CHAPTER 4

DYNAMICS OF GRANULAR SOLIDS MOTION IN THE TRANSVERSE PLANE OF ROTATING CYLINDER

4.1 Introduction

The motion of a bed of granular solids in the transverse plane of a rotating cylinder can exhibit a series of motions (Henein et al., [1983a]), namely: slipping, slumping, rolling, cascading, cataracting and centrifuging which have been explained in detail in Chapter 2. The slipping, slumping and rolling modes of bed motion have been investigated experimentally by a number of authors, including Henein et al. [1983a] and McTait [1998]. The transition from slumping to the rolling mode of bed motion was found to occur at a rotational Froude number between $10^{-3}$ and $10^{-4}$ by Henein et al. [1983a] and between $10^{-4}$ and $10^{-5}$ by McTait [1998].

The specific mode of bed motion was also found to be uniquely determined by (i) the hold-up as a fraction of the cylinder volume, (ii) the rotation speed, (iii) the cylinder diameter, and (iv) the particle size and shape. It was found that the transition from a slumping to a rolling bed occurred at a lower rotational Froude number for (i) a higher fractional hold-up, (ii) particles having a spherical shape rather than an irregular shape, (iii) smaller particles, (iv) cylinders with larger diameter, and (v) granular solids with a lower static angle of repose. In a slumping bed, it was observed that the granular materials is lifted as a rigid body by the cylinder wall such that the inclination of the bed surface increases continuously until it reaches an upper angle of repose; then detaches from the upper surface of the bed, and falls as a discrete avalanche toward the lower half of the bed.
It can be observed from the literature that the work carried out by various investigators are mainly at a macroscopic level and further they have interpreted the process through empirical parameters derived from experiments. *No attention has been paid to the effect of the dynamic behaviour of the granular solids due to surface flow of granular solids, the effect of transient forces such as collision forces acting on the particles, the total kinetic energy of the system, particle velocities etc.* These factors are crucial to generate a reliable model based on first principles for the purpose of design with negligible empirical input.

In this chapter we use Discrete Element Method to obtain the relevant dynamic information on motion of granular solids subjected to rotational movement based on discrete element simulation. *Even though the software code DEMCYL is capable of solving three-dimensional motion of granular solids in a rotating cylinder, results in this chapter are aimed for a 2-dimensional situation viz., the simulation of granular solids in the transverse plane of the rotating cylinder. Since this approach provides adequate interpretation of the process without requiring unjustified computer time. We have restricted our analysis to 2-dimensional, since this would be adequate to gain enough information about the process while also reducing the computer time considerably.*

The dynamic information sought for includes the individual particle velocities, their trajectories, surface flow velocities, velocity vector plots, total kinetic energy and dynamic angle of repose. The two important process parameters considered in this chapter are fill fraction and rotational speed. The transition from slumping-rolling behaviour has been explained based on bed turnover time. The effect of the coefficient of friction on dynamic angle of repose, active layer depth as a function of rotational speed and fill fraction are also discussed.
4.2 Simulation of a packed bed of granular solids as an initial condition

The transverse plane of a horizontal cylinder is a circle. This is represented as a set of spherical particles at a distance equal to the cylinder radius from the origin of the Cartesian coordinate system chosen. The origin of the coordinate system is at the center of the circle as shown in Figure 4.1.

![Coordinate system chosen for simulation](image)

Figure 4.1 Coordinate system chosen for simulation

The initial condition, \( t = 0 \) of granular solids in the transverse plane of the rotating cylinder is assumed to be a packed bed of granular solids. Since the initial conditions of granular particles in a packed bed cannot be specified a priori, the calculations were carried out at two stages. Given the fill fraction, particle sizes
and their distribution, the number of particles for each size range was determined using equation (4.1)

\[ n_p(i) = \frac{4 f_w A}{\pi d_p(i)^2} \]  

(4.1)

where \( f_w \) is the percentage of fraction of the particles of size \( d_p(i) \) and \( A \) is the cross sectional area of the bed. An orthogonal grid is then generated with the size of the grid equal to the diameter of the largest particle and the particles are placed at the center of these grids so that they don't overlap over each other. Uniform sized particles are given a particular colour. Figure 4.2 shows one such initial distribution generated for two different sized particles at equal number concentrations.

