CHAPTER 2

INTERIOR PERMANENT MAGNET SYNCHRONOUS MOTOR

2.1 INTRODUCTION

Interior Permanent Magnet Synchronous Motor offers high efficiency by utilizing torque due to permanent magnet and reluctance. The computer simulation of IPMSM model is very useful to design a machine and torque controller for high performance servo applications, before building a prototype machine. This chapter deals with a suitable model for IPMSM from the reported literature. Moreover, the following assumptions are made so that the fundamentals of this study can be presented with better clarity.

1. High-resolution position sensor and speed sensor are utilized in the drive system resulting in negligible speed error.
2. The net sustainable loss for the machine is assumed to be constant resulting in negligible core loss.
3. The complete drive system operates within the limit of constant torque region and speed varies from zero to base speed.
4. Windage and friction losses are assumed negligible.
5. The dc bus voltage is maintained constant at all time.
2.2 EVOLUTION OF IPMSM

Before the development of power electronic technology, the cage-rotor IPMSM drives were typically line started (asynchronously started) in the presence of rotor cage. The cage bars provide the induction torque for starting and PM provided synchronous torque for running. The cage bars also act as damper winding to maintain synchronism under sudden variation in load torques. The difference in permeability between magnet and rotor core results in significant magnetic saliency which produces reluctance torque at synchronous speed. When the field strength of the magnet is too strong, the magnet creates considerable braking (pulsating) torque during its asynchronous mode of operation. Hence, the cage-rotor IPMSM may fail to synchronize. These problems are naturally very detrimental to the precise speed control application.

Development in the area of machine design and digital electronics allows the IPMSM designer to remove the starting cage from the rotor and achieve better performance along with vector control. Enhancements of drive performance and constant torque operation down to low speed have been possible by the use of vector control. IPMSM can also offer a wide constant power operation using the field weakening technique. However, PM material has the property of low permeability so that regulating the field requires a large amount of d axis current at high speed. This feature greatly limits the speed range (Boldea 2006). On the other hand, field-rotor IPMSM has a very good field regulation capability. The combination of adjustable field winding excitation and fixed PM excitation provides very good field regulation capability for constant power operation at high speed. However, this benefit is achieved at the cost of excitation loss and maintenance of slip ring and brushes (Xiaogang and Lipo 2000).
2.3 IPMSM MODEL

The phase coordinate model of IPMSM consists of electrical and mechanical dynamic equations. These equations are set forth in the following sections.

2.3.1 Electrical Dynamics

Consider the structure of IPMSM as shown in Figure 2.1. It has 3 phase concentrated stator windings displaced from each other by 120° and inserted a permanent magnet on the rotor. The difference in reluctance between magnet and rotor core makes the inductance along the direct (d) axis smaller than transverse (q) axis \( L_{sd} < L_{sq} \) i.e., in contrast to the saliency followed from standard synchronous machine (Bianchi 2005).

![Figure 2.1 Structure of 4 pole, 3 phase, IPMSM](image)
The dynamic equations of the IPMSM can be obtained from salient pole synchronous machine in the absence of field and damper windings. Thus, the voltage and stator flux linkage equations (Pillay and Krishnan 1989) can be written as

\[
\begin{align*}
\mathbf{\dot{v}_s} &= [r_s \mathbf{I}_s] + [D\lambda_s] \\
\mathbf{\dot{\lambda}_s} &= [L_s(\theta)] [\mathbf{I}_s] + [\lambda_p(\theta)]
\end{align*}
\]  

Equation (2.1) implies that the successful prediction of machine behavior mainly depends on the estimation of parameters, namely, stator inductances and magnet flux linkage. There are many methods available to calculate the parameters such as finite element (Bianchi 2005) and equivalent circuit method (Honsinger 1982). In this research work, these parameters are determined analytically from the geometrical information, properties of magnetic material and winding layout of the machine given in Appendix 1.

