Chapter 7
Conclusion and Future Works

Every real life problem has a solution, that has been the motivating stone for this thesis. The thesis deals with modelling situations that involve dependence between the random quantities of interest. Most of the natural phenomena in real life involve dependence between the consecutive observations. The assumption of dependence between the variables becomes impossible when the phenomenon or the characteristic under study involves some repetitions. Most of the modelling problems neglect the dependence that is redundant in them. Here we tried to model some of the scenarios where there is dependence.

In chapter three, we considered a multistate system with Markov dependence between the states occupied by the system. We assumed that the lifetimes and the repair times in each state follow Phase type distribution with the parameters depending on the state. We also assumed that the repairs will have an impact on the lifetime or the repair time. We modelled the impact of the repairs on the lifetimes or the repair times by the Cox multiplicative model. The number of repairs on the system is assumed to be the concomitant variable and the regression factor is assumed to be depending on the state. An expression for long run cost per cycle for the system is derived. An algorithm that simplified the procedures for finding the optimal number of repair, minimizing the expected cost per cycle, was also developed. Finally the algorithm was illustrated with a numerical example.

Eventhough we were able to derive an expression for the long run expected cost per unit time in the case of general multistate systems which do not assume any stochastic ordering between the lifetimes or repair times, the development of the algorithm was based on the assumption that the lifetime or the repair times are stochastically monotone random variables. Developing a simple method or algorithm
for finding the optimal policy under the given condition is still interesting. We exploited the Phase type distribution to model the lifetimes and the repair times in each state. Exploiting general situation involving any distribution can also be dealt with in the future. Even in the case of approximating the lifetimes or the repair times by a Phase type distribution, estimating the number of recurrent states to be used is an open problem for almost a decade. Deriving the statistical inference procedures for the model is also an area to be explored. We assumed that the ageing factor depends only on the state which it is occupying. But systems in which the ageing factors depend on the state occupied till now is an interesting problem. Optimization based on the time dependent factors like availability is also a problem to be tackled in the future.

In chapter four, we introduced the concept of protection into the reliability modelling scenario. Till now we did not consider problems with the shocks or the failures that may cause the malfunctioning or even the nonfunctioning of the repair facility. It has been assumed that there will not be any reliability concerns regarding the repair facility. We considered the situation where the repair had to be restarted from the scratch when an interruption happens to the repair facility. Long run cost for the completion of the repair has been developed assuming that the $k$ states are unprotected while the remaining $n-k$ are protected states, shocks do not have any impact on the repair facility when the repair is in these states. Since high cost is involved with protecting the states, an optimal policy regarding the time of introduction of the repair facility is considered in the chapter. The results are illustrated with the help of a numerical example.

It has been assumed that the repair time in each state is an exponentially distributed random variable. Generalizing this into the general framework is interesting and this is to be opened upon. Also we assume a sequential transition between the states. This is also restrictive. We also did not consider situations where we take away the protection after some time if the system is performing reasonably well.
In chapter five, we proposed two new models of start-up demonstration having two phases with the condition for the corrective action in the first model, being specified number of consecutive failures, and in the second model, being specified number of random failures. Expressions for various measures of interest like the probability of acceptance of the product, probability of rejection of the product, expected number of start-ups etc were developed. We assumed a Markov dependence between the consecutive trials. We considered a numerical example. The probability of acceptance of the product can be used to derive an optimal policy in both cases.

One of the possible generalizations is one with more than two phases and the results can be obtained in a similar fashion as we had done for the two-phase case. The models can be generalized to $m$ Markov dependent case by proceeding as in Aston and Martin (2005).

In chapter six we obtained the joint distribution of runs and occurrence of outcomes there for multi-state outcomes, there by generalizing Han and Aki (1999) and Chadjiconstantinidis et. al (2000).Probability generating functions for the joint distribution of occurrence of events and the runs was also derived. Probability generating functions of the distribution of occurrence of the events and the runs were also derived. The results were in tune with Han and Aki (1999). Expected values of the joint variables and the marginal variables are also derived. We derived the expression for the waiting times also. The recurrence relation, we derived, can be used to evaluate the probabilities in the case of various types of counting as illustrated. A more general Markov Multinomial distribution was also derived Throughout the development of the chapter we assumed that the consecutive trials are Markov dependent.

All the results discussed in the chapter can be extended to the case of $l$ dependent variables discussed by Aki and Hirano (2000). Patterns and scans are area
of interest, generalizing the results to the case of patterns and the scans can also be considered in future.