, as already
mentioned, everything happens as if the flow were entirely determined by its
properties at the onset of the interaction. This observation led Chapman et al.
(1958) to formulate the free interaction concept.

![Diagram of static pressure distribution]

**Figure 2.3** Typical Wall Static Pressure Distribution Observed in
Ramp, Shock Reflection and Step Flow; Adopted from
Lange (1954)

### 2.2 FLOW SEPARATION IN NOZZLES

A flow exposed to an adverse pressure gradient of sufficient
strength can cause the boundary layer to separate from the wall. In the
previous section we examined the influence of such adverse pressure
gradients generated by obstacles. A similar condition occurs when a nozzle is
operating in an overexpanded condition. A nozzle flow is said to be
overexpanded when the theoretical wall exit pressure $P_{e,v}$ (the wall pressure
obtained when the flow is ejected in to vacuum ambient conditions) is below
the ambient pressure $P_a$. 

The phenomena of separation was noticed during the course of early investigations of flow in C-D nozzles conducted by Buchner (1904), Prandtl (1907), Meyer (1908), Flugel (1917), Stanton (1926) and Stodola (1927). The separation or breakaway of an over-expanded gas from the nozzle wall boundary was noted and was explored to a limited extent by steam turbine engines. These findings are discussed by Stodola (1927) in his book on steam and gas turbines.

The experimental results for one convergent and three C-D nozzles were compared with one-dimensional nozzle theory by Krull (1952). Their result shows that C-D nozzles have low thrust coefficients when overexpanded, but not as low as was predicted from 1-D theory. They showed that the performance of a convergent nozzle can be calculated from 1-D theory within 1 or 2 percent, while the performance of a C-D nozzle differs widely from theory when overexpanded and cannot be foreseen because of the unpredictable behaviour of the flow in the divergent section. The C-D nozzles with higher divergence angles had the highest ratio of actual to theoretical thrust coefficients when the nozzles were overexpanded but the lowest thrust coefficients at the design pressure ratio.

After World War II, this problem became increasingly important during the efforts in rocket engine design. The first well-known investigations of flow separation for hot fired nozzles were performed at the California Institute of Technology, by Forster and Cowles (1949), and Summerfield et al (1954). Tests using a small nitric acid/aniline engine resulted in the separation correlations relating the location of separation in conical and two-dimensional nozzles as function of chamber pressure. These investigators ascertained that the location of separation could be predicted reasonably well by assuming that the flow is turned through a wedge angle, $\theta$, in the region of separation and then applying two-dimensional-shock theory to determine the
pressure rise across the shock. In this approach, the pressure downstream of the shock is taken to be the back pressure, \( P_b \). The interesting, and potentially significant, discovery was made that the experimental data were best fitted if the wedge angle was assumed to have a constant value which was almost independent of the divergence angle of the nozzle, nozzle length, pressure ratio, and gas temperature. This result was quite surprising because one would expect the flow to be turned approximately parallel to the axis of the nozzle. Thus, one could guess that the turning angle would be nearly that of the divergence angle of the nozzle. As a consequence of these investigations, the authors observed flow separation within the nozzle, as soon as the wall pressure at the nozzle exit was lower than about 0.35-0.4 times the ambient pressure, depending on the pressure ratio \( p_e/p_{w,e} \). The corresponding formula was soon called the “Summerfield criterion” i.e. \( p_s \approx 0.4 p_a \). For several years, this value served as guide for locating the point of separation. However, considerable evidence which indicates that this ratio is not a constant has since been accumulated, as will be made apparent in the following discussion.

In the past it has been assumed that a rocket nozzle operating with complete expansion (nozzle exit plane pressure equal to ambient pressure) will produce the greatest thrust for a given nozzle inlet condition and propellant. It has been proposed by Durham (1955) that it is actually possible to obtain a slight increase in thrust from an underexpanded nozzle (nozzle exit plane pressure greater than ambient pressure) as compared to the completely expanded nozzle with the same inlet conditions. This phenomenon is explained briefly as follows:

For underexpanded nozzles the thrust developed consists of two parts – the momentum thrust, and the pressure thrust. In the case of isentropic
flow, the thrust for a given nozzle inlet condition will be maximum when the pressure thrust is zero – that is, when complete expansion to ambient pressure takes place within the nozzle. In the actual, non-isentropic case, however, some degree of underexpansion will reduce the energy loss due to friction within the nozzle and the pressure thrust may more than compensate for the reduction in momentum thrust caused by the underexpansion.

The optimization of a propulsive nozzle involves a variety of conditions and parameters. These include: the environment in which the nozzle is to operate; the propellant selection; the thrust requirement; the mass flow rate; and the property to be optimized such as weight, length or exit diameter.

It is well known as shown by Shapiro (1953) that in the absence of friction a perfect nozzle gives maximum thrust if the exit pressure is equal to ambient pressure. The particular problem of length optimization for axisymmetric frictionless flow has been investigated by Rao (1958) with the aid of the calculus of variations. Although this method leads to the minimum length nozzle for a prescribed thrust, it has not been extended to solve the minimum surface area problem, which is probably the most important of these extremal problems. Moreover, the effect of wall friction cannot be included. Since one type of optimum nozzle shape cannot satisfy the many conceivable requirements, it is desirable to have a procedure which encompasses the various optimum problems.

Although Summerfield’s separation criterion was improved by many authors (Arens 1963, Schmucker 1974), mainly by including the Mach number influence, his phenomenological observations were confirmed by numerous tests that were carried out in the following decades, even for different working gases and nozzle contours (Campbell 1960, Farley 1960,
Lawrence 1967, Sunley 1964). These studies were followed by the work of Lawrence and Weynand (1968), who suggested a separation criterion using the plateau pressure rather than the ambient pressure $P_a$. A separation criterion used at the European industry and scientific institutes describes the pressure ratio $P_s/P_a$ as function of the local flow Mach number at the wall near the separation point. This analytical model was derived from various cold- and hot-firing tests of overexpanded nozzles. Former experiments of separated flows indicated that there is a random movement of the separation line in overexpanded rocket nozzles, resulting in the possible generation of side loads. The correct prediction of side-loads with models is uncertain, and still a subject of on-going research. Side-load model described by Schmucker (1974) assume that the separation in the nozzle has a maximum tilt angle with the centerline. The minimum and maximum separation points are predicted using the upper and lower separation data that result from the applied separation model. Once the flow was separated no reattachment occurred; that is why this standard type of flow separation is called the free-shock separation.

Musial and Ward (1959), showed that for the nozzle pressure ratios available with air breathing engines at Mach numbers greater than 2, it is advantageous to expand the flow in a C-D exhaust nozzle. The thrust to be gained by expanding the nozzle flow increases with nozzle pressure ratio; however, the performance of these nozzles operating in the overexpanded region is poor.

Quiescent air tests and wind tunnel tests by Fradenburgh et al (1954) and Ashwood et al (1957) have indicated overexpansion of the flow in the nozzle at pressure ratios lower than design with a resultant thrust loss. In an effort to increase the off design thrust, additional tests have been made by
Musial et al (1959) employing secondary air injection to induce separation in the nozzle. Musial et al (1959) presents the internal pressure distribution and thrust characteristics of the nozzles over a range of pressure ratios from 3 to 105 with freestream Mach numbers of 0, 2, 2.5, 3 and 3.5. Also included are results of attempts to increase off design nozzle performance by secondary air injection and by mechanical means.

Thrust data by Musial and Ward (1959) at supersonic freestream conditions indicated that only a small percentage of the ideal nozzles thrust would be available at NPRs below design. However, the overexpanded primary nozzle thrust loss was decreased by injecting large quantities of secondary air near the nozzle exit. In most cases no net gain thrust resulted from secondary air injection when the nozzle thrust was compared with the ideal thrust of both the primary and secondary airflows.

A contribution by Campbell and Farley, in 1960, made a serious attempt at defining from their experimental results the pressure distribution, and hence thrust correction, in the separated flow region of nozzles. Flow separation in supersonic flows is, of course, not limited to the field of rocket nozzles. When a supersonic flow meets a forward facing step, a ramp, or an incident shock, the pressure rise in the boundary layer can be strong enough to cause the flow separation (Moretti 1963).