![Figure 4.2: Initial Distribution of the particles](image)

The initial velocities of individual particles were chosen randomly and the force of gravity is allowed to act on each particle. The time step was chosen according to the stability criteria explained in Chapter 3. Initially the particles fall under gravity since there is no contact force, but after some time the contact forces
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also come into play. The bordering shape of the rotating cylinder, which first stops
the pure vertical motion and secondly causes the particles to interact with each
other, limits the motion of the particles. The total kinetic energy of the system is
calculated as follows,

\[ KE = \frac{1}{2} \sum_{i=1}^{N_p} m_i v_i^2 \]  

(4.2)

where \( m_i \) is the mass, \( v_i \) is the velocity and \( N_p \) is the total number of particles in
the system. The simulation was continued till all the particles came to rest; that is
until the total kinetic energy of the system becomes negligible as shown in Figures
4.3 (a) and (b). The location and linear and rotational velocities of particles at this
stage are chosen as the initial conditions for the next stage of simulation.

Figure 4.3(a): Initial Packed bed
Figure 4.3(b): Total kinetic energy with respect to time
4.3 Simulation of granular solids in the transverse plane of the rotating cylinder

Starting with the initial conditions explained in Section 4.2, the cylinder wall is allowed to rotate at a specified rotational velocity. Various simulation parameters used in this work are summarized in Table 1.

Table 4.1: Process parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (m)</td>
<td>1.5 m, 7.5x10^{-2}m</td>
</tr>
<tr>
<td>Rotational speed (rpm)</td>
<td>1 - 80</td>
</tr>
<tr>
<td>Particle radius (mm)</td>
<td>5-10</td>
</tr>
<tr>
<td>Particle density kg/m(^3)</td>
<td>1300</td>
</tr>
<tr>
<td>Fill fraction (%)</td>
<td>10% - 30%</td>
</tr>
<tr>
<td>Number of particles</td>
<td>500-3000</td>
</tr>
<tr>
<td>Normal spring constant (Pa)</td>
<td>5.0e+05</td>
</tr>
<tr>
<td>Normal energy dissipation coefficient (s(^{-1}))</td>
<td>100</td>
</tr>
<tr>
<td>Tangential energy dissipation coefficient (s(^{-1}))</td>
<td>20</td>
</tr>
<tr>
<td>Coefficient of Friction (-)</td>
<td>0.1-1.0</td>
</tr>
</tbody>
</table>

The spherical solid particles representing the wall of the cylinder undergo only rotational motion, and these particles contribute only to the contact force calculations of the particles in the solid bed. When the cylinder rotates, the particles in the solid bed undergo collisions with other granular particles as well as with the cylinder wall resulting in translational and rotational motion. The following parameters are obtained from the simulation results.

4.3.1 Dynamic angle of repose

The included angle made by the upper surface of the granular solid bed with the horizontal line of a rotating cylinder is defined as the dynamic angle of repose. Hence from the simulation data, the maximum and minimum values of x-position of the particles are found and a line is drawn joining these two points as
shown in Figure 4.4. The dynamic angle of repose \( \Theta \), can be then obtained as the angle that this line makes with the x-axis by employing the following equation

\[
\tan \Theta = \frac{m_2 - m_1}{1 + m_1 m_2}
\]  

(4.3)

where \( m_1, m_2 \) are the slopes of the respective lines.

4.3.2 Velocity profiles

The velocity profile of the particles at any instant is best exhibited by a vector plot indicating the magnitude and the direction of the velocities of the particles by small directed line segments. The x, y coordinates of all the particles in the granular bed at time \( t \) and \( t + \Delta t \) are stored in two different files.
The graphics software **ORIGIN 6.0** was used to draw the velocity vector plot using **XYXY option**, where the extreme X and Y columns determine the XY coordinate of the tail of the vector and the inner set of X and Y columns determine XY coordinate of the head of the vector. One such typical profile is shown in Figure 4.5.

