Abstract

This thesis deals with modeling circuits using nonlinear elements like the diode and BJT transistor apart from linear elements like resistors, capacitors and inductors. For each circuit, based on KCL and KVL, and the basic nonlinear voltage-current relations for diodes and transistors, we set up the nonlinear differential equations that govern the dynamics of the circuit. The linear part of the system of nonlinear differential equations is separated from the nonlinear part, always using a tag $\epsilon$ multiplying the nonlinear part. $\epsilon$ is supposed to be small enough to guarantee Taylor expansions of the solutions to the system in its powers. We then equate the coefficients of same powers of this tag $\epsilon$ to arrive at an infinite sequence of differential equations for the approximants of the state variables of the circuit. All of this comes under the standard methods of perturbation theory as used in quantum and classical mechanics. The differential equations satisfied by a given approximant are linear in the approximant but generally nonlinear in the lower order approximant. This permits a recursive solution to the various approximants in terms of the lower approximants. The lowest order approximant called the zeroth order term obeys a linear differential equation with the driving input voltage appearing on the right side. We assume that this input is a sinusoid or a linear combination of sinusoids of different frequencies. Application of perturbation theory as discussed above leads to a Volterra representation of the input-output relations which cause the generation of higher harmonics in the output, i.e., frequencies given by integer linear combinations of the input frequencies. A spectral analysis of the output states then gives the strength of each higher harmonic component which provides a clue to estimating the parameters of the transistor or diode. The unknown parameters of a diode are the leakage current $I_0$ and the cutin voltage $V_T$. For the transistor we have apart from the collector and emitter leakage currents $I_{C0}, I_{E0}$ and the base emitter and base collector cutin voltage $V_T$, the parameters $\alpha_C, \alpha_E$ which give information about the fraction of the collector current entering the base emitter junction and the fraction of the emitter current entering the base collector junction.
In the first part of this work, we obtain the Fourier series of the output signal of the circuit (amplifier or rectifier) for sinusoidal input using this technique. We then use this Fourier series representation to obtain least squares frequency-domain algorithms for estimating the diode and transistor parameters. The general idea is to give as input a superposition of \( p \) sinusoids whose amplitudes are fixed but frequencies are variable and apply the KAM method after its discoverers Kolmogorov, Arnold and Moser. KAM is the abbreviation of the famous theorem on averaging for quasi-periodic motion due to Kolmogorov, Arnold and Moser. They used the technique to analyze the trajectories of ‘n’ body gravitational problem. Averaging can be used to determine qualitative characteristics of the motion, like, the average time spent by the trajectories inside a set. In this work, we use KAM method to calculate the Fourier components averaging of the output waveform at any given frequency.

Another part of the thesis deals with nonlinear circuits with randomly fluctuating parameters due to noise. The existing perturbation-based models for the nonlinear circuits omit the effect of electrical noise that exists in the circuits, and deteriorate the signal through the circuits. We propose such a model which takes into account a) nonlinear behavior, and b) noise effects in a circuit. We present a general model for a nonlinear circuit, in which, the circuit parameters (e.g. resistance and capacitance) are subject to random fluctuations due to noise, which vary with time. The fluctuating amplitudes of these parameters are assumed to be Ornstein-Uhlenbeck (O.U.) processes and not the white noise owing to temporal correlations. The nonlinear circuit is represented by a system of nonlinear differential equations depending upon a set of parameters that fluctuate slowly with time. To model these fluctuations, we use the theory of Ito’s stochastic differential equations (SDEs). Then the driving force of the circuit dynamics is in accordance with the general perturbation theory decomposed into the sum of a strong linear component and a weak nonlinear component by the introduction of a small perturbation parameter. The circuit states are expanded in the powers of this small perturbation parameter and recursive solutions to the various approximates obtained. Finally, the approximate expressions for the output states are obtained as stochastic integrals with respect to Brownian motion processes. The proposed method is applied to a half-wave rectifier circuit which is built out of a diode, a resistor and a capacitor. The diode is represented by a nonlinear voltage-current equation, and resistance and capacitance are subject to random fluctuations due to noise, which vary slowly with time. The results, obtained using the proposed method, are compared with those obtained via
the conventional perturbation-based deterministic differential equations model for a nonlinear circuit. Hence, the noise process component, present at the output, is obtained.