In general, shock separated boundary layers may be divided into two main groups; the free and the restricted separated boundary layer. By free shock separated layers, one means that type of separation where the flow downstream of the separation region is free to adjust to any direction that may result from the shock boundary layer interaction process. Such separation occurs ahead of sufficiently large steps or in overexpanded nozzles of sufficiently large divergence.
Some purely analytical considerations, which only refer to the governing equations, are used to further analyze some influencing parameters. The momentum equation in wall-tangential direction for any point at the wall can be simplified as follows:

$$\mu \left( \frac{\partial^2 u}{\partial y^2} \left|_w \right. \right) \frac{dp_w}{dx} = dx$$

(2.6)

A separation point is defined as the point of zero wall friction:

$$\tau_{\omega, sep} = \mu \left( \frac{\partial u}{\partial y} \right)_{w, sep} = 0$$

(2.7)

The axial velocity at the wall $u_w$ is always zero because of the no-slip condition, and is always positive in the mean flow outside the boundary layer. Recirculation only occurs downstream of the separation point, therefore it can be assumed that near the separation point, $u$ grows with $y$ as shown in Figure 2.4. As a consequence of this, $\left( \frac{\partial^2 u}{\partial y^2} \right)_w$ must be positive in the separation point. Considering Equation (2.6), it can be stated that an adverse pressure gradient is required in the separation point.

In a supersonic flow with a favourable pressure gradient, as it is common in rocket nozzles, this pressure rise is provided by the oblique shock which turns the flow from the wall, see Figure 2.5. The abrupt pressure rise delivered by the shock is usually softened by the subsonic part of the boundary layer, so wall pressure measurements show a strong, adverse pressure gradient which is approximately constant over a well defined separation region between $x_s$ and $x_p$ (See Figure 2.5).
Figure 2.4 Profiles of the Axial Velocity (Left), its First (Centre) and Second (Right) Derivative with Respect to the wall Normal Direction in the Separation on Point

Figure 2.5 Free-shock Separation in Overexpanded Rocket Nozzles, Wall Pressure Profile, and Phenomenology: ——, Compression Waves/Shock, and ----, Boundary/Shear-Layer Edge
Assuming a constant length of this separation region, e.g. 
\( x_p - x_s \approx \text{const} * \delta \), Equation (2.6) directly connects the value of the separation criterion \( P_s / P_p \) to the state of the boundary layer.

\[
\frac{P_s}{P_p} = 1 - \frac{\text{const} * \delta * \mu}{P_p} \left( \frac{\partial^2 \mu}{\partial y^2} \right)_w
\]  
(2.8)

By analyzing the effect of arbitrary parameters on the right side of this equation, its influence on the separation criterion \( P_s / P_p \) can be detected. For example, an increase of the dynamic viscosity \( \mu \) with the Reynolds number kept constant would result in a delayed separation. An influence much closer to the application in rocket nozzles is the wall cooling, which also delays flow separation because of its dominating influence on boundary layer shape and velocity profile.

Free shock separation has also been investigated by Reshotko and Tucker (1955), however, these investigators have not recognized the experimentally observed difference between the separation and final pressures. This failure, to note that separation occurs at pressure lower than that which is ultimately reached, forces an adjustment in the “separation parameter” of reference Reshotko and Tucker (1955), depending on the mode in which the separation is produced – e.g. a different parameter is used for separation ahead of steps and that ahead of wedges.

At the separation point the wall pressure quickly rose to a plateau pressure \( p_p \), which is slightly lower than the ambient pressure \( p_a \). The source of this pressure rise was shown to be an oblique shock that originates from the separation point, the wall pressure increases slowly from \( p_p \) to \( p_{w,e} \). The wall
pressure in the exit plane $p_{w,e}$ is, in general, not equal to the ambient pressure $p_a$, but slightly smaller. Nevertheless, the simplification $p_{w,e} = p_a$ is frequently used in the literature. A sketch of the described phenomena is shown in Figure 2.5.

The name restricted shock separated boundary layers, on the other hand is given to all other cases of separation – that is, where the flow downstream of the separation region must adjust not only to the imposed pressure but also to a definite direction. An example of such separation is the case of an oblique (or normal) shock striking the boundary layer and calling for certain pressure and directional adjustment.

A RSS has been analyzed by Gadd (1953), however the various assumptions used and certain discrepancies in the experimental data (from which the semi-empirical information was obtained) throw some doubt in his results.

A comprehensive compilation of available turbulent flow separation data for over expanded supersonic nozzles is presented with a discussion of correlation techniques and prediction methods by Morrisette and Goldberg (1978). Data are grouped by nozzle types: conical, contoured and two dimensional wedge. Correlation of conical nozzle separation is shown to be independent of nozzle divergence half angle above about $9^\circ$ whereas the contoured nozzle data follow different correlation curve. Zero pressure gradient prediction techniques are shown to predict adequately the higher divergence angle conical separation data, and an empirical equation is given for the contoured nozzle data correlation. Flow conditions for which the correlations are invalid are discussed and bounded.
For vehicles such as proposed hypersonic research aircraft, which have after bodies, that act as the nozzle separation location will affect lift and stability characteristics as well as thrust. Obviously knowledge of the point of separation is essential for performance prediction.

During cold-flow subscale tests for the J-2S engine in the early 1970s, (Frey 1998) a different kind of separated nozzle flow at strongly overexpanded conditions was observed, which had not been known yet. In this flow regime, which only occurred at pressure ratios corresponding to power levels between 34 and 54%, the pressure downstream of the separation point shows an irregular behavior and reaches values even above the ambient pressure. This is attributed to a reattachment of the separated flow to the nozzle wall, inducing shocks and expansion waves, which result in wall pressure peaks with values above ambient pressures. Because of the very short separated region, this flow regime is called restricted-shock separation. A phenomenological sketch of the flowfield and the corresponding wall pressure is shown in Figure 2.6. The physical phenomenon of flow separation can be divided into two simple phenomena. The first is the turbulent boundary layer separation from the nozzle wall, which is characterized by the ratio of the nozzle wall pressure just after the separation, $p_\text{w}$, to the nozzle wall pressure just before separation, $P_1$. This pressure ratio is referred to as the critical pressure ratio $P_\text{cr} = P_1/P_p = P_1/P_2$. The second phenomenon is connected with the flow in the separated zone, which is characterized by a minor pressure gradient along the wall. The analysis of model experimental data on separated turbulent supersonic flows shows that the pressure ratio $P_\text{cr}$ is equal for separated flows before an obstacle, and for separated nozzle flows. For turbulent flows $P_\text{cr}$ shows a slight dependence on Reynolds number, but depends strongly on Mach number. Furthermore, investigations
of separated flows in rocket engine nozzles showed that $P_c$ is also a function of wall temperature, gas composition, and nozzle wall roughness.

Figure 2.6 Restricted-shock Separation in Overexpanded Rocket nozzles, Wall Pressure Profile, and Phenomenology: ——, Compression Waves/Shock; ......, Expansion Waves; and ----, Boundary/Shear-Layer Edge

The observation of reattachment flow in the J-2S subscale nozzle was confirmed by numerical simulations by Chen et al in 1994. In addition, their calculations revealed a trapped vortex behind the central normal shock. Recent cold-flow tests performed by Mattsson et al (1998) showed a similar behavior for a subscale nozzle within a certain pressure range. Together with
the J-2S subscale test, these are the only publically reported cases of restricted shock separation.

Generic methods for boundary layer separation cannot capture the entirety of events inside a nozzle. A theoretical model proposed by Romine (1998) fills this gap. For shocks with moderate Mach number (less than 2.25), Romine postulates that the jet flow emerging from the shock is above ambient pressure and adjusts to the ambient pressure via a gradual under expansion. The magnitude of the under expansion is equal to that of the overall over expansion. It is important to note that this argument applies in the vicinity of the centerline of the nozzle, where the shock is normal, and not on the walls. On the walls, Romine postulates that flow adjusts to the ambient pressure almost immediately past the shock. The under expansion is evident in the computational Mach number contours of Hunter (1998) although he did not mention it explicitly.

Potential origins for aerodynamic side loads are 1) a globally asymmetric separation line 2) pressure pulsations at the separation location and downstream 3) an aeroelastic coupling 4) a transition of separation pattern in thrust optimized or parabolic nozzles and 5) external flow instabilities (also called buffeting).

A review of different models, which have been developed based on the origins just listed, is given by Frey and Hagemann (1998, 2000). Although most of the models were able to predict the side load level for one certain nozzle contour type, none of them were able to give correct predictions for a greater variety of different nozzle types. This indicates that the magnitude of side-loads strongly depends on the nozzle contour and that this contour type has to be included into new models.
Side loads have been observed both in subscale and full-scale rocket nozzles during transient operations like startup or shutdown and during stationary operation with separated flow inside the nozzle. The side loads, which act in a direction perpendicular to the main thrust direction, represent a severe design constraint for new rocket engine concepts. Side loads in rocket nozzles can have different origins, and hence different models have been developed for the predictions.