### 2.3.1.1 Stator inductance

Xiaogang and Lipo (1995) and Toliyat and Lipo (1995), have presented the modified winding function theory to determine the stator inductance of synchronous machine. Since the IPMSM dynamics are similar to synchronous motor, the same concept is extended further to IPMSM also. Unlike dq model, the machine inductances are considered to be time varying, while secondary parameters such as leakage inductances ($L_{ls}$) are treated as constant.

According to the modified winding function theory, the mutual inductance between any two stator windings ‘a’ and ‘b’ can be expressed in general form (Krause 2004) as,
where the higher harmonics inductances are neglected. The parameters \( L_A \) and \( L_B \) are given by

\[
L_A = K_0 \left[ \langle n_a \rangle \langle n_b \rangle - \langle n_a \rangle \langle n_b \rangle \right], \quad L_B = K_1 \sqrt{A_{ab}^2 + B_{ab}^2}, \quad \gamma_B = \tan^{-1} \frac{A_{ab}}{B_{ab}}
\]

(2.3)

where \( n_a \) and \( n_b \) are the turn functions between phase a and b, respectively. Operator \( <> \) is defined as the mean value of winding function over \([0, 2\pi]\). The turn functions can be figured out directly from the winding distribution arrangements of windings ‘n_a’ and ‘n_b’ as shown in Figure 2.2.

The calculation of phase inductance requires the integration along the entire stator inner surface. Mathematically, the integration of the product \( n_a n_b \) over period \([0, 2\pi]\) is calculated directly from the waveforms of two turn functions as shown in Figure 2.2 as given by

\[
\langle n_a n_b \rangle = \frac{1}{2\pi} 200 \left[ \left( \theta_1 \right) + \left( \theta_3 - \theta_2 \right) + \left( \theta_5 - \theta_4 \right) + \left( \theta_7 - \theta_6 \right) \right]
\]

(2.4)
In summary, the angular dependency of the stator inductance is given by

\[
L_s(\theta) = \begin{bmatrix}
L_{ls} + L_A - L_B \cos(2\theta) & L_A - L_B \cos\left(\theta - \frac{\pi}{3}\right) & L_A - L_B \cos\left(\theta + \frac{\pi}{3}\right)
\end{bmatrix}
\begin{bmatrix}
L_A - L_B \cos\left(\theta - \frac{\pi}{3}\right)
\end{bmatrix}
\begin{bmatrix}
L_{ls} + L_A - L_B \cos\left(\theta - \frac{2\pi}{3}\right) & L_A - L_B \cos\left(\theta + \frac{2\pi}{3}\right)
\end{bmatrix}
\begin{bmatrix}
L_{ls} + L_A - L_B \cos\left(\theta + \pi\right)
\end{bmatrix}
\begin{bmatrix}
L_A - L_B \cos\left(\theta + \pi\right)
\end{bmatrix}
\begin{bmatrix}
L_{ls} + L_A - L_B \cos\left(\theta + 2\pi\right)
\end{bmatrix}
\begin{bmatrix}
L_{ls} + L_A - L_B \cos\left(\theta + 2\pi\right)
\end{bmatrix}
\]

(2.5)

The aim of this work is to extend the modified winding function approach to IPMSM for evaluating the coefficients of stator inductance in the presence of stator coil asymmetry.
2.3.1.2 Stator flux linkage

The radial flux density produced by four pole magnet can be approximated as in Figure 2.3 and the same can be expressed in Fourier series (El-Refaie et al 2006) as,

\[ B_r(\theta) = \sum_{n=1}^{\infty} B_{2n-1} \cos((2n-1)\theta) \] (2.6)

where

\[ B_n = \frac{4 B_r(0)}{\pi} \sin\left(\frac{1}{2}n\tau_m\right) \]

where \( B_{2n-1} \) denotes the magnitude of \( n^{th} \) spatial harmonic component of flux density with respect to the fundamental harmonic. The magnets used are Sintered Neodymium Iron Boron (NdFeB) with open circuit remanent flux density \( B_r \) of 0.945T. Then, the flux linkage of phase ‘a’ can be expressed as

\[ \lambda_{p,a}(\theta) = \frac{2\pi}{0} \int_{0}^{\tau_m} B_r(\theta) r_i \ell_s d\theta \] (2.7)
Evaluating the integral using equation (2.6) yields,

\[ \lambda_{p,a} (\theta) = 2r \ell_s \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} B_{2n-1} \sin((2n-1)\theta) \]  

(2.8)

In essence, the stator flux linkage matrix can be written as

\[ \lambda_p (\theta) = \sum_{n=1}^{\infty} \lambda_{2n-1} \begin{bmatrix} 
\sin((2n-1)\theta) \\
\sin((2n-1)(\theta - \frac{2\pi}{3})) \\
\sin((2n-1)(\theta + \frac{2\pi}{3})) 
\end{bmatrix} \]  

(2.9)

**Table 2.1 Parameter of IPMSM**

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>( L_{ls} )</td>
<td>-</td>
<td>0.2 mH</td>
</tr>
<tr>
<td>( L_A )</td>
<td>0.0102 H</td>
<td>0.0103 H</td>
</tr>
<tr>
<td>( L_B )</td>
<td>-0.0045 H</td>
<td>-0.0047 H</td>
</tr>
<tr>
<td>( L_{md}=3/2 )</td>
<td>8.55 mH</td>
<td>8.4 mH</td>
</tr>
<tr>
<td>( L_{mq}=3/2 )</td>
<td>22.05 mH</td>
<td>22.5 mH</td>
</tr>
<tr>
<td>( L_{sd}=L_{md}+L_{ls} )</td>
<td>8.75 mH</td>
<td>8.6 mH</td>
</tr>
<tr>
<td>( L_{sq}=L_{mq}+L_{ls} )</td>
<td>22.25 mH</td>
<td>22.7 mH</td>
</tr>
<tr>
<td>( \lambda_p )</td>
<td>0.115 wb</td>
<td>0.105 wb</td>
</tr>
</tbody>
</table>

The parameters of the IPMSM are evaluated and the results are tabulated in Table 2.1. Evaluation of inductance using Boldea method (2006) is also presented in the same table and found that the results are closer to the machine geometry method. However, the leakage inductance is evaluated by
using Boldea method. These parameters are used in simulation of IPMSM and design a high performance torque control over the base speed.

### 2.3.2 Mechanical Dynamics

Consider the mechanical dynamics governed by the simple relationship as,

$$J \dot{\omega}_r + B \omega_r = T_e - T_L$$

(2.10)

In order to complete the derivation of IPMSM model, the expression for electromechanical torque is essential. In three phase quantities, the electromechanical torque with the assumption of linear magnetic condition is given by

$$T_e(\theta) = \frac{P}{2} \left( \frac{1}{2} i_s \frac{\partial L_s(\theta)}{\partial t} i_s + i_s T_s \lambda_p(\theta) \right)$$

(2.11)

### 2.4 MOTOR DYNAMICS IN dq COORDINATE

The phase coordinate model is feasible for complex digital simulations in the presence of static power converters and non-sinusoidal emf distributions. When a simple model is chosen for parameter estimation and control design, the machine phase variables in equation (2.1) can be transformed into dq variables in the rotating reference frame with the help of a well-known Park’s transformation matrix to yield
\[ v_{sd} = r_s i_{sd} + D\lambda_{sd} - \omega_r \lambda_{sq} \]
\[ v_{sq} = r_s i_{sq} + D\lambda_{sq} + \omega_r \lambda_{sd} \]
\[ \lambda_{sd} = L_{sd} i_{sd} + \lambda_p \]
\[ \lambda_{sq} = L_{sq} i_{sq} \]  

(2.12)

Transforming equation (2.11) into dq coordinate systems equation (2.13) is obtained.

\[ T_c = \frac{3}{2} \frac{P}{2} \left[ \kappa_p i_{sq} + \left( L_{sd} - L_{sq} \right) i_{sd} i_{sq} \right] \]  

(2.13)

Using (2.10), (2.12) and (2.13), the machine model can be simulated in Matlab/Simulink environment.

### 2.5 VECTOR CONTROL OF IPMSM

Pillay et al (1989) and Mongkol and Xu (2006) explained that the vector control technique could be easily realized in hardware as well as in a simulation model using a current controlled Voltage Source Inverter (VSI). The complete implementation scheme of the vector controlled IPMSM drive is shown in Figure 2.4. There, the torque command is input to the drive system. The motion controller implements feedback control, based on mechanical sensors or estimator information. The controller output commands the electrical variables to obey. The electrical control block converts its input commands into commands for the power converter and sometimes utilizes feedback of voltage or current. The power converter block imposes the desired electrical signals onto the IPMSM with connected load.
2.5.1 Classification of Control Strategy

In torque control, one of the following performance criteria can be optimized while varying the speed from zero to rated value. Optimization of each performance criteria leads to the following control strategies for IPMSM drive.

(a) Zero d-axis Current
(b) Maximum Torque Per Ampere (MTPA)
(c) Maximum Torque Per Flux (MTPF)
(d) Maximum Efficiency (ME)
(e) Unity Power Factor (UPF)
(f) Constant Mutual Flux Linkages (CMFL)
The zero d axis current control strategy (Morimoto et al. 1990) is widely used in the industry as it forces the torque to be proportional to current magnitude for the IPMSM. The MTPA control strategy (Ching-Tsai and Shinn-Ming 2005) provides the maximum torque for a given current. This, in turn, minimizes copper losses for a given torque. However, the MTPA control strategy does not optimize the system for net loss. The MTPF control strategy (Jawad and Mohseni-Zonoozi 2003) provides maximum torque for a given flux. The MTPF control strategy achieves fast response by regulating the stator flux vector. The MTPA/MTPF control strategy is utilized in high performance applications where efficiency is important, and is one of the most studied and utilized control strategy by IPMSM drive researchers. The UPF control strategy (Krishnan 2005) optimizes the system’s Volt Ampere requirement by maintaining the power factor at unity. The ME control strategy (Mademlis et al. 2004) minimizes the net loss of the motor at any operating point. This control strategy is particularly appealing in battery-operated motion control systems as it enables to extend the life of the system. The CMFL control strategy (Chandana et al. 2003) limits the air gap flux linkages to a known value, which is usually the magnet flux linkage. This is to avoid saturation of the core.

### 2.5.2 High Performance Control Strategy

All high performance control strategies for IPMSM are based on an electrical model of the machine. In most cases, the parameters of the machine are assumed to be constant. In reality machine parameters vary as a function of temperature and current. Phase resistance of a motor varies with temperature and frequency. The inductance is a function of current. The non-linear magnetic properties of the core are the reason behind the variations of inductance as a function of current. The magnet’s flux density changes as a
function of temperature (Sebastian 1995). Another parameter that varies in some applications is the dc bus voltage input to the drive (Krishnan 2005). The variation of the dc bus voltage affects the speed after which flux weakening must be initiated. As a result, parameter estimation favors high performance of drive.

Resistance can be estimated by measuring the temperature of the core. Inductance can be estimated if the inductance versus current relationship is accurately known. The variations of the magnet flux density can be estimated if the temperature of the magnet is measured. However, this is a difficult and costly process due to the movement of the rotor. An online estimation of these parameters can be incorporated in the model being used to control the machine. Bus voltage can be monitored at all times and variations if any, in bus voltage can be corrected by suitable robust tuner circuit (Shin 1998).

2.6 CONCLUSION

In this chapter, the literature related to the modeling of IPMSM is presented. The traditional method of machine modeling in abc form is considered. A simple method for evaluation of machine inductance from the geometry is illustrated. A short review on various control strategies and the impact of parameter variations on high performance control is presented.