![Figure 4.5: A typical vector plot](image)

**4.3.3 Surface velocity of the granular bed**

The entire transverse plane of the rotating cylinder was divided into segments as shown in Figure 4.6. The average velocity for each sector was calculated as
where $v_{x_i}, v_{y_i}$ are the x and y component of the velocities of the individual granular particles inside each sector. The surface sectors are then identified and the velocities corresponding to these sectors are fixed as the surface velocities.

\[
\begin{align*}
    v_x &= \frac{1}{n_s} \sum_{i=1}^{n_s} v_{x_i} \\
    v_y &= \frac{1}{n_s} \sum_{i=1}^{n_s} v_{y_i}
\end{align*}
\] (4.4)

Figure 4.6  Discretization of the cross section into segments

4.3.4 Active layer depth

According to Henein et al [1983a], the rolling bed is characterized by two regions: the non-shearing plug flow region and the shearing active layer of particles cascading down the top surface. In the plug flow region, the particles rotate as a rigid body with a velocity, which varies linearly with distance from the
center of the cylinder. At the region near the free surface, particles cascade down due to rapid collisions and gravity. The demarcation between these two regions is a stagnation plane where particles are momentarily stationary before reversing the direction. Such demarcation is clearly shown in Figure 4.5. From the velocity profile plot the active layer is approximated as an area where the particles cascade down along the free surface and this is shown by the velocity vectors accelerating downwards.

4.3.5 Evolution of kinetic energy of the system with respect to time

The amount of energy dissipated during collisions between particle-particle and particle-wall interactions plays a prominent role in characterizing the dynamics of granular motion. Dissipation of energy attributed to the particle distributions and velocity distributions include kinetic energy, potential energy etc. Since the kinetic energy plays an important role, a bookkeeping was carried out to store the fluctuations of kinetic energy as defined by equation 4.2. At every instant of time the kinetic energy was calculated for each particle and was averaged out with the total number of particles to get the total kinetic energy of the granular solid bed during the rotational motion of the cylinder.

4.4 Simulation of the various modes of the granular solid bed

Employing the procedures described in the previous sections, we simulated the dynamics of granular material motion with the diameter of the rotating cylinder set to 1.5 m. Results have generated for mono sized spherical particles having diameters 6 mm assuming an initial fill fraction of 20 % at various rotational speeds. Figures 4.7(a) to 4.7(e) depict the six different modes of motion, namely slipping, slumping, rolling, cascading, cataracting and
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Centrifuging which were obtained through simulation by varying the rotational velocity of the rotating cylinder along with their corresponding velocity profiles.

Figure 4.7: Six different modes of motion obtained through DEM simulation
4.4.1 Slipping mode

Figures 4.8 (a) and (b) show the profile of granular solid bed and the corresponding velocity profile when the rotational velocity of the cylinder was maintained at 4 rpm. These plots were obtained after the cylinder had undergone 10 rotations while reaching the steady state. It can be clearly seen from the velocity profile that, the granular solid bed remains nearly stationary as evidenced by a look of movement of particles. The fact that the particles were almost stationary was also verified through the Figure 4.8 (a) where it is clearly visible that mixing of particles was absent and the solid bed was equivalent to a packed bed at an elevated position. This mode of granular solids motion was further confirmed by plotting the dynamic angle of repose and the total kinetic energy of the granular solid bed with respect to time as shown in Figure 4.8(c) and Figure 4.8(d).

