2.1 Introduction

Rotating cylinders play a prominent role in the processing of granular materials in metallurgical and chemical industries in which operations such as reduction of iron oxide, calcination of limestone and petroleum coke are carried out. The widespread usage of rotating cylinders in processing can be attributed to such factors as its ability to handle varied feed stocks having large variations in particle sizes and ability to maintain distinct environments; for example reducing conditions within the solid bed coexisting with an oxidizing atmosphere in the freeboard (a unique feature of the rotating reactor that is not easily achieved in other reactors) in the direct reduction of minerals.

In order to improve the performance of the processes taking place inside the rotating cylinder, a better understanding of the transport phenomena in the granular medium inside the rotating cylinder is required. During particle motion, solid particles inside the cylinder undergo various processes like heat exchange, drying, heating, chemical reaction etc. Hence it is very essential to have a fundamental understanding of the processes occurring inside a rotating cylinder, so that it can be designed to function under optimum process conditions.
Since the present work focuses on granular bed motion in the radial direction of a rotating cylinder, this chapter presents a comprehensive review of the literature relating to solid bed motion in the radial direction of the horizontal rotating cylinder. Commencing with a description of the general dynamics of the solid bed movement in a rotating cylinder, a detailed review of the theoretical and experimental studies related to transverse solid bed motion, mixing and segregation behaviour is presented.

2.2 Dynamics of granular motion in rotating cylinders

In normal industrial practice, rotating cylinders partially filled with granular solids having a volumetric hold-up of 10% to 50% of the total volume of the cylinder are used. The cylinder is inclined along its axial length at a few degrees to the horizontal and is rotated along its horizontal axis as shown in Figure 2.1 Horizontally rotating cylinders are slightly tilted along their axis to facilitate the movement of solid particles.

![Diagram of granular bed motion in a rotating cylinder](Spurling, 2000)
The movement of particles inside the solid bed can be resolved into two components, namely, (i) in the transverse direction and (ii) in the axial direction. The transverse movement of the granular medium is perpendicular to the cylinder axis and is responsible for the homogeneity of the solid bed, whereas the axial movement of particles takes care of the shape of the solid bed and the residence time of the particles inside the various zones. The granular materials are slowly conveyed along the cylinder length as a result of continuous circulation and force of gravity down the slope. Earlier works by Sullivan et al. [1927], Seaman [1951] and Boateng [1993] show that the axial motion of the particles is mainly due to transverse movement, since for every particle turnover in the radial cross section there is an axial material advance. Eventhough there is a linkage between particle motion in the transverse direction and particle velocity in the axial direction, the literature deals with these two types of bed motion as independent phenomena. Only in recent past an attempt has been made to quantitatively link the axial velocity to the transverse motion (Perron and Bui [1990]).

2.3 Granular bed motion along the cylinder axis

The bulk motion of a granular bed under steady state operating conditions in a rotating cylindrical reactor has been experimentally investigated by several groups of researchers using laboratory and pilot plant scale devices, commencing with the work of Sullivan et al. [1927] and Seaman [1951]. The movement of solid particles in the axial direction determines the residence time, which is an important parameter for the design of rotating cylindrical reactors. The first attempt to study the residence time of solid particles in a rotating cylinder was made by Sullivan et al. [1927]. Earlier researchers made attempts to predict the mean axial velocity and residence time of the granular materials focusing mainly on the angle of repose of the solid bed, horizontal slope of the rotating cylinder and the angular velocity of the cylinder. Seaman [1951]
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proposed a theoretical analysis for the relationship for residence time determined by axial solids velocity namely;

\[ V_s = 0.955 \frac{R \omega \beta}{\sin \epsilon} \]  

(2.1)

where \( \beta \) is the slope of the cylinder, \( R \) is inner radius of the cylinder, \( \epsilon \) is the dynamic angle of repose and \( \omega \) is the angular velocity of the cylinder.

Perron and Bui [1990] summarized various relations connecting the mean solids axial velocity to slope of the cylinder, rotational speed, loading and dynamic or static angle of repose. By applying a dimensional analysis on the velocity profile for the bed cross-section, they proposed a new formula to predict axial velocity; namely,

\[
V_s = \frac{\psi}{L\pi R} \frac{1 - \psi}{\pi} \left( H^2 \cos \epsilon + 2H \sqrt{2HR - H^2} \right) \left( \frac{1 - \cos \theta}{2 \tan \frac{\theta}{2}} \right) \frac{1}{\theta} \left( \frac{1 - \cos \theta}{2 \tan \frac{\theta}{2}} \right) \]

(2.2)

In this equation, \( H \) is the actual bed depth, \( \theta \) is the bed depth in angular measure, and \( \psi \) and \( \pi \) are exponents determined by dimensional analysis. This expression is unique in that rather than yielding only a mean axial velocity it allows quantitative predictions for local axial velocity through its inclusion of various local parameters: for example, the local transverse velocity profile. However, the validity of the expression has not been clearly established even at the laboratory or pilot scales, and the problems of scale-up to the industrial level still remains formidable.
Nonetheless the most commonly used correlation is still that proposed by Seaman [1951].