Uncontrolled flow separation in nozzles of rocket engines is not desired because it can lead to dangerous lateral forces. Different origins for side loads were identified in the past. Meanwhile, it is proven that in thrust optimized or parabolic nozzles, a major side load occurs as a result of the transition of separation pattern from free shock separation to restricted shock separation and vice-versa. Reasons for the transition between the separation patterns are discussed in detail by Hagemann et al. (1998) and the cap-shock pattern which is identified to be the cause of this transition, is closely analyzed. It turns out that this pattern can be interpreted as an inverse Mach reflection of the internal shock at the nozzle axis. To prove the transition effect as main side load driver, a subscale test campaign has been performed by Hagemann et al. (2000). Two different nozzle contours a thrust optimized and a truncated ideal nozzle with equal performance data were also tested by Hagemann et al. (2002). Highest side loads were measured in the thrust optimized nozzle, when the separation pattern changes from free to restricted shock separation. Side loads measured in the truncated ideal nozzle were only about one-third as high as in the thrust optimized nozzle.

For lower pressure ratios $P_c/P_a$ when the flow is strongly over expanded and separates inside the nozzle a cap shock pattern may occur inside the nozzle. In that case the cap shock pattern induces a radial
momentum towards the wall, which might promote a reattachment of the separated flow and hence results in the FSS → RSS transition. Figure 2.7 illustrates this momentum balance. Thus, it was concluded by Hagemann et al (2003) that transition from FSS to RSS and vice versa only depended on the type of nozzle contours, and not on the scale or the combustion gas properties.

As mentioned by Frey and Hagemann (2000), the cap-shock pattern is only observed in the plume of thrust-optimized or parabolic rocket nozzles with a highly two-dimensional flowfield structure in the nozzle. The strong expansion along the circular arc contour in the nozzle throat with high initial angles up to $38^\circ$ and the subsequent recompressing contour result in a high-Mach-number flowfield along the centerline, limited by an internal shock. This internal shock does not exist in truncated ideal nozzles, where the cap-shock pattern has never been observed. In conical nozzles a weak internal shock exists, but it intersects the centerline shortly downstream of the throat, hence preventing the formation of a high-Mach-number flow near the axis. This explains why the cap-shock pattern has never been observed in conical nozzles either. As a consequence, it can be supposed that the internal shock is responsible for the cap-shock structure.

**Figure 2.7 Momentum Balance Across Cap-Shock System**

and Ciezki (2003) etc. further added vital flow information that lead to the knowledge of side load origin. One significant information was provided by Frey and Hagemann (1998 and 2000) who found out from their computational studies that the normal shock (at the triple point) and the separation shock location (on the nozzle wall) moved downstream towards the nozzle exit at different rates at different power levels. This irregular movement of the shock locations helped to explain that certain nozzle operating conditions can result in an imbalance in momentum flow passing through the separation and reflected shocks initiating flow reattachment downstream of separation resulting in RSS condition.

An experimental investigation has specifically been carried out by Verma et al. (2005) to study surface flow details during the shut down phase in a thrust optimized parabolic (TOP) rocket nozzle with Goertler vortex formation. The entire range of NPRs exhibiting RSS condition are accompanied by Goertler vortex formation over the entire nozzle circumference during which both the separation and reattachment shocks show fluctuating characteristics with varying intensity at different NPRs. This fluctuating nature of shocks leads to the origin of several side-load peaks during shut down sequence. Further, the length of the region between point of incipient separation and physical separation seems to have a strong influence in initiating reattachment downstream of separation which in turn seems to be strongly influenced by the cross-over axial locations of both the normal and separation shocks.

The most potential origin of aerodynamic side loads are, as suggested by Deck and Nguyen (2004), the random pressure pulsation of the separation line and an asymmetric separation condition. Wave and Coffey (1973) and Terhardt et al. (1999) identified another type of side load behaviour
that originates as a result of transition of flow condition inside from FSS to RSS and vice-versa with change in NPR. After studying the structure of exhaust flow from overexpanded rocket nozzles extensively, Terhardt et al (1999) argued that this transition process was typical of TOP nozzles where the strong overexpansion along the circular arc contour in the nozzle throat, with high initial angles up to 38 degrees and the subsequent recompression contour results in a high Mach number flow field along the centerline, limited by an internal shock. As a result at certain NPR the momentum balance of flow passing the separation shock and the reflected shock (originating from the triple point of internal shock and Mach disk) determines the transition phase in such nozzles where in the separation front suddenly jumps downstream leading to a side-load condition. It becomes, therefore, mandatory to study this transition phase in such nozzles in more detail since understanding this side load process may help in lighter nozzle thruster masses and hence gain in payload.

Flow inside rocket nozzles, especially parabolic nozzles, provide a wide range of flow conditions varying from separated to reattached flows, jump in separation front etc that present challenging avenues of study related to unsteady nature of flow under such conditions. Many physical aspects of these complex flow inside rocket nozzles need to be addressed and understood. Past studies Kistler (1964), Verma and Koppenwalner (2002) on two dimensional and three-dimensional supersonic flows have shown the shock wave boundary layer interaction to be associated with unsteady behaviour which can generate severe pressure fluctuations. These dynamic wall pressure measurements clearly indicate that the separation shock shows a large scale streamwise oscillatory motion.

It is understood that the boundary layer separation in turbulent supersonic flows is not a stationary process. It seems to be triggered by the
major scales of turbulence and also influences the recirculation region downstream. A focused work dedicated to the investigation of the flow separation phenomena in rocket nozzles and corresponding side loads was described and exemplified by Östlund et al (2004).

The separated flow in a C-D nozzle is studied computationally for a nozzle of an exit to throat area ratio \( A_e/A_t \) of 1.5 and over a range of nozzle pressure ratios by Xiao et al (2007). Their computations use the Reynolds averages Navier–Stokes equations with a two equation \( k-\omega \) turbulence model to examine the flow physics of asymmetric separated flow in a symmetric nozzle. The experimental results from Papamoschou and Zill (2004) are used as benchmark to assess the computational results. Their result shows that the flow separates by the action of a lambda shock, followed by a succession of expansion and compression waves. For \( 1.5<NPR< 2.4 \), the computation reveals the possibility of asymmetric flow structure.

### 2.3 SHOCK PATTERNS IN THE EXHAUST PLUME

Different shock patterns can be observed in the plume of overexpanded nozzles, depending on the nozzle type (plane of axisymmetric) and contour (thrust optimized, truncated ideal, etc). A very interesting shock pattern occurs in the plume of axisymmetric thrust optimized or parabolic nozzles (e.g. Vulcain or Space Shuttle Main Engine (SSME)) at low pressure ratios \( P_e/P_a \); the cap shock pattern. Being the only one of all observed patterns that can substantially affect the wall pressure and hence influence thrust and side force behaviour, it is worth taking a closer look at this specific plume.

The internal shock does not exist in truncated ideal nozzles and the cap-shock pattern has never been observed (Terhardt et al 1999). In conical
nozzles a weak internal shock exists, but it intersects the centerline shortly downstream of the throat, hence preventing the formation of a high Mach number flow near the axis. This explains why the cap-shock pattern has never been observed in conical nozzles either.

![Figure 2.8 Shocks in the Plume of Overexpanded, Plane Two-Dimensional Nozzles: Regular Reflection at the Centerline (Left) and Mach Disk (Right)](image)

At the nozzle exit, the lower pressure of the exhaust gases is adapted to the higher ambient pressure by means of an oblique shock. A widespread opinion about the exhaust plume of overexpanded, attached nozzle flows is that there is either a regular reflection of this shock at the center line or a Mach reflection, resulting in a Mach disk. This is correct for the case of a two-dimensional plane nozzle producing a uniform exit flow as shown in Figure 2.8. Which one of these shock patterns will occur, mainly depends on the exit pressure ratio $p_e / p_a$ and exit Mach number $M_e$. Frey and Hagemann (1998 and 2000) wrote that, generally, a strong overexpansion and low exit Mach numbers promote the formation of a Mach disk, whereas regular reflection is reached for weaker overexpansion and higher exit Mach numbers.

In a similar, but axisymmetric form, these two shock patterns can be observed in conical and truncated ideal nozzles. However, a third shock
pattern can exist in thrust-optimized nozzles such as the Vulcain or SSME nozzle. Typical for these nozzles is a flow region near the centerline with highest Mach numbers, also commonly named Kernel, which is bounded by the internal shock. During the startup and shutdown, a cap-like shock pattern with a trapped vortex is visible in the plume of these nozzles. As soon as the chamber pressure exceeds a certain critical value, the cap shocks are converted into a Mach disk. At even higher chamber pressures, a regular shock reflection is reached. Figure 2.9 shows a phenomenological sketch of both the cap shock and the Mach disk. It is important to note, that the cap shocks are stationary and stable, if the chamber pressure is not increased above the critical value. So the cap shocks are not only a transient, but a stable, stationary flow regime for the full-flowing nozzle.

Cap shocks seem to occur only in the plume of thrust-optimized nozzles. They have never been observed in either conical or truncated ideal nozzles. This implies that the high Mach number flow near the centerline limited by the internal shock in thrust-optimized nozzles might be responsible for this special shock pattern. In fact, a triple point exists, where the internal shock, the small normal shock and the cone-shaped oblique shock meet, indicating a concentration between the internal shock and the existence of a cap shocks.

There are three major differences between cap shocks and a Mach disk:

1. Across the small normal shock of the cap-shock pattern (points d-e in Figure 2.9-(b)), the pressure rises to approximately ambient pressure, whereas downstream of a Mach disk, the pressure is always higher than the ambient.

2. A cap-shock pattern always includes a cone-shaped shock that is inclined away from the center line (points c-f in Figure 2.9-
(b)), whereas the oblique shocks occurring upstream of the Mach disk are always inclined toward the axis.

3. A stable vortex is trapped downstream of the small normal shock in the cap shock pattern, whereas there is no recirculation behind a Mach disk.

The existence of a trapped vortex behind the small normal shock was also observed by Chen et al (1994). Up to now, experimental validation for its existence is missing; however, independent numerical simulations, which were performed by Chen et al (1994), and by Frey et al (1998) respectively, confirmed the existence of this vortex.

The aerodynamic study of supersonic jets exhausting from C-D nozzles is one of the most challenging problems in space and aeronautical applications. Various physical phenomena involved in this fluid dynamics problem are directly linked to the performance of jet engines. Though off design operations with either overexpanded or underexpanded exhaust flow induce performance losses, in many cases such regime cannot be avoided. The imperfect matching between the ambient pressure and the exit nozzle pressure leads to the formation of a complicated shock wave structure. The flow gradually adapts to the ambient conditions as it passes through the system of shock waves. For several decades, numerous investigations of the structure of supersonic jets have been undertaken but the subject is quite complicated and not yet clearly understood. The transition from regular to Mach reflection in a supersonic planar jet operating under overexpanded conditions has been studied numerically by Hadjadj et al (2004). The results demonstrate that a hystereses phenomenon is observed as the jet/ambient pressure ratio decreases and increases, causing a change in the angle of
incidence of the nozzle lip shock and as consequence, the transition from regular to Mach reflection and back.

(a) Mach disk

(b) Cap-Shock pattern with trapped vortex

Figure 2.9  Shock Patterns in the Exhaust Plume of a Thrust-Optimized Nozzle with Internal Shock

2.4  REATTACHMENT OF THE SEPARATED FLOW

In the preceding section shock patterns in the exhaust plume were analyzed for a full-flowing nozzle. If the chamber pressure is lowered, flow separation will occur, and the shock patterns will move into the nozzle. In thrust-optimized nozzles the cap-like form of the shocks will be mainly
preserved, whereas in truncated ideal and conical nozzles the Mach disk will occur down to quite low chamber pressures.

As stated before, a cone-shaped oblique shock, which is inclined away from the centerline, exists in thrust-optimized nozzles as a part of the cap-shock pattern. Because of its inclination, this shock deflects the flow away from the centerline in the radial direction. Under the circumstances of flow separation within the nozzle, this means that a momentum toward the nozzle wall is produced. If this momentum is greater than the momentum induced by the separation shock, which turns the boundary layer away from the wall toward the centerline, the flow will be forced to reattach to the wall. Consequently, the probability of RSS rises if the cone-shaped shock is long and the separation shock is short because, then, the momentum toward the wall is high.

Up to now, the observation of RSS in nozzles has been reported only twice in literature (Frey et al 2000). In both cases thrust optimized-subscale nozzles were used, which were fed with cold air. This led to the widespread but erroneous assumption that this type of flow separation could only occur in subscale and cold flow nozzles. No attention was paid to the nozzle design process. However, many cold-flow subscale experiments with both truncated ideal and conical nozzles were performed, where only FSS without reattachment occurred. This is not surprising because truncated ideal nozzles do not feature an internal shock, which could lead to a cap-shock pattern and thus to a radial momentum toward the wall that can make the flow reattach. In conical nozzles the various internal shock reflections at the centerline destroy the high Mach number flow, and the Kernel is closed shortly downstream of the throat. Therefore, neither cap-shock pattern nor RSS can occur in this nozzle family either. As a consequence, the conclusion
can be made that the RSS phenomenon is triggered by the nozzle contour and not by nozzle size or working gas.

A typical characteristic of RSS is the irregular wall pressure profile downstream of the reattachment point, which reaches values higher than the ambient pressure $P_a$ because the flow is attached and supersonic downstream of the reattachment point. In this flow shocks and expansion waves can hit the wall as indicated in Figure 2.6, resulting in an unsteady wall pressure.

In the two published cases where RSS was observed in experimental setup, the separation point is located further downstream than in the free shock case, as will also be shown next. The reason for this, however, is not a lower separation pressure ratio $p_s/p_p$, but a much lower value for the plateau pressure $p_p$. This result is not surprising because the flow downstream of the plateau point is supersonic.

Not only the switch over from cap-shock pattern to Mach disk, but also the appearance of RSS shows a strong hysteresis effect, which was observed in both the J-2S subscale experiments and in Chen et al.’s (1994) numerical simulation. According to their results, RSS is more likely to occur when the chamber pressure is lowered than when it is increased.

To understand this hysteresis effect, two points have to be analyzed. First, the position of the small normal shock mainly depends on the pressure ratio $p_c/p_a$ because it increases the pressure to the ambient value as was stated before. On the other hand, there is a hysteresis effect concerning the flow separation process itself, (Sunley and Ferriman 1964) which means that the separation pressure ratio $p_s/p_p$ is higher for increasing pressure ratio
If we assume fixed chamber and ambient pressure, but a slight deviation in the separation pressure ratio $p_s/p_p$ due to the hysteresis of flow separation, the position of the small normal shock will remain constant, whereas the separation point can vary, as shown in Figure 2.10. For $[\delta(p_c/p_a)/\delta t] > 0$, $x_s$ will be smaller than for $[\delta(p_c/p_a)/\delta t] < 0$, which also means that for a decreasing pressure ratio the cone-shaped shock that produces a momentum toward the wall is longer and the separation shock is shorter than in the case of decreasing pressure ratio. As a consequence, the net momentum toward the wall, which can cause reattachment, will be greater if $[\delta(p_c/p_a)/\delta t] < 0$ and the probability of restricted shock separation will be higher.

![Figure 2.10 Hysteresis of Flow Separation and its Influence on Reattachment: Increasing Chamber Pressure Produces FSS (Left), whereas Decreasing Chamber Pressure Results in RSS (Right)](image)

### 2.5 CRITERA FOR FLOW SEPARATION PREDICTION IN ROCKET NOZZLES

Table 2.1 gives the some separation criteria found in the literature.
Table 2.1 Separation Criteria Found in the Literature

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Author</th>
<th>Separation criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Arens and Spiegler (1963)</td>
<td>( \frac{P_s}{P_a} = \frac{P_c}{P_a} \left( \frac{(P_c/P_a)^{(\gamma-1)/\gamma} - (u_s^<em>/u_s)^2}{1 - (u_s^</em>/u_s)^2} \right)^{-\gamma/(\gamma-1)} )</td>
</tr>
<tr>
<td>2</td>
<td>Crocco-Probstain (1952)</td>
<td>( \frac{P_p - P_s}{\gamma P_s} \left[ \frac{k_{1s}}{M_s^2} + \frac{\gamma - 1}{2} (k_{1s} - 1) \right] = \sqrt{1 - \frac{k_{2p}^2}{k_{2s}^2}} \left( 1 - \frac{\delta^*}{\delta} - \frac{\delta^{**}}{\delta} \right) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( k_1 = \sqrt{1 - \frac{\delta^<em>}{\delta} - \frac{\delta^{**}}{\delta}} ) ( k_2 = \left( 1 - \frac{\delta^</em>}{\delta} - \frac{\delta^{**}}{\delta} \right)^{-1} )</td>
</tr>
<tr>
<td>3</td>
<td>Donaldson and Lange (1952)</td>
<td>( \frac{P_p}{P_s} = 1 + k_3 \frac{\gamma}{2} M_s^2 c_f ) where, ( k_3 ); constant determined from analysis of experimental data</td>
</tr>
<tr>
<td>4</td>
<td>Frey and Hagemann (2000)</td>
<td>( \frac{P_s}{P_p} = \left[ 1 + \frac{2\gamma}{\gamma + 1} (M_i^2 \sin^2(\beta) - 1) \right]^{-1} ) with ( \beta = 44.5^\circ - 4.7^\circ M_s )</td>
</tr>
<tr>
<td>5</td>
<td>Kalt and Badal (1965)</td>
<td>( \frac{P_s}{P_a} = \frac{2}{3} \left( \frac{P_c}{P_a} \right)^{-1} ) ( P_a )</td>
</tr>
<tr>
<td>6</td>
<td>Kudryavtsev (1975)</td>
<td>( \frac{P_a}{P_p} = 1 + \left( \frac{0.192}{\sin \alpha} - 0.7 \right) \left( 1 - \frac{M_s}{M_{\text{design}}} \right) ) proposed for air in conical nozzles, ( \alpha &lt; 15^\circ )</td>
</tr>
<tr>
<td>7</td>
<td>Lawrence and Weynand (1967)</td>
<td>( \frac{M_p}{M_s} = 0.8 )</td>
</tr>
</tbody>
</table>
Table 2.1 (Continued)

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Author</th>
<th>Separation criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Mager (1956)</td>
<td>$M_s^2 = K M_i^2$, where $K = 0.55$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_s = 1 + \frac{\gamma M_i^2}{2} (1 - K) $</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1 + \left[\frac{\gamma - 1}{2}\right]M_i^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_p = 1 + 0.328 \frac{\gamma M_s^2 \theta_p}{1 + \left[\frac{\gamma - 1}{2}\right]M_s^2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>where deflection angle [\theta_p \approx \left(\sqrt{M_i^2 - 1/\gamma M_i^2}\right)[(P_p/P_i) - 1]\</td>
</tr>
<tr>
<td>9</td>
<td>Östlund (1999)</td>
<td>$\beta = -3.764^\circ M_s + 42.878^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta = 1.678^\circ M_s + 9.347^\circ$</td>
</tr>
<tr>
<td>10</td>
<td>Schilling (1962)</td>
<td>$P_s = 0.582 \left(\frac{P_c}{P_a}\right)^{0.195}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>; suggested for short bell nozzles</td>
</tr>
<tr>
<td>11</td>
<td>Schmucker (1974)</td>
<td>$P_s = (1.88 * M_s - 1)^{0.64}$</td>
</tr>
<tr>
<td>12</td>
<td>Stark (2005)</td>
<td>$P_s = \frac{\pi}{3 * M_s}$</td>
</tr>
<tr>
<td>13</td>
<td>Summerfield (1954)</td>
<td>$P_s = 0.35 \sim 0.4$</td>
</tr>
<tr>
<td>14</td>
<td>Tagirov (1985)</td>
<td>$P_s = \left(\frac{\sqrt{A_1^2 + A_2^2 \left(1 + \frac{\gamma - 1}{2} M_s^2 \right)} - A_1}{A_2}\right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_1 = \frac{\gamma + 1}{4} (A_3 - 1) * M_s^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_2 = 1 + \frac{\gamma - 1}{2} * A_3 * M_s^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_3 = \left(\frac{M_p}{M_s}\right)^2$</td>
</tr>
<tr>
<td>15</td>
<td>Zukoski (1967)</td>
<td>$P_s = \frac{2}{M_s + 2}$ ; Forward-Facing step</td>
</tr>
</tbody>
</table>
2.5.1 Data Correlation

Schilling (1962) collected a total of 409 data points from the industry as of 1962. These were grouped in sets for six conical nozzles, short contoured nozzles with exist half-angles from 7 to 13.5°, and long contoured nozzles with exit half-angles from 4 to 6°. The data were initially shown in the usual form of $P_s/P_a$ vs $P_c/P_a$; however, the data scatter was very large, forcing Schilling to look for other correlation parameters. Green (1953) gave a correlation of various worker’s results, largely achieved by swamping the previous scatter in an independent hyperbolic function, where the term $(P_a - P_s)/P_c$ vs $P_c/P_a$ was suggested based on that form being in the thrust equation. Schilling (1962) found there to be a dramatic collapse of all of the data by using Green’s parameter.

A focused study on the overall jet shock structure, which is responsible for the separation shock, that yielded a direct calculation of the flow separation pressure and location is presented by Romine (1998). The author observed that the data trends of the conical nozzles and the long contoured nozzles are similar. The magnitudes of Green’s parameter from the 5-, 10-, and 15-deg conical nozzles and the long contoured nozzles vary by only ±3.5% from the short contoured nozzles. This illustrates the minimal effects that nozzle half-angle and boundary layer characteristics have on the flow separation phenomena. Contoured nozzles will have a similarly small effect as the nozzle lip curves towards the separated jet and restricts entrainment inflow inside the nozzle, thus altering the local ambient pressure downstream of the separation point.
2.5.2 Correlation Parameters

A number of correlation have been suggested by various investigators. Early workers in the field used

\[
\frac{P_t}{P_a} = f\left(\frac{A_t}{A^*}\right) \tag{2.9}
\]

and

\[
\frac{P_s}{P_t} = f\left(\frac{P_t}{P_a}\right) \tag{2.10}
\]

Green (1953) suggested a modification of Equation (2.10):

\[
\frac{P_a}{P_t} - \frac{P_s}{P_t} = f\left(\frac{P_a}{P_t}\right) \tag{2.11}
\]

which through the use of the redundant parameter $P_a/P_t$, resulted in apparent suppression of scatter. However, this reduction in scatter was actually accomplished through a conversion to a higher reference value.

Investigations by Ahlberg et al (1961) have suggested

\[
\frac{P_a}{P_s} = f\left(\frac{P_t}{P_a}\right) \tag{2.12}
\]

Although the first of these Equation (2.11) has the merit of being written in more convenient terms for the designer, with $P_t$ being a function of engine design and $P_a$ a function of flight environment, the following equation as used by Lawrence (1967) and (1968) is more closely related to the local conditions at boundary-layer separation and also used in the technical paper by Morisette and Goldberg (1978).
\[
\frac{P_a}{P_s} = f(M_s)
\]  
(2.13)

Lawrence (1968) achieved better correlation with Equation (2.13) than with Equation (2.12) and suggested it to be the preferable correlation. The two equations are actually related through the isentropic relation

\[
M_s = f\left(\frac{P_i}{P_t}, \gamma\right)
\]  
(2.14)

with \( \gamma \) having only a small effect at lower Mach numbers where data are available for a range of \( \gamma \).

2.5.3 Free Shock Separation Criteria

The theoretical prediction of FSS is the case, which has been most extensively studied in the past since, historically, almost all experiments have been performed in conical and truncated ideal nozzle contours only featuring this separation pattern. Experimental data have been used to develop a number of empirical and semi-empirical criteria in order to give the nozzle designer a prediction tool for the separation point, although knowing that in reality there is no exact point of separation because it fluctuates between two extreme locations. But even today, an exact prediction cannot be guaranteed because of the wide spectrum of parameters involved in the boundary layer – shock interaction such as nozzle contour, gas properties, wall temperature, wall configuration and roughness.

Probably the most classical and simple criteria for FSS purely derived from nozzle testing is the one given by Summerfield et al (1954) which is based on extensive studies on the separation phenomenon in conical nozzles in the late 1940’s:
\[ \frac{P_i}{P_a} \approx 0.4 \quad (2.15) \]

A first approach to include the Mach number influence was published by Arens and Spiegler (1963). However, the major formula derived turned out to be too complex for engineering application. Based on experiments with conical and truncated ideal nozzles, Schilling derived in 1962 (Östlund 2002) a simple expression accounting for the increase of separation pressure ratio \( \frac{P_i}{P_a} \) with increasing Mach number,

\[ \frac{P_i}{P_a} = k_1 \left( \frac{P_c}{P_a} \right)^{k_2} \quad (2.16) \]

with \( k_1 = 0.582 \), and \( k_2 = -0.195 \) for contoured nozzles, and \( k_1 = 0.541 \), and \( k_2 = -0.136 \) for conical nozzles. In 1965, based on Schilling’s expression Kalt and Badal chose \( k_1 = 2/3 \), and \( k_2 = -0.2 \) for a better agreement with their experimental results. NASA adopted a correlation similar to the one of Schilling for truncated contoured nozzles as a state of the art indication at the mid 1970’s (Östlund 2002).

The first semi-empirical criterion was derived by Crocco and Probstein (Östlund 2002), which is based on a simplified boundary layer integral approach. The criterion accounts for the properties of the boundary layer, the gas and the inviscid Mach number at the onset of separation. The criterion showed a variation of agreement with experimental data. NASA therefore recommended a 20% margin to the predicted separation point. At the same time Schmucker (1974) derived the empirical criterion

\[ \frac{P_i}{P_a} = \left( 1.88 M_i - 1 \right)^{-0.64} \quad (2.17) \]

It showed a similar agreement with experimental data as Crocco and Probstein (Östlund 2002), and is therefore still widely used.
A number of predictions and correlations of the separation pressure ratio for turbulent boundary layers are shown in Figure 2.11. Also shown is the pressure ratio across a normal shock wave. The rather poor agreement of the above criteria is observed. This explains the NASA advice of a 20% margin and also points out the necessity of new and more reliable criteria. One of the major reasons for the rather poor agreement is that all above criteria include two separate mechanisms involved in the pressure rise of the flow in one single expression. This fact was realised already in the 1960’s by Arens and Spiegler (1963), and by Lawrence (1967). The latter suggested that the pressure recovery $P_i/P_a$ should be subdivided into two parts, one part for the critical pressure rise, $P_i/P_p$, over the separation shock and a second for the pressure rise in the recirculation zone, $P_p/P_a$.

The pressure rise $P_i$ to $P_p$ is caused by shock-wave boundary layer interaction, as described in section 2.1.3. This is a general mechanism, not
restricted to nozzle flow separation, which has been extensively studied. As an example, Zukoski (1967) found the following simple relation to be in good agreement with experimental results for high Reynolds numbers:

\[
P_{i}/P_p = (1 + 0.5M_{i})^{-1}
\]  

for the Mach number range of \( M_i = 1.4 \text{--} 6.0 \) and \( R_{\text{e}a} > 10^5 \). According to the author, this correlation also agrees with the plateau pressure values measured in overexpanded conical nozzles in the Mach number range \( M_i = 2.0 \text{--} 5.5 \).

The drawback of the Zukoski (1967) criterion is that it does not include the dependency of the specific heat ratio observed in experimental data and should thus only be used for gas flow with \( \gamma = 1.4 \), since the experiments were performed with air. A first attempt to account for the specific heat ratio dependency by using oblique shock relations was proposed by Summerfield et al (1954). From experimental data they found that the flow deflection angle \( \theta \) of the separated flow was nearly constant \( \theta \approx 15^\circ \) for the nozzles tested. With this value and the use of oblique shock theory the pressure rise for different gas mixtures can thus be calculated. This observation has also been confirmed in later synthesis of nozzle flow separation data, from a number of experiments performed with both hot and cold gas flows. However, the data also indicate that the Summerfield criterion with a constant \( \theta \) value is too simple. In fact the data rather indicate a linear dependence of the Mach number on both the deflection angle \( \theta \) and the shock angle \( \beta \) itself. Based on this and data from the VOLVO subscale tests (Östlund 2004) proposed an empirical criterion based on oblique shock relations:
\[
\frac{P_i}{P_p} = \left\{ 1 + \gamma M_i^2 \sin^2(\beta) \left[ 1 - \frac{\tan(\beta - \theta)}{\tan(\beta)} \right] \right\}^{-1}
\] (2.19)

with \( \beta = -3.764 M_i + 42.878 \) (\(^{\circ}\)) and \( \theta = 1.678 M_i + 9.347 \) (\(^{\circ}\)) for the Mach number range \( 2.5 \leq M_i \leq 4.5 \). Östlund (2002) used linear expressions for both \( \theta \) and \( \beta \) in the correlation since he found that a criterion only based on the shock angle \( \beta \) (and \( \theta \) calculated with the \( \theta - \beta - M \) relation) experiences a minimum already for a modest extrapolation above \( M_i = 4.5 \). Frey and Hagemann (2000) have proposed a similar criterion based only on the shock angle \( \beta \) as:

\[
\frac{P_i}{P_p} = \left[ 1 + \frac{2\gamma}{\gamma + 1} (M_i^2 \sin^2(\beta) - 1) \right]^{-1}
\] (2.20)

with \( \beta = -4.7 M_i + 44.5 \) (\(^{\circ}\)) for the Mach number range \( 2.5 \leq M_i \leq 4.5 \), which produces a similar result as the criterion by Östlund (Equation 2.19 reduces to Equation 2.20 with the use of the \( \theta - \beta - M \) relation). However, it does not give the correct trend of \( P_i/P_p \) for higher Mach numbers. At \( M \approx 4.8 \) the function has a minimum and \( P_i/P_p \) suddenly increases with the Mach number.

To be able to describe the pressure distribution throughout the nozzle a criterion for the pressure ratio \( P_p/P_a \) is also needed. The only reported models for the recirculating flow in the literature are the ones by Kudryavtsev and the one by Malik and Tagirov (Östlund 2002), both for conical nozzles operated with air. The model by Kudryavtsev is purely
empirical. He found that in conical nozzles with a half angle $\alpha < 15^\circ$ the pressure rise in the recirculating zone could be approximated as:

$$\frac{P_p}{P_a} = \left[ 1 + \left( \frac{0.192}{\sin \alpha} - 0.7 \right) \left( 1 - \frac{M_i}{M_a} \right) \right]^{-1} \tag{2.21}$$

where $M_a$ is the average exit Mach number defined by the nozzle expansion area ratio $\varepsilon$. Whereas, in conical nozzles with a half angle $\alpha > 15^\circ$ he found that the pressure rise $P_p / P_a \approx 1$, i.e. independent of the Mach number.

Because shock induced separation of the boundary layer is of particular interest, a number of analytical and experimental studies have been concerned with this phenomenon. The problem of shock induced turbulent boundary layer separation was analyzed by Reshotko and Tucker (1955) using an approximate method. Reshotko and Tucker (1955) developed a semi analytical prediction method for the onset of turbulent separation. In this method, empirical incompressible separation criteria are used in conjunction with a compressibility transformation to predict separation for the compressible case. A relationship between the local Mach number and velocity profile (form factor) provides a means of determining a Mach number ratio across the discontinuity for shock-induced separation. The above analysis can also be applied to the case of favourable or adverse pressure gradient ahead of the shock when the appropriate form factors are known. Qualitatively it is known that the form factors for adverse and favourable pressure gradients are respectively greater and less than the flat plate form factor. Thus for favourable pressure gradients ahead of the shock, a stronger shock is required, whereas for adverse pressure gradients ahead of the shock a weaker shock would separate the turbulent boundary layer. The form of the result suggests that the Mach number ratio across the shock is a characteristic
parameter for defining shock induced separation. The experimental data for forward facing steps and for wedges are well described by the curves of constant Mach number ratio (ratio of Mach number ahead of to that behind discontinuity) $M_{2}/M_{1} = 0.76$ and $M_{2}/M_{1} = 0.81$ respectively. Calculation of $P_{s}/P_{a}$ as a function of $M_{s}$ is shown in Figure 2.11 for this method using the Mach number ratio of 0.762 with $\gamma = 1.4$.

An investigation was conducted by Campbell and Farley (1960) to obtain nozzle performance data with relatively large scale models at pressure ratios as high as 120. Thrust ratios for all nozzle configurations are presented over the range of pressure ratios attainable and extrapolated, when possible, to design pressure ratio and beyond. Design thrust ratios decreased with increasing nozzle divergence angle according to the trend predicted by the $(1 + \cos \alpha)/2$ parameter. Decreasing the nozzle divergence angle resulted in sizable increases in thrust ratio for a given surface area ratio (nozzle weight), particularly at low nozzle pressure ratios. A simplified method of thrust determination is developed that permits the calculation of nozzle thrust in the separated flow region from unseparated static pressure profiles. Separation data for all nozzle configurations tended to generalize at a Mach number ratio of about 0.76 or 0.77 for both hot and cold flow and for all NPRs for which separation data could be determined.

Arens and Spiegler (1963) utilized the assumption, that the pressure rise associated with separation must be sufficient to stagnate a characteristic velocity in the boundary layer $u^*$. This theory is also shown in Fig. 2.11 for a value of $u^*/u_s = 0.6$. Arens and Spiegler (1963) points out that this theory assumes the separation peak pressure to be equal to the ambient exhaust pressure and does not account for any compression associated with the mixing
region downstream of separation. The theories of Reshotko and Tucker (1955) and of Arens and Spiegler (1963) showed an increasing effect of specific heat ratio on the separation pressure ratio as Mach number increases.

A review of the older literature, and correlation of experimental results in a large variety of nozzles, is given by Morrisette and Goldberg (1978). Their primary conclusion is that zero pressure gradient separation predictors like the method of Reshotko and Tucker (1955) give reasonable predictions for nozzles with turbulent separation and large divergence angles. The ratio $P_s/P_a$ is a declining function of the shock Mach number $M_s$ and as a rule of thumb, is roughly 0.5 for $M_s \approx 2$ and 0.3 for $M_s \approx 4$. Nozzles with laminar separation exhibited higher separation pressure ratios. Separation in nozzles with low local wall angles, such as low divergence conical nozzles and contoured nozzles, deviated from the above predictions. The close proximity of the wall to the separating shear layer has been cited as a possible reason for the discrepancy.

### 2.5.4 Restricted Shock Separation Criteria

The prediction of RSS has only been addressed in the last few years; (Frey 1999 and Östlund 1999). The key point for the prediction of RSS is to predict the location where the transition from FSS to RSS takes place. The driving force for reattachment of the flow is when the radial momentum of the separated jet is directed towards the wall, which can occur with a cap-shock pattern, whereas no reattachment is possible if the momentum is directed towards the centre-line, which is always the case with a Mach disc. Thus, by quantifying the momentum balance of the jet, the transition point can be determined. On this basis Östlund and Bigert (1999) proposed a simple empirical criterion for the prediction of transition from FSS to RSS, which
relates the FSS-RSS transition to the axial position where the small normal shock at the centre-line coincides with the RSS separation front.

Frey and Hagemann (1999) have developed a more sophisticated and physical model. In this model the FSS shock system is always prevailing before a possible reattachment is defined. Based on numerical flow field data, the cap shock pattern is re-calculated by a shock-fitting technique. By calculating the momentum balance across the cap shock pattern and the corresponding direction of the jet downstream of the cap shock pattern, the driving force for reattachment is evaluated and the location where the transition takes place is determined.

Both models account for the sudden pressure drop of the plateau pressure and the subsequent jump of the separation point when the flow reattaches and the separated region becomes enclosed by supersonic flow. Due to the complexity of the flow downstream of the reattachment point, which is characterized by subsequent compression and expansion waves, no models for this pressure recovery process exist so far. Instead a constant value of the plateau pressure based on test data experience is often used. This value is kept until the RSS is transformed back into FSS and FSS criteria are applicable again. This transformation occurs either when the cap-shock is converted into the Mach disc or when the enclosed separation zone is opened up at the nozzle exit.

2.6 JET FLOW STRUCTURE

The boundary condition encountered by an expanding nozzle flow is a constant ambient pressure. The process of pressure adjustment produces a complicated shock structure in the separated jet that alternates from over
expansion to over compression, which is shown by the repeating shock diamonds or cells (Figure 2.12).

![Figure 2.12 Jet Flow Structures](image)

The first one or two cells usually contain a Mach disk across the axis terminating at a triple point. The initial triple point has an incident shock that originates at the separation point on the nozzle wall, a reflected shock, a normal shock or Mach disk, and a slipstream separating supersonic and subsonic flows aft of the triple point. Of primary importance is the first complete cell after the flow has separated. Subsequent cells are disrupted by shear layers at the boundary and slipstream and by vorticity gradients from the shocks; thus, their shock structure is not as sharply defined. The adverse pressure gradient necessary to separate the mainstream flow and the nozzle boundary layer from the wall is supplied by the separation shock. The shock originates at the separation point, but its strength and location are dictated by the overall adjustment process of the complete first cell, where all parts are controlled by the ambient pressure boundary condition.

Lawrence and Weynand (1968) pointed out that the pressure downstream of separation is a consequence of entrainment in the “dead air”
spaces between the high speed jet and the walls of the nozzle. In general, the wider the divergence angle of the nozzle and the shorter the length for entrainment (length of nozzle downstream of separation) will less that $P_2$ (pressure just downstream of separation) will differ from the atmospheric pressure. A study of separated flow in two dimensional axisymmetric nozzles having wall contours was investigated by Lawrence and Weynand (1968). Their experiments indicate that $P_2$ is only slightly different from $P_a$ for conical nozzles having divergence half angles of 15º or wider. However, the difference is significant for divergence half-angles less than 10º. They also found that for the conditions of most of their study, a Mach number ratio $M_2/M_1$ of 0.8 was appropriate.

The pressure rise across the separation shock determines the angle of the shock with respect to the walls. Lawrence and Weynand (1968) showed that if the divergence angle of the walls at the point of separation is narrow (10º), oblique (or conical) shocks span the stream and the flow remains an integral supersonic jet for some distance beyond separation. However if the divergence angle of the walls at separation is wide (25º), the oblique separation shocks will be bridged by a normal shock.

2.7 THE REGIME OF FLOW SEPARATION

Figure 2.13 shows the various ways in which a gas flowing down a nozzle reacts in order to attain the prevailing back pressure conditions. Curve A on the plot shows the relationship between the overall pressure ratio, $P_a/P_c$ and the nozzle area ratio $A_e/A_i$ for correct isentropic expansion conditions. If for a given nozzle the back or ambient pressure is reduced below the correct expansion value, the change is met by the formation of Prandtl-Meyer expansion waves at the nozzle exit. Conversely, if the back
pressure is increased, compression waves at the nozzle exit raise the gas pressure to meet this new condition. There is, however, a limit to the compression shock strength which can be maintained at the nozzle exit, and as the back pressure increases sufficiently to cause this limit to be exceeded, the shock moves back into the nozzle and the gas downstream separates from the nozzle wall to form a free jet.

![Figure 2.13 The Regime of Separation](image-url)
For example, considering a nozzle of area ratio 10, correct expansion at the nozzle exit is obtained with a pressure ratio \( P_a/P_e \) of 0.0072. If this ratio is increased, to say, 0.13, it is found in practice that isentropic expansion will only take place down the nozzle to an area ratio of about 2.6, where oblique shocks raise the pressure ratio to the nozzle exit value on the separation curve B. Further flow down the nozzle will be separated from the nozzle wall. It will be noted that the chosen overall pressure ratio of 0.13 can be attained by a normal shock wave at the nozzle exit, but this condition is not normally encountered in the type of nozzle used with rocket combustion chambers.

2.8 COMPUTATIONAL STUDIES ON SUPersonic FLOW THROUGH A CONTOUR NOZZLE

Moretti and Abet (1966) in their work state that the equations of motion governing steady, inviscid flow are of mixed type that is hyperbolic in the supersonic region and elliptic in the subsonic region. These mathematical difficulties may be removed by using the so called time dependent method, where the governing equations become hyperbolic everywhere. The steady state solution may be obtained as the asymptotic solution for large time. This technique has been used to compute C-D nozzle flows. While the results of the proceeding calculations are for the most part good, the computation times are rather large. Considerable efforts have been devoted by many authors to the understanding of these problems.

Allman and Hoffman (1981) reported that, it appears that one can distinguish between two categories of flow field. (a) Flow fields where the domain of influence of the sonic line in the hyperbolic region does not include portions of the body surface. (b) Flow fields where portion of the body
surface falls within the aforementioned domain of influence. To a first approximation the problem of categories (a) can be analyzed by methods on the assumption that the existence of a sonic line is of secondary importance, while the problem of category (b) show an important influence of the shape of the body in the sonic region, an analogous distinction applies when more accurate methods of analysis (numerical methods are developed). This had led either to poor accuracy in the flow field which resulted in the prediction of inaccurate nozzle discharge coefficients or to complex numerical procedures which requires substantial amount of computer time. In order to eliminate these limitations, it is necessary to stabilize the numerical procedure in the vicinity of the shocks and to establish realistic flow conditions at the entrance.

Back and Cuffel (1966) on their previous investigation of gas flow through conical nozzles have shown deviation from one dimensional isentropic flow, as indicated by measured wall static pressures. These deviation results from radial velocity components caused by the taper and the curvature of the nozzle. Local velocity measurements in the throat region have indicated the two dimensionality of the flow. Exact solution of two dimensional isentropic flow equations are virtually non existent for the flow regimes throughout the supersonic nozzle.

Back et al (1972) presented some procedures for designing maximum thrust nozzle contours are based on the calculus of the variations. A major drawback to this procedure is that if the nozzle configuration or the gas dynamic model is changed, then the entire optimization analysis and corresponding computer program must be reworked. Present investigation is to develop an efficient method for the design of maximum thrust nozzle contours by direct optimization methods. In the present investigation the nozzle configuration is that of a conventional contour nozzle specified by a
second degree polynomial. The gas dynamic model assumes the isentropic flow of a perfect gas, and the design constraint is that the nozzle has specified length.

Back et al (1969) describes that, for the maximum thrust, the contour is given by the second degree polynomial \( y(x) = a + bx + cx^2 \). Specifying the three coefficient \( a, b \) and \( c \) uniquely defines the nozzle contour. These coefficients are determined by specifying the attachment angle \( \theta_a \), the exit radius \( y_e \), and by requiring that the polynomial contour attaches continuously with continuous wall slope. To the circular arc initial expansion contour consequently \( \theta_a \) and \( y_e \) determines a unique nozzle configuration. The optimization procedure searches through the allowable ranges of these two parameters to determine the particular set of values that yields the maximum nozzle thrust.

Back et al (1965) states that the general philosophy of a direct optimization method is to begin with an arbitrary starting point (base point). The optimization method then selects a search direction and a step size for moving the base point towards the optimum. The number of function evaluation required for each base point move and the number of the base point determines the efficiency of the method since a functional evaluation requires much more computational effort than the optimization logic.

Bohachevsky and Rubin (1966) reported that the problem of describing the shock layer in front of a blunt body has received considerable attention. The primary difficulty arises because of an incomplete knowledge of the boundary condition on the shock layer, location, and the strength of the bow shock and the location of the sonic line are not known. Several methods have been proposed for circumventing these difficulties. However most of these techniques heavily depend on assumption of axisymmetric flow or a smooth body contour.
Carpenter and Casper (1999) describes that the inverse method begins with an assumed shock shape and determines the corresponding body. Since in the inverse problem the body shape obtained is extremely sensitive to the assumed shock shape, the procedure is unsuitable for contours with sharp corners. For the same reasons the method is difficult to apply to unsymmetrical flow fields, since the required shock shape is complex. So far the inverse technique has been applied successfully to asymmetric flow fields when the required shock shape is complex.

Carpenter and Casper (1999) states that the difficulty of the unknown boundary condition can be avoided by solving the unsteady problem for which the governing equations are hyperbolic. The body is accelerated from rest and the asymptotic (for large time) behavior of the unsteady flow fields is determined. This procedure originally was used by Godunov and co-workers. However, the application of their elaborate differencing scheme to three dimensional flows appears impractical.

Hence in the present work the method suggested by Bohachevsky and Rubin (1966) for the computation of supersonic flows with detached shock wave is followed. In the computation process a generalization of the finite-difference scheme first proposed by Lax was employed. The simplicity of the scheme was a major factor in the success of the present work.

The solution of the difference equation is straightforward. The values of the flow quantities are advanced in time using the finite difference analogues the ensuing simplicity of the program logic makes it relatively easy to apply the method to flows about complex geometries and to extend the method to include non-equilibrium effects.

The magnitude of the time step $\Delta t$ used in the advance is controlled by the stability requirements. The Courant-Friedrichs-Lewy (CFL) stability
conditions apply to linear hyperbolic partial differential equations. The computation reported by Bohachevsky and Rubin (1966) suggest that these conditions are sufficient in his investigation of the one dimensional flow. Lax points out that they appear necessary and to obtain best results, \( \Delta t \) should have the largest possible value which permits a stable computation.

For the success of the aforementioned problem the finite difference scheme, which is to be used must have the following two properties. (1) It must remain uniformly valid across any discontinuity (i.e.) It should prevent the infinite derivative at the shock. (2) It must satisfy the conditions across any shocks that develop inside the flow field.

After the pioneering work of Chen et al (1991) whose calculations revealed a trapped vortex behind the central normal shock, there has been considerable interest in numerical approaches.

Computational studies of two-dimensional over expanded nozzles by Wilmoth and Leawitt (1987) and by Hamed and Vogiatzis (1997 and 1998) assessed the accuracy of turbulence models for predicting the flow field and thrust performance. The works agree on the basic structure of the separation shock, which consists of the incident shock, Mach stem (normal shock), and reflected shock. Thrust predictions were in good agreement with experiments except at pressure ratios associated with separated flow. A combined experimental and computational work by Hunter (1998) offers one of the most comprehensive treatments of this flow. His experimental results on a two dimensional nozzle with \( A_e / A_r = 1.8 \) showed two distinct separation regimes: three dimensional separation with partial reattachment for nozzle pressure ratio NPR \( \leq 1.8 \) and fully detached two dimensional separation for NPR \( \geq 2.0 \). Hunter (1998) claims that this transition was not the result of markedly different onset conditions or stronger shock-boundary layer interaction, but instead came about through the natural tendency of an
overexpanded nozzle flow to detach and reach a more efficient thermodynamic balance. As a result, the thrust of the separated case is much higher than that given by inviscid analysis. Notable in Hunter’s (1998) experiments and simulations was the much higher NPR required to situate the shock at a given area ratio compared to the inviscid prediction. For example, to place the normal shock just outside the nozzle exit a NPR of 3.4 was required, versus NPR 1.8 predicted in the inviscid case.

At first glance, it is often surprising that nozzle separation is accompanied by an increase in thrust efficiency over the attached case, but it should not be surprising. A major effect of boundary-layer separation is that it redefines the “effective” geometry of a fluid dynamic system by displacing the inviscid flow. In a CD nozzle, separation moves the jet detachment point upstream, causing a change in the effective nozzle geometry to one that is shorter and has a lower expansion ratio. By acting like a natural adjustment mechanism, separation allows an overexpanded nozzle flow to reach a more efficient thermodynamic balance. For a given NPR, this alleviates overexpansion and improves thrust efficiency.

Numerical simulations of a truncated ideal, two parabolic, and three dual bell nozzles were performed by Gross et al (2004) for different pressure ratios. The simulations by German Aerospace Centre (DLR) indicated that FSS is the only separation mechanism possible for the truncated ideal contour nozzle and this is attributed to the absence of a pronounced internal shock wave.

Computational simulation of shock induced separated turbulent flow is a challenging task. Turbulence modeling remains a major factor for both the accuracy and efficiency of the computations. Hamed and Vogiatzis (1997 and 1998) computed the flow in an overexpanded C-D nozzle and compared the results using several turbulence models. Their results indicate
significant spread of the computed shock location and pressure distribution by the different algebraic and two-equation turbulence models.

For rocket nozzles, Chen et al (1994) examined the flow structures of the start up and shut down processes using a Navier-Stokes solver. The configuration they studied was a subscale nozzle of a J-25 rocket engine (that is a precursor of the U S Space shuttle main engine). Later Nasuti and Onofri (1998) simulated the flow transients during the start up of the Vulcain nozzle (designed for the European rocket Ariane V). Both inviscid and viscous laminar flow models were considered, which permitted the demonstration that very peculiar flow field configurations, which are characterized by two main vertical regions, may occur during the start up of the nozzle. The first region is due to the boundary layer separation from the wall, whereas the second has an inviscid origin. The major properties of the transient flows developing in a supersonic nozzle driven by a shock tube have been investigated by Mouronval (2005) using a fifth order weighted essentially non-oscillatory (WENO) scheme.

Hunter (1998) used a NASA Langley Reynolds-averaged Navier-Stokes computational fluid dynamics code (PAB3D) to calculate such flows. They used two-equation $k-\varepsilon$ turbulence closure with a nonlinear Algebraic Reynolds Stress (ARS) model. Their computational results in general agree well with experimental data but the method is still relatively complex and presents numerical stiffness, which may require very fine grid resolution and much computation. Xiao et al (2005) used $k-\omega$ turbulence model with and without the inclusion of a lag model to study the turbulent separated nozzle flows. They found that inclusion of the lag model significantly improves the results where there is strong shock induced separation. They also noticed that significant differences exist between the computational results and the experimental data for NPR values less than 2.4. Hunter (1998) found the same behaviour in his two-dimensional computations and pointed out that the
differences were due to the fact that the flow became very three dimensional at the low NPR values.

Hunter (2004) used a two-dimensional non-axisymmetric experimental configuration in a 16-ft transonic wind tunnel to conduct an investigation more recently. Hunter’s (2004) experiment had the advantage of testing the symmetry of shock separation. The shock wave Mach number ranged from 1.1 to 2.1. The main objective of his investigation was to understand the static behavior of separated nozzle flows as a first step to resolve the complicated relationship between overexpansion, shock induced separation (SIS) and nozzle thrust efficiency.

Hunter (2004) found that there were two distinct regimes for SIS depending on the pressure ratio: for a pressure ratio less than 1.8, shock wave Mach number $M_{sw} < 1.5$, the separation was three dimensional unsteady and confined to a bubble; for a pressure ratio larger than 2.4, $M_{sw} > 1.6$, the separation was two dimensional, steady and fully detached. The transition from three to two-dimensional structure in separation was not because of shock boundary layer interaction but as a natural tendency of an overexpanded nozzle flow to detach and reach a more efficient thermodynamic balance.

Hunter (2004), in addition to his experimental investigation, performed Computational Fluid Dynamics (CFD) calculations on his two-dimensional C-D nozzles with a time averaged Navier-Stokes solver coupled with $k – \varepsilon$ turbulence closure and nonlinear algebraic Reynolds stress models. A wall tripped point was located near the inlet of the inflow duct to form a turbulent boundary layer in the test nozzle. A large number of pressure ratios (PRs) ranging from 1.25 to 8.78 were calculated for a detailed comparison with his experimental data, including the Schlieren images. At low PRs, say PR≤2.4, the comparison of the wall static pressure along the centerline
between the experimental data and the CFD results was rather crude. This was mainly because of the three-dimensional effect in the flow structure, as mentioned earlier, and partly because of the unsteadiness in the flow field, which the two dimensional steady computations neglected. At high PRs, the comparison was in good agreement because the flow was two dimensional and steady, satisfying the computation conditions.

2.9 PRESENT INVESTIGATIONS

From the above survey it is evident that, even though there is a quantum of information available in the open literature about the performance of C-D and Laval nozzles with straight entry, almost all of them are for nozzles with either stagnation state at the entrance or with very low entry velocity. Further, such investigations are not directly applicable to a slanted entry nozzle run by a supersonic stream because they do not provide asymmetries at the inlet. Therefore, there is an essential need to investigate the performance of straight as well as slanted entry nozzles with supersonic flow upstream of the entrance. The present work aims at these investigations at some a specified supersonic Mach number. Also, the slant angle of the nozzle entrance varied in few steps.

Operation of a C-D nozzle with supersonic inlet Mach numbers, at lower-pressure ratios than the design pressure can cause the internal flow to separate and possibly be unstable. The present investigation aims at understanding the flow characteristics of straight and slanted entry C-D nozzles with design Mach number of 2.94 exposed to a supersonic nozzle flow.