![Granular bed in slipping mode](image)
Figure 4.8 (b): The velocity profile in slipping mode

Figure 4.8(c): Dynamic angle of repose of the granular solid bed at 4 rpm
**4.4.2 Slumping mode**

As the rotational velocity of the cylinder was increased further, the slumping of the granular bed commences. For most of the time, the entire granular bed rotates at the same rotational velocity, thus undergoing solid body rotation. Further an avalanche is triggered when the bed surface reaches the static angle of repose $\Theta_s$. During the avalanche, material from the upper part of the bed surface slides rapidly down. At the end of avalanche period the bed surface was inclined at a lesser angle $\Theta_d$ called the dynamic angle of repose. Solid body rotation of the whole bed ensues, until the surface inclination again reaches the angle $\Theta_s$ when avalanching occurs and the cycle is repeated. Davidson et al (2000) has given the following relationship for characterizing the slumping cycle time:
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\[ t_{13} = t_{12} + \frac{(\Theta_s - \Theta_d)\pi}{180\omega} \]  \hspace{1cm} (47)

where \( t_{12} \) is the avalanche time, \( t_{13} \) is the total cycle time and \( \omega \) is the rotational velocity of the cylinder. The schematic view of one slumping cycle is shown in the Figure 4.9 (a) to 4.9 (c).

**Figure 4.9**: Cyclic slumping or avalanching
(a) Initial stage of a slump at an angle \( \Theta_s \)
(b) Final stage of a slump at an angle \( \Theta_d \) and
(c) Initial stage of the next slump at an angle at \( \Theta_s \).

**Figure 4.9(d)**: The granular solid bed profile at the beginning of avalanching in slumping mode.
It has been shown by Henein (1983a) that the slumping frequency is dependent on the rotational speed, particle size and cylinder diameter. The slumping mode of the granular solid bed motion is shown in Figures 4.9(d) where the rotational velocity of the cylinder is kept at 10 rpm. Figure 4.10(a) and 4.10(b) shows the velocity profile of the granular solid bed at the beginning of avalanching of the surface particles and the end of the avalanching of the particles respectively, in the slumping mode.

Figure 4.10(a): Vector plot of the granular solid bed during solid body rotation in slumping mode
Figure 4.10 (b) Vector plot of the granular solid bed at the beginning of avalanching in slumping mode.

The trajectory of a single particle in the slumping mode is shown in figure 4.10 (c), which shows that, the particles will have much movement. The corresponding kinetic energy distribution and the dynamic angle of repose variation are shown in Figures 4.10 (d) and 4.10(e). The angles $\Theta_1$ and $\Theta_2$ are found to be approximately 17.5° and 15.1° and the avalanching duration is found to be around 3s and the predicted cycle time based on equation (4.7) turns out to be 3.3s which agrees with the predicted value of 4s given by Davidson et al (2000) for the same process parameters.
Figure 4.10(c): Trajectory of a single particle in slumping mode

Figure 4.10(d): Kinetic energy distribution during slumping mode
Figure 4.10(e): Dynamic angle of repose with respect to time during slumping mode.
4.4.3 Rolling mode

As the rotational speed of the cylinder increases further, a transition to the rolling mode takes place. This type of motion is characterized by a uniform static flow of a particle layer on the surface (active layer) while the larger part of the bed (plug flow region) is transported upwards by solid body rotation with the rotational speed of the wall. The bed surface is nearly level. This type of motion enables good intermixing and hence this mode has been investigated extensively in the literature.

Figure 4.11 (a): Velocity profile of the rolling bed
Figure 4.11 (a) depicts the velocity profile of the granular solid bed after 10 rotations, with the rotational speed of the cylinder set at 15 rpm. The active layer is clearly visible in the picture. A small stagnation zone can also be observed and the bulk of the granular solids move with the cylinder wall.

Figure 4.11(b) shows the profile of the granular solid bed after 10 rotations and as can be observed significant intermixing of particles has occurred. Figure 4.11(c) shows the kinetic energy distribution and Figure 4.11(d) depicts the variation of dynamic angle of repose. The trajectory of a single particle shown in figure 4.11 (e), describes the path of the particle in the rolling mode.