Sai et al. [1990, 1992] conducted experiments in rotating cylinder containing ilmenite particles to study the residence time distribution of low density particles, hold-up and bed depth profile and their study supported the conclusion of Sullivan et al. [1927]. Chatterjee et al. [1983] has also reported the same observation. All these studies are based on the assumption that the system is under steady state and only a single type of material is used for deriving the empirical relationships.

A number of industrial situations give rise to transient behaviour in the solids motion, such as start-up, irregularities in the granular feed rate and fluctuations in the physical properties of the granular solid. In order to devise an effective control strategy, and successful process models, it is therefore important to understand the unsteady solids motion resulting from sudden changes in the operating variables, namely the rotation speed, the solids feed rate, and the slope of the kiln axis. There is little published work dealing with the dynamic case except for a small set of laboratory scale experimental measurements of the variation in the discharge rate of solids with time following a step change in each of the operating variables, as reported by Sriram and Sai [1999]. They validated the equations of Perron and Bui [1990] for different experimental conditions covering the steep changes in the feed rate of solids. Ang et al. [1998] developed an empirical equation for the residence distribution for a binary feed mixture of solid particles.

Recently Spurling [2000] has carried out an extensive study, both theoretical and experimental, of the motion of a rolling bed of sand flowing through a laboratory scale cylinder inclined along its length and slowly rotating about its axis, to study the effect of the discharge dam geometry on the steady state hold-up profile. Also, experimental measurements have been made of the discharge flow rate and axial
hold-up profile as functions of time following a step up or down in one of three variables; viz., (i) the mass feed rate, (ii) the rotation speed, and (iii) the axis of inclination, and has been verified with the work of Sai et al. [1992] and Sriram and Sai [1999].

2.4 Granular bed motion in the transverse plane

The motion of a bed of granular solids in the transverse plane of a rotated cylinder can take different forms, as described by Henein et al. [1983a]. As the cylinder rotation speed is increased from zero, six distinct modes of bed behaviour viz., slipping, slumping, rolling, cascading, cataracting and centrifuging are observed, as shown schematically in Figure 2.2.

![Diagram of different modes of solids motion](image)

**Figure 2.2:** Schematic view of the different modes of solids motion (Boateng, [1993])
2.4.1 Slipping Mode

At very low rotational speeds, especially when the friction between the granular bed and the cylinder wall is low, the granular bed behaves as a rigid body. The bed motion is observed to take one of two forms (i) the granular bed remains at rest and the granular solid continuously slides at the wall, or (ii) the granular solid repeatedly moves upward with the cylinder until the bed surface reaches a maximum inclination and then slips at the wall back to a minimum inclination and then resumes rotation.

2.4.2 Slumping Mode

In this mode, the granular solid is lifted like a rigid body by the cylinder wall such that the inclination of the bed surface increases continuously until it reaches an upper angle of repose, then detaches from the upper surface of the bed, and falls as a discrete avalanche toward the lower half of the bed. Following the avalanche the inclination of the surface of the granular solid drops to an angle of repose that is less than the static angle of repose of the granular solid. The slumping frequency is observed to increase with increasing rotation speed, eventually leading to the rolling mode. The change between the slumping and rolling modes of bed motion is not always clearly defined, rather the bed behaviour is found to go through a transition in which the bed changes unpredictably between rolling and slumping behaviour.

2.4.3 Rolling Mode

The bulk of the bed rotates as a rigid body about the cylinder axis at the same rotation speed as the cylinder wall. On the bed surface there appears a thin layer of continuously falling particles forming a plane free surface that is inclined at the dynamic angle of repose of the granular solid to the horizontal plane. Solid particles
mix more effectively in the rolling mode. In the rolling mode, bed material can be divided into two distinct regions, namely a 'passive region or plug flow region' where the particles are carried upward by the cylinder wall, and a relatively thin 'active region or cascading layer' where the particles flow down the slipping upper bed surface as shown in Figure 2.3. In the passive region, granular mixing is negligible and the mixing mainly occurs in the active region. At higher mixer rotational speeds, the continuous flow rolling regime is obtained, in which a thin layer of particles flows down the free surface while the remaining particles rotate as a fixed bed. Transverse mixing in this case depends on the dynamics and results from the shearing and collisional diffusion within the layer.