Figure 4.11 (b): Granular solid bed during rolling mode
Figure 4.11 (c): Kinetic energy distribution

Figure 4.11(d): Dynamic angle of repose with respect to time.
4.4.4 Cascading

As the rotational speed further increases, the bed surface begins to arch and cascading sets in. The granular solid bed is almost kidney shaped and the height of the arch of the kidney-shaped bed increases with increasing rotational speed. The results obtained from the simulation data when the rotating speed of the cylinder is increased to 30 rpm is shown in Figures 4.12 (a). The corresponding trajectory of a single particle, vector plot, kinetic energy distribution and dynamic angle of repose obtained through simulation are shown in figures 4.12 (b) to 4.12 (e) respectively.
Figure 4.12 (a): Granular solid bed during cascading mode

Figure 4.12 (b): Trajectory of a single particle in cascading mode
Figure 4.12 (c): Velocity profile of the cascading mode
Figure 4.12 (d): Kinetic energy distribution

Figure 4.12 (e): Dynamic angle of repose with respect to time.
4.4.5 Cataracting mode

As the rotational speed increases, the cascading motion is so strongly pronounced that individual particles detach from the bed and are thrown off into free space of the cylinder. The release of particles from the solid bed to the free surface is the characteristic feature of cataracting motion. The simulated results of the cataracting mode of the granular solid bed are shown in Figures 4.13(a). The corresponding figures for the velocity profile is shown in figure 4.13(b). The trajectory of a single particle is depicted in figure 4.13(c). kinetic energy distribution is given in figure 4.13(d). The rotational speed of the cylinder is 35 rpm.

Figure 4.13(a): Granular solid bed during cataracting mode
Figure 4.13 (b): Velocity profile of the cataracting mode

Figure 4.13 (c): Trajectory of a single particle in cataracting mode.
4.4.6 Centrifuging

With increasing rotational speed, the number of particles thrown off from the bed increases and the length of trajectories also increase. With further increase in the rotational speed, some of the particles begin to adhere to the wall and after a few rotations the whole solid bed rotates with the cylinder wall as a uniform film. This type of mode is known as centrifuging and is usually obtained at high rotational speeds. Typical plots of the granular solid bed, velocity profile, trajectory of a single particle and kinetic energy distribution of the centrifuging motion when the rotational speed of the cylinder is set at 80 rpm are shown in the following series of figures 4.14 (a) to 4.14 (d).
Figure 4.14 (a): Granular solid bed during centrifuging mode

Figure 4.14 (b): Velocity profile of the centrifuging mode
Figure 4.14 (c): Trajectory of a single particle in centrifuging mode.

Figure 4.14 (d): Kinetic energy distribution
Figures 4.15 (a) to 4.15 (f) show the vector plots of various stages which lead to the centrifuging mode.

**Figure 4.15 (a):** Vector plot of centrifuging motion after 1 rotation

**Figure 4.15 (b):** Vector plot of centrifuging motion after 2 rotations
Figure 4.15 (c): Vector plot of centrifuging motion after 3 rotations

Figure 4.15 (d): Vector plot of centrifuging motion after 5 rotations
Figure 4.15 (e): Vector plot of centrifuging motion after 7 rotations

Figure 4.15(f): Vector plot of centrifuging motion after 9 rotations

The dynamics of granular solid bed at different rotational speeds are presented in Figures 4.16 (a) to 4.16(e) after 2, 4, 6, 8, 10 rotations. It can be clearly seen from the Figures that good mixing occurs only in the rolling and cascading modes. Since Cascading, Cataracting and Centrifuging motions are less preferable modes of operations for rotating cylinders, further studies are confined only to investigating the rolling mode.
Figure 4.16 (a): After 2 rotations with different rotational speed
Figure 4.16 (b): After 4 rotations with different rotational speed
Figure 4.16 (c): After 6 rotations with different rotational speed
Figure 4.16 (d): After 8 rotations with different rotational speed
After 10 rotations with different rotational speed
4.5 Results and Discussion

4.5.1 Characterization of the Transition behaviour

Henein [1983a, 1983b] proposed the Froude number as a suitable parameter to define the transition from one mode to another. The Froude number is defined as

$$Fr = \frac{R \omega^2}{g}$$

where $R$, $\omega$ and $g$ are the cylinder radius, rotational speed of the cylinder and acceleration due to gravity respectively. Henein's [1983a, 1983b] experiments showed that low values of $Fr$ less than $10^{-5}$ are consistent with avalanching. Then there is a transition region where $10^{-5} < Fr < 10^{-4}$. Values of $Fr$ more than $10^{-4}$ are consistent with the rolling mode. Mellmann [2001] developed mathematical models to predict the transition behaviour and he has presented the results as a bed behaviour diagram and he observed that the bed behaviour diagram depends strongly on the fill fraction and rotational speed. He also presented the Froude number values for transition behaviour. To confirm the transition values of the Froude number, simulation runs are made for the following operating parameters for simulating the transition behaviour of the granular solid bed from one mode to the next mode.

- Cylinder diameter 7.5 cm
- Particle size - 3 mm with density 1300 Kg/m$^3$
- Fill fraction 20% which corresponds to around 750 particles
- Rotational speed is varied from 1 rpm - 80 rpm
The simulations are carried out and by graphically observing the velocity profile for various rotational speeds and the Froude numbers are calculated using the equation $Fr = \frac{\omega^2 R}{g}$. And these values are tabulated in Table 4.2 along with the predicted values Mellmann [2001]. The values predicted by the simulation closely matches with the values predicted by Henein et al. [1983a, 1983b] and Mellmann [2001].

**Table 4.2 Froude numbers as predicted by simulation**

<table>
<thead>
<tr>
<th>Types of motion</th>
<th>Froude number as predicted by Mellmann [2001]</th>
<th>Froude number as predicted by the present simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slipping</td>
<td>$0 &lt; Fr &lt; 10^{-4}$</td>
<td>$3.4 \times 10^{-4}$ (2 rpm)</td>
</tr>
<tr>
<td>Slumping</td>
<td>$10^{-3} &lt; Fr &lt; 10^{-3}$</td>
<td>$7.5 \times 10^{-4}$ (3 rpm)</td>
</tr>
<tr>
<td>Rolling</td>
<td>$10^{-4} &lt; Fr &lt; 10^{-2}$</td>
<td>$5.3 \times 10^{-3}$ (8 rpm)</td>
</tr>
<tr>
<td>Cascading</td>
<td>$10^{-3} &lt; Fr &lt; 10^{-1}$</td>
<td>0.03 (20 rpm)</td>
</tr>
<tr>
<td>Cataracting</td>
<td>$10^{-1} &lt; Fr &lt; 1$</td>
<td>0.20 (50 rpm)</td>
</tr>
<tr>
<td>Centrifuging</td>
<td>$Fr \geq 1$</td>
<td>1.01 (110 rpm)</td>
</tr>
</tbody>
</table>

Davidson et al [2000] predicted the transition pattern from slumping to rolling by plotting the slump cycle time against $1/\omega$ and demonstrated a linear relationship between these variables. They also obtained the avalanche time $\tau_2$ (explained in section 4.3.2) in the order of 1-2 s. To verify Davidson's results, the following methodology is adapted in the present work.
From the trajectory plots as shown in the Figure 4.17, the slump cycle time is calculated as the difference between in time between consecutive maximum x-positions. This is calculated at various times for each representative particle and then they are averaged out. These results are plotted in the same way as that of Davidson [2000] and are shown in Figure 4.18 for various fill fractions. The avalanche time is in the order of 0.5-0.8 s for the simulation runs.
Ding et al., [2002] presented theoretical models based on overall material balance and geometric parameters to calculate bed turnover time in both the slumping and rolling modes and they suggested that the transition from slumping to rolling occurs when the two turnover times are equal. According to Ding, [2002] the bed turnover during rolling mode is given as
where $\omega$ is the rotational speed, $L$ is the half chord length, $R$ is the radius, $h$ is the shortest distance between the drum center and bed surface and $\delta_m$ is the maximum depth of the active region.

The simulation to check this equation is carried out for the following process parameters:

**Table 4.3:** Process parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Assigned Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder Radius</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Particle Size</td>
<td>6 mm</td>
</tr>
<tr>
<td>Fill Fraction</td>
<td>10% - 30%</td>
</tr>
<tr>
<td>Rotational Speed</td>
<td>5 - 30 rpm</td>
</tr>
</tbody>
</table>

From the simulation data the values of $L$, $h$ and $\delta_m$ are obtained and substituted in the equation 4.9 to obtain the bed turnover time $(t_{br})_r$. From the trajectory plots of the representative number of particles, the bed turnover time is calculated as $(t_{br})_r$ as predicted by simulation = \[ \frac{\text{Simulation time}}{\text{Average no. of cycles}}. \] For example, from the figure 4.17, we obtain $(t_{br})_r$ as predicted by simulation = \[ \frac{90 \text{ s}}{9 \text{ cycles}} \approx 13 \text{ s}. \] The predicted bed turnover time from simulation are presented in table 4.4, along with the values obtained from equation (4.9).
From the table it can be observed that the simulation results are deviating from the results of Ding et al [2002]. The deviations may be due to the following reasons.

Results obtained by the present simulation are based on the transient analysis of the granular solid bed dynamics, whereas derivation by Ding et al [2002] is based on the steady state assumption and on the geometrical pattern of the granular solid bed.

Only uniform spherical particles are taken for the present simulation, whereas the work of Ding et al [2002] is based on a particle size distribution and for a different cylinder diameter.

Table 4.4: Bed turnover time as predicted by equation 4.9 and by DEM simulation

<table>
<thead>
<tr>
<th>Fill Fraction (%)</th>
<th>Angular Velocity (rpm)</th>
<th>( (t_{br})_p ) by equation 4.9 (s)</th>
<th>( (t_{br})_r ) by simulation (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>15</td>
<td>1.17</td>
<td>13</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td>0.931</td>
<td>11.1</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>1.105</td>
<td>9.5</td>
</tr>
</tbody>
</table>

4.5.2 Characterisation of active layer

Since active layer has been declared as the zone, that is responsible for the mixing of solids under going a rolling mode, several studies have been directed to characterizing the active layer depth.

Henein et al. [1983a,b] noted that the active layer depth decreased for smaller particles, decreased with increase in bed depth and increased with the
rotational velocity of the cylinder. Boateng [1993] found that the active layer proportion increased with increase in velocity and decreased with increased drum loading.

Recently Van Puyvelde et al. [2000] characterized the active layer depth based on their experimental results for various rotational speeds and fill fraction. They concluded that the only parameters, which affected the active layer percentage, were the cylinder loading and cylinder velocity. They also found out that particle sizes and cylinder diameter did not affect the active layer depth.

The active layer percentage as predicted by Van Puyvelde [2000] is given as

\[ \%AL = 9.81\ln(N) + 0.438e^{0.08744(50-L)} \]  

(4.10)

The simulation tests are carried out for the same process parameters Table 4.3. The active layer depth from the simulation runs are calculated as explained in section 4.3.4. The results are presented in Table 4.4:

**Table 4.5**: Percentage of Active layer both simulation and equation 4.10

<table>
<thead>
<tr>
<th>Fill Fraction (%)</th>
<th>Rotational Speed (rpm)</th>
<th>% AL (Equation 4.10)</th>
<th>% AL (present study)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>15</td>
<td>41</td>
<td>40</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td>39</td>
<td>38</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>35</td>
<td>32.3</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
<td>33.2</td>
<td>32.9</td>
</tr>
<tr>
<td>30</td>
<td>20</td>
<td>31.9</td>
<td>31.8</td>
</tr>
</tbody>
</table>
It is clearly evident that the DEM simulation carried out in this study is able to predict the active layer depth of granular solids undergoing rolling motion accurately.

The variation of active layer depth with respect to the rotational speed and fill fractions are presented in Figures 4.19 and 4.20. It can be observed that active layer depth increases with rotational speed and decreases with fill fraction, which agrees well with results of Henein et al., [1983a, 1983b]

Figure 4.19: Active layer depth vs rotational speed.
Figure 4.20: Active layer depth vs fill fraction

Increasing the percentage of fill signifies that although more material goes into shearing, the amount of material sheared is distributed over a longer chord length and hence smaller increment in thickness. This results in the observed decrease in percent active layer depth. The decreasing percent active layer depth with increased percent fill is therefore related to geometrical constraints.
4.5.3 Dynamic angle of repose

The variation of dynamic angle of repose with respect to rotational speed obtained through DEM simulation carried out in this work is plotted in Figure 4.21 for various fill fractions. It can be seen that the dynamic angle of repose increases linearly with the rotational speed.

![Figure 4.21: Dynamic angle of repose vs rotational speed](image)

To study the effect of friction between cylinder wall and granular particles, simulation runs were made by varying the coefficient of friction. The values of dynamic angle of repose are plotted in figure 4.22 for various values of coefficient
of friction. The dynamic angle of repose increases rapidly with an increase in the coefficient of friction but levels off for friction coefficients greater than 0.5. These simulation results agree very closely with the observations of Yamane et al. [1998].

![Graph showing the dynamic angle of repose vs coefficient of friction](image)

**Figure 4.22:** Dynamic angle of repose vs coefficient of friction

### 4.5.4 Surface velocity during granular motion

Particle velocity is one of the most important variables that affect individual particle dynamics. Boateng and Barr [1997] measured surface velocity with fibre optic probes and found that the velocity profiles which were roughly parabolic centered about the mid chord length, agrees with the previous experimental works by Singh [1978]. However, these symmetric profiles were not consistent among all the materials tested nor for the same material at different
rotation rates. Decreasing the fill percent at constant rotation rates, caused the velocity profiles to become skewed towards the base, meaning the particles accelerated beyond the mid chord position. MRI studies by Nagakawa et al. [1993] likewise showed roughly parabolic velocity profiles skewed towards the base of the cylinder. One such typical plot is shown in Figure 4.23 which clearly shows the parabolic velocity profile.

The maximum surface velocity is plotted against rotational speed in Figure 4.24 and it can be seen that there is a linear relationship between these two quantities as predicted by Yamene et al. [1998].

![Diagram: Normalised surface velocity along the chord length]

**Figure 4.23:** Normalised surface velocity along the chord length
Figure 4.24: Maximum surface velocity vs rotational speed
4.6 Conclusion:

Using the software code DEMCYL presented in Chapter 3, the simulation of granular solids in the transverse plane of a horizontal rotating cylinder is carried out in this Chapter. The results obtained are presented both at the micro-level and macro-level. The procedure to obtain individual particle velocities, their trajectories, velocity vector plots, dynamic angle of repose, surface layer velocities, active layer depth are presented. This is followed by results showing the six forms of solid body motion with respect to rotational speed along with trajectory plots and kinetic energy distribution for each mode.

For slumping mode, the slump cycle time is validated with the results of Davidson et al., [2000]. Prediction of the transition behaviour based on Froude number from simulation is compared with the results of Mellmann [2001] and the agreement is found to be satisfactory. The bed turnover time against the frequency \( \frac{1}{\omega} \) for various fill fractions demonstrated a linear relationship as predicted by Davidson et al., [2000].

The active layer depth obtained through simulation are compared with the active layer depth predicted by Van Puyvelde et al., [2000a] and the agreement is found to be very good. It has also been found that active layer depth does not depend on the particle size and cylinder diameter as observed by Van Puyvelde [2000a].

It has also been found that the active layer depth increases with rotational speed and decreases with degree of fill as predicted by Henein et al., [1983a, 1983b]. The surface velocity profiles showed a parabolic nature and the maximum surface velocity increased with rotational speed in a linear way as predicted by Yamane et al. [1998].
The dynamic angle of repose is found to increase with the increase in rotational speed. The dynamic angle of repose is also found to increase with coefficient of friction, but after a value of 0.5 for coefficient of friction is almost constant as predicted by Yamane et al. [1998].