Figure 2.3: Rolling bed motion: top plane- active (shear) layer, bottom plane - plug flow (non-shearing region), (Boateng, [1993])
2.4.4 Cascading Mode

As the rotation speed is increased further, the particles in the upper corner of the rolling bed are lifted higher before detaching from the cylinder wall, and the bed surface assumes a crescent shape in the cylinder cross-section. This mode of material motion is termed as cascading mode.

2.4.5 Cataracting Mode

On further increasing the rotational speed, centrifugal forces become increasingly important in the motion of particles along the bed surface, the curvature of the cascading surface becomes highly pronounced and particles are projected into the freeboard space from the upper corner of the bed.

2.4.6 Centrifuging Mode

At a Froude number of unity, the granular solid is confined to the inner wall of the cylinder by centrifugal forces. According to Nityanand et al. [1986], the critical rotational speed at which a particle at the cylinder wall starts centrifuging can be calculated from the equation

\[
\eta_c = \frac{30}{\pi} \sqrt{\frac{2g}{D}}
\]  

(2.3)

where \( g \) is the acceleration due to gravity and \( D \) is the diameter of the cylinder.
2.5 Dynamics of granular material motion

Rutgers [1965] has provided a criterion for the dynamic similarity of the rotating cylinder by quantifying the transverse bed motion in terms of rotational Froude number,

\[ Fr = \frac{\omega^2 R}{g} \]  

(2.4)

where \( \omega \) is the rotational speed of the cylinder, \( R \) is the inner radius of the cylinder and \( g \) is the acceleration due to gravity. The slipping, slumping and rolling modes of bed motion have been investigated experimentally by a number of authors, including Henein et al. [1983a, 1983b] and McTait [1998]. The change from the slumping to the rolling mode of bed motion was found to occur at a rotational Froude number between \( 1 \times 10^{-4} \) and \( 1 \times 10^{-3} \) by Henein et al. [1983a] and between \( 1 \times 10^{-5} \) and \( 1 \times 10^{-4} \) by McTait [1998]. Also the mode of bed motion was found to be uniquely determined by (i) the hold-up as a fraction of the cylinder volume, (ii) the rotation speed, (iii) the cylinder diameter, and (iv) the particle size and shape. It was found that the transition from a slumping to a rolling bed occurred at a lower rotational Froude number for the cases of (i) a higher fractional hold-up, (ii) particles of spherical rather than irregular shape, (iii) smaller particles, (iv) cylinders of larger diameter, and (v) granular solids with a lower static angle of repose.

McTait [1998] used sand and glass ballotini, of mean particle size 0.6 and 0.22 mm respectively, rotated in cylinders of diameter 0.105, 0.194, 0.288 and 0.5 m with either smooth or rough walls. The bed motion was classified from observations made through a transparent end plate of the cylinder and the slumping mode of
motion was characterised from the digital images captured by a video camera. It was found that the transition between the slumping and rolling modes of bed motion occurred at a higher rotation speed for a cylinder with rough walls. The scale-up parameters proposed by Henein et al. (1983a) were tested against the experimental observations.

Rajchenbach [1990] identified hysteresis in the transition between slumping and rolling modes of bed motion in a granular bed of 0.3 mm diameter glass spheres, rotated in a cylinder of diameter 190 mm. When the rotational speed was increased, the transition from slumping to rolling took place at a rotational speed of 0.5 rpm, whereas when the rotation speed was decreased, the transition from rolling to slumping occurred at a lower rotation speed of 0.25 rpm.

Henein et al. [1983b] proposed mathematical models governing the transition between various modes of motion such as (i) slipping to slumping (ii), slipping to rolling, (iii), slumping to rolling and (iv) rolling to cascading. It was proposed that the transition from slumping to rolling would take place when the time taken for the shear wedge to slump down the bed surface was equal to the time required for the surface of the bed to increase from the lower to the upper angle of repose.

Ding et al., [2001] used non-invasive PEPT (Positron Emission Particle Tracking) technique to follow the particle trajectory and velocity in the rolling mode. A mathematical model based on the thin-layer approximation was proposed to describe solids motion in the active layer. Reasonable agreement was obtained between the model predictions and experiments. A new parameter termed as the solids exchange coefficient was proposed to characterize particle exchange between the passive and active regions. A theoretical expression for this parameter was also derived. This expression, upon application of the thin-layer approximation, reduces to give an explicit relationship between the solids exchange coefficient and cylinder
operating parameters such as rotational speed and fill percentage, as well as the bed material rheological properties. The solids exchange coefficient was also shown to give a possible scale-up rule for rotating cylinders operated in a rolling mode.

Mellmann [2001] has reviewed the results on transverse bed motion and developed mathematical models to predict the transitions between different forms of transverse motion of granular solid bed. His predictions for the transition behaviours based on the values of the Froude number are presented in Table 2.1. He has also represented the transverse motion of bed materials as a bed behaviour diagram where the wall friction coefficient and the Froude number are plotted against the filling degree and it represents the ranges of individual forms of motion and their limits.

Table 2.1: Different modes of solids motion in terms of Froude Number (Mellmann [2001])

<table>
<thead>
<tr>
<th>Types of motion</th>
<th>Froude number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slipping</td>
<td>0&lt;Fr&lt;10^{-4}</td>
</tr>
<tr>
<td>Slumping</td>
<td>10^{-4}&lt;Fr&lt;10^{-3}</td>
</tr>
<tr>
<td>Rolling</td>
<td>10^{-3}&lt;Fr&lt;10^{-2}</td>
</tr>
<tr>
<td>Cascading</td>
<td>10^{-2}&lt;Fr&lt;10^{-1}</td>
</tr>
<tr>
<td>Cataracting</td>
<td>10^{-1}&lt;Fr&lt;1</td>
</tr>
<tr>
<td>Centrifuging</td>
<td>Fr≥1</td>
</tr>
</tbody>
</table>

2.6 Particle mixing and segregation in transverse plane

Since particle mixing is maximum in the rolling mode, studies related to mixing and segregation have been reported mainly for the rolling regime in transverse direction. Also it has been found that the active layer is predominantly responsible for
the mixing and segregation of solids (Henein et al. [1983a, 1983b], Boateng and Barr [1996, 1997], Boateng [1993], Chakraborty et al. [2000]).

The mixing and segregation of particles in a horizontal cylinder, partially filled with granular solid and slowly rotated about its axis, has been investigated experimentally by a large number of authors, commencing with the work of Oyama [1939], Weidenbaum and Bonilla [1955] and Roseman and Donald [1962]. In general, experiments were performed in a cylinder that was closed at both ends and rotated in batch mode, at a rotation speed resulting in the slumping, rolling or cascading modes of bed motion.

Experimental measurements of radial mixing of mono-sized particles of equal density in a slumping bed has been reported by Clement et al. [1995] and Metcalfe et al. [1995], and in a rolling bed by Khakhar et al. [1997a]. Clement et al. [1995] used a video camera to track the motion of a single particle of diameter 1.5 mm in a short cylinder of diameter 160 mm and depth 15 mm. The particle was found to move at random between different radial positions in the bed. Metcalfe et al. [1995] investigated the radial mixing of table salt of mean diameter 0.6 mm, rotated in a short cylinder of diameter 144 mm and length 24 mm. A simple model for the mixing process was proposed which involves (i) a geometric part corresponding to transport of the slumping wedge from the top to the bottom of the bed, and (ii) a dynamic part, assuming random mixing within the wedge. Good agreement was found between the model and experiment, but the model was found to consistently overestimate the mixing rate.

Khakhar et al. [1997a] used a video camera to investigate radial mixing in a thin cylinder, of diameter 69 mm and length 15 mm, between different colours of (i) angular sugar crystals of diameter 1 mm, and (ii) spherical sugar balls of diameter 1.8 mm. The rate of mixing per revolution was found to increase as the hold-up was
decreased and to be higher for spherical particles. A continuum model for mixing based on an Eulerian model for granular flow, was derived in which the particle motion was assumed to have a random diffusional component perpendicular to the direction of flow. The model was found to be in reasonable qualitative agreement with experimental observations for a range of values of hold-up.

A number of authors have reported that differences in the size, density and shape of particles would result in particle segregation in the falling layer of the circulating bed. According to Henein et al. [1983a, 1983b], Nityanand et al. [1986] segregation is a combination of three mechanisms, namely, (i) percolation, (ii) vibration and (iii) flow segregation. Particle percolation is the process where by smaller particles in a mixture trickle through the voids in a bed of larger particles, thus forming a mixture with larger particles on top and smaller particles on bottom. This type of segregation is a function of the packing characteristics of the bed, particle size and shape. According to Bridgwater [1976], when there is a difference in particle size, there is a chance for percolation to occur. Segregation by vibration typically occurs when a mixture of coarse and fine particles is subjected to prolonged vibration. As a result of the vibrations the smaller particles segregate to the bottom while the larger particles would segregate to the top. Segregation by flow occurs when granular solids are set in motion over an inclined surface. Coarse particles travel down an inclined surface further than fine particles and spherical particles flow easier than angular particles.

During percolation, more dense particles are found to filter downwards through the layer while larger, less dense, particles are displaced upwards. Hence it was proposed that percolation occurs because there is a larger probability of smaller more dense particles falling downwards into voids between particles at lower positions in the flowing layer. Particle segregation in the shear layer was found to produce segregation patterns in the axial and radial directions. In the radial direction,
a horizontal core of finer or denser particles was found to form in the granular bed, whereas in axial segregation either (i) alternate bands of coarse and fine, or light and heavy, particles formed down the length of the granular bed, or (ii) bands of fine, or light, particles formed adjacent to the ends of the cylinder and in a horizontal core along the length of the bed. The rate of radial segregation was found to be of an order of magnitude faster than axial segregation, leading to the conclusion that axial segregation progressed with the radially segregated bed as its initial condition.

Nityanand et al. [1986] have investigated radial segregation in binary and ternary mixtures of spherical acrylic beads of diameter 4, 6.4 and 9 mm, rotated in a cylinder of diameter 0.2 m or 0.4 m, to produce the rolling mode of bed motion. The effect of varying the following experimental parameters was investigated: (i) the hold-up, between 23 % and 52 % of the cylinder volume, (ii) the rotation speed, between 1.4 and 100 rpm, and (iii) the mass fraction of fines in the mixture, between 10 % and 49 %. The granular bed was initially well mixed, and measurements of the segregation process were made using a high speed video camera through a transparent end plate to the cylinder. It was found that:

- The smallest size of particles segregated to form a central core, of similar shape to the bed cross-section, just below a falling layer of coarse particles, and the size of the core was found to be proportional to the mass fraction of the finest component in the granular bed.
- In a ternary mixture, the annulus of granular solid around the core was found to be a well mixed mixture of beads with diameters 6.4 mm and 9.5 mm respectively.
- In all cases the segregation process was found to reach equilibrium within a single turnover of the granular bed. The segregation rate was quantified by
counting the rate at which the finer beads were depleted from a control volume, defined to be equal to the outer annulus of the segregated bed.

- The segregation kinetics was found to be zero order, independent of the hold-up, and the rate of segregation was found to be higher for (i) a larger cylinder diameter, (ii) a higher rotation speed and (iii) a larger ratio of the largest and smallest particle sizes.

Pollard and Henein [1989] reported similar measurements for 3 narrow size ranges of irregular limestone particles with a mean particle size of 2.8, 4.6 and 6 mm. The behaviour of the granular bed was found to be similar to that described for spherical particles, except that the segregated core was found to contain a small fraction of larger particles. Henein et. al. [1985, 1987] investigated radial segregation by sampling the composition of the granular bed in a cylinder of diameter 0.4 m rotated to produce the slumping or rolling mode of bed motion. It was found that the bulk density of a binary mixture of two particle sizes had a minimum value for a certain ratio of the two components, and the density of the segregated core was shown to be close to this value for a granular bed composed of a binary mixture of limestone particles.

Khakhar et al. [1997b] investigated radial segregation between spherical particles of steel and glass, both about 1.6 mm in diameter, but with a weight ratio of about 3.8, in a cylinder of diameter 75 mm. The effect of varying two experimental variables was investigated: (i) the number fraction of the dense particles, and (ii) the hold-up. It was found that the dense particles formed a central core of similar shape to the bed cross-section irrespective of the hold-up. The core size was found to increase, and the boundary of the core to become more sharply defined, as the number fraction of dense particles was increased.
There is little experimental evidence to quantify the effect of particle shape on radial segregation, or segregation in a granular solid made up of many different particle sizes, and in the limit, a continuous particle size distribution.

A number of authors have proposed quantitative models for granular flow, mixing and segregation in the cross-section of a cylinder. McCarthy et al. [1996] extended the model of Metcalfe et al. [1995], for a slumping bed, by using particle dynamics to model the effect of segregation on rearrangement of granular solids inside a falling wedge, but no attempt was made to validate the model experimentally. Boateng and Barr [1996, 1997], Boateng [1993] and Khakhar et al. [1997a, 1997b] derived Eulerian models for particle mixing in the rolling mode and reported good quantitative and qualitative agreement with their experimental observations.

The velocity profile across the rolling bed at mid chord has been measured by Nakagwa et al. [1993], using magnetic resonance imaging and by Boateng [1993], using a fibre optic method. The results of both the authors agree considerably. It has been commonly assumed that the velocity profile in the active layer was symmetric about the mid chord. But Boateng [1993] observed that for larger cylinders the velocity profiles become skewed. Henein et al. [1983 a, b] noted that the active layer depth decreased for smaller particles; decreased with increasing bed depth and increased with the rotational velocity. Boateng [1993] found that the proportion of active layer increased with increasing velocity and decreased with increased cylinder loading.

The model of Yang and Farouk [1997] predicted that the active layer thickness increased for finer particles. This was not consistent with the results of Henein et al. [1983a, 1983b] and could be due to the use of different experimental materials and/or conditions. Boateng and Barr [1997] studied the effect of the end zone on the active layer velocity and proportion. They showed that near the end piece
of a cylinder the angle of repose of the material was enhanced and this resulted in a significantly higher velocities in the active layer at the end piece compared to velocities in the bulk.

Hogg and Fuerstenau [1972] described the mixing in the transverse section of a rolling bed as a combination of the convective and diffusive mixing mechanisms. Lehmberg et al. [1977] measured the dynamics of mixing of solids in an experimental cylinder but did not quantify the mixing rate. Their experiments involved a 310 mm cylinder and they mixed 0.80 mm sand, which consisted of a batch of approximately 90% white and 10% black. They used a rotational velocity of 2 rpm and found that the bed became fully mixed 42 seconds after the rotation was started.

Pershin and Mineav [1989] simulated the mixing in a transverse section of a rolling bed using a concentric layer model. This model considered mixing when material moved from one sub layer to the next and only one transaction was allowed per bed rotation. This model had the obvious drawback of assuming that mixing depended only on the number of rotations and not on the dimensions of the bed. Furthermore, this model focused on material moving closer to the center of the bed such as is the case for segregation.

Woodle and Munro [1993] studied the mixing rate from a statistical point of view and found the rate of mixing to be constant until the bed became fully mixed. There was a random fluctuation in this fully mixed value due to random distributions of the material. They used cylinder loadings of 3% to 15% for 12mm particles and found that the bed became fully mixed in 17 to 46min.

Boateng and Barr [1996, 1997] and Boateng [1993] devised a model of mixing and segregation using granular mechanics. The focus of their work was on the fully segregated bed and heat transfer from the gas phase to the bed, which had
important implications for calcining in rotary kilns but was not useful to predict the
dynamic mixing behaviour in a rotating cylinder. The modelling of both the mixing
rate and the final amount of mixing of solids have been proposed by Van Puyvelde et
al. [2000a,2000b] based on the dynamic data collected from experimental results of
Van Puyvelde et al. [1999].

As can be seen from the above review, several models has been proposed in
the literature but each model is limited by its individual experimental conditions.
Numerous variables in the models, such as cylinder size, the material size and type,
the rotational velocity and the analysis method make it difficult to compare the results
of one model with those of the other. This difficulty is further enhanced by the lack of
a full disclosure of experimental conditions.

2.7 Modelling the flow of granular materials

According to Hogue and Newland [1994], the methods to simulate the
behaviour of granular materials may be classified by two approaches, namely;
Continuum Mechanics Method (CMM) or Macroscopic Modelling and Discrete
Element Methods (DEM) or Microscopic Modelling.

2.7.1 Macroscopic modelling or Continuum theory

Primary challenge in granular flow modelling is not in setting up the
conservation equations for mass, momentum and energy but in establishing the
stress/strain relationship for the particulate mass as this relationship depends on the
flow regime and vice versa. Davies [1986] has compared the observed behaviour of
granular materials, subjected to shear stress, to other common types of flow behaviour
as shown in Figure 2.4.
The figure 2.4 shows the shear stress as a fraction of a dilation factor $\lambda$, defined as,

$$\frac{1}{\frac{1}{v^*} - 1}$$

(2.5)

where, $v$ is the volume concentration of solids, $v^*$ is the minimum possible void fraction that the material can maintain. For granular materials in static condition, the particles flow together into a rigid grid, which means that some degree of stress can be sustained without inducing flow. However as the stress approaches some critical level, the particles begin to ride up on one another and the grid commences dilation (Boateng and Barr [1996,1997]). At the critical stress, the dilation, $\lambda$, reaches a...
maximum and the material begin to flow. Once this occurs the shear stress shows an incipient steep decline with increasing strain rate and it is this initial behaviour, which distinguishes granular flow from that of other flows. Beyond a certain rate of strain, the stress begins to increase again and the relation between the shear stress and the rate of strain turns non-linear. These fundamental aspects of the flow behaviour of granular materials or bulk solids similar to that in rotary kilns have been reviewed by Savage [1979].

The mechanisms of momentum transfer and hence stress generation for granular flows include the following:

- Static stresses resulting from the rubbing between particles, which is independent of strain rate.
- Translational stresses resulting from the movement of particles to regions having different velocity.
- Collisional stresses resulting from inter-particle collision, which result in transfer of both momentum and kinetic energy.

The relative importance of these three mechanisms will depend on both the volume concentration of solids within the solids bed, i.e., the dilatancy factor, $\lambda$, and the rate of strain. The static contribution dominates at high particle concentration and low strain rates. In this situation, the particles are in close contact and the shear stresses are of the quasi-static, rate independent Coulomb-type as described in soil mechanics literature (de Jong [1964], Spencer [1964], Mandl and Luque [1970] and Roscoe [1970]).

Conversely at low particle concentrations and high strain rates, the mean free path of the particles is large compared to particle diameters and interchange of particles between adjacent layers moving at different mean transport velocities may
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dominate stress generation, analogous to turbulent viscosity in fluid flow. At moderate particle concentration and high strain rate, collision between particles will dominate stress generation because in this situation there are rarely any void spaces of sufficient size for the interchange of particles over significant distance. The case pertaining to low and moderate particle concentration has been termed as the grain inertia regime (Bagnold [1954]) and under these conditions the dynamics of the actual particle collision becomes important.

Reviews of continuum methods are widely available in the literature (Runesson and Nilsson [1986], Polderman et al. [1987], Johnson and Jackson [1987], Savage and Lun [1988], Savage [1988], Campbell [1990], Gu et al. [1992], Adams and Briscoe [1993], Abu Zaid and Ahmadi [1993]). Khakhar et al., [1997a, 1997b] analyzed the flow behaviour macroscopically using continuum model by taking averages across the active layer to obtain average velocity variation along the layer.

2.7.2 Microscopic modelling or discrete element modelling

The discrete element method (DEM) is based on the Lagrangian approach for the simulation of the motion of granular material on the microscopic level of particles. It means that DEM can be used to calculate quantities that are difficult to obtain experimentally and can be used to improve continuum mechanics method. With the development of powerful computing machines and various numerical techniques more and more attempts have been made to study granular flows using discrete element methods (Kenji Yamane et al., [1995], Chakraborty et al., [2000], Rajamani et al., [2000]). The DEM can be divided into three main classes

- Statistical Mechanics models
- Classical Newtonian dynamics models
- Hybrid models
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Statistical Mechanics models: Statistical mechanics models use stochastic components in particle displacements, for example the Monte Carlo method (Devillard, [1990], Camp and Allen [1996]), the cellular automata method (Baxter and Behringer [1990]) or the random walk approach (Caram and Hong [1991]).

Monte Carlo method was first introduced into the field of granular materials to study the size segregation of binary mixtures undergoing vertical vibrations (Rosato et al., 1987). But this method has several limitations, for example; (i) no physical time scale enters the model since the collision time is assumed to be zero (ii) the normal restitution coefficient has to be zero in order to minimize the potential energy during each particle move. It is therefore difficult to relate the physical material properties to the random walk process which might be the reason why this method has not attracted much attention in the field of granular materials in recent years.

The use of cellular automaton models to study granular materials dates back to the introduction of the concept of self-organized criticality, where sandpile avalanche statistics were used (Bak et al., 1987). In these models, space is discretized into cells which have the size of the particles and can either be occupied or empty. The particle dynamics is modeled by specifying a set of particle collision rules which apply when certain conditions are fulfilled, e.g. the local surface angle (slope) exceeds a threshold value. Theses rules were later refined by deriving them from experiments to study the outflow rate (Baxter and Behringer, 1990), stagnation zones (Baxter and Behringer, 1991] and the segregation process during particle outflow from two-dimensional hoppers (Fitt and Wilmott, 1992]. But as with most models working on lattices, the surface angle is mostly given by the topology of the underlying lattice and only identical particles were studied so far. A direct connection of the update time and the physical time is also missing which greatly reduces the scope of the cellular automaton approach. It has been argued (Baumann, 1997) that
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the discretization of the particle velocities leads to contradictions with experimental results and to unphysical behaviour.

Random walk model by Caram and Hong, [1991] focuses on the overall geometry of highly packed granular particles through which voids diffuse under the influence of gravity (Hong, [1993]). It is based on a random walk process on a discrete lattice and was mostly used to study the particle outflow from two-dimensional hoppers (Caram and Hong, 1992]. The model involves many simplifications and has certain limitations: (i) the void diffusivity which enters the model is not correlated to any physical or geometrical quantity, (ii) the voids maintain their integrity as they move upwards whereas they disperse in real granular systems and (iii) stresses do not appear in the model but play a crucial role.

Classical Newtonian dynamics models: Classical Newtonian dynamics models use equations of particle dynamics derived form Classical Newtonian mechanics for each particle. These models can be divided into two groups namely;

- Event-driven methods (EDM)
- Time-driven methods (TDM)

The event driven method (EDM) or hard sphere method is based on instantaneous collisions, which means that the state of the particles is updated only when the event occurs, i.e., when the particles collide. Energy dissipation during collisions is defined by the coefficient of restitution, which can be set according to Newton's law of restitution, or Poisson's hypothesis. However, the principle of energy conservation may be violated under certain conditions by Newton's law and Poisson's hypotheses (Hogue and Newland [1994]). This method has been used to study simple shear flow of frictional spheres (Lun and Bent, [1994]), one-dimensional
particle dynamics with energy input (Du et al., 1995) and the collision process in two dimensions (McNamara and Young, 1996). But for large particle numbers and low restitution coefficient, the system can undergo an infinite number of collisions in finite time leading to a kind of clustering termed as inelastic collapse and this has been found in one-dimensional (McNamara and Young, 1992) and two-dimensional systems (McNamara and Young, 1993). In such cases the event driven algorithm breaks down and requires further refinement to take care of the inelastic collapse.

**Time driven methods (TDM) or soft sphere method** (pioneered by Cundall and Strack [1979]), is best suited when the time of collision between real particles is larger than the time of the mean free path of the particles. In this case the current state of the particles at a particular time is updated after a fixed time step, which is smaller than the smallest time of impacts. The state of the particles is obtained by the time integration of the Newton’s equations of motion for the translational and rotational motion for each particle in the granular medium. The inter-particle forces acting on the system are of key importance, apart from the external forces like the frictional force, damping force and the gravitational force. At every time step, the forces and momentum acting on each particle is tracked and the velocities and accelerations are assumed to be constant during that time step. Only the collisions and the contact forces affect the neighbouring particles. Particles are treated as contacting elastic bodies which may overlap with each other. Contact forces depend on the overlap geometry, materials properties and the dynamics of the particles.

A characteristic feature of the soft sphere model is that they are capable of handling multiple particle contacts, which is of importance when modelling quasi-static systems like granular flows in rotating cylinder. Hence this method is chosen for the present work to study the dynamics of granular material motion in a rotating cylinder. A good review of soft sphere method or discrete element simulation, is presented by Algis Dzingys and Bernhard Peters [2001], Cleary [1998a, 1998b,
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Hybrid models combine ideas of statistical mechanics and Classical Newtonian Dynamics models. The particle motion is simulated using hard sphere approach using pseudo-random coefficients of restitution for energy dissipation.

2.8 Experimental work

In recent years experimental techniques such as Positron Emission Topography (PET), Positron Emission Particle Tracking (PEPT), Magnetic Resonance Imaging (MRI) and Gamma Ray Tomography (GRT) have been used to study the granular dynamics in rotating cylinders.

2.8.1 Positron Emission Topography:

The Positron Emission Topography (PET) has now become a widely used technique in medicine, and was developed at Birmingham University by Hawkesworth et al. [1991], Parker et. al. [1993, 1994]. PET is a technique in which, radioisotopes that emit positrons are injected into the system under consideration say, a rotating cylinder. Positrons are similar to electrons, but they have a positive charge, whereas electron has a negative charge. The collision of a positron and an electron in the system results in the emission of gamma rays, which can be detected and used to note the location of activities of each particle inside the cylinder. The newer models of positron emission topography scanners allow for more accurate and fast performance.
2.8.2 Positron Emission Particle Tracking:

Positron emission particle tracking (PEPT) is a technique for following the motion of a radioactively labeled tracer particle. It uses a radio nuclide which decays by positron emission and detects the back-to-back rays produced when a positron annihilates with an electron. The tracer position is calculated by triangulation from a small number of back to back ray pairs, to within 2 mm 20 times per second for a slow moving tracer particle, and to within 5 mm 250 times per second for a tracer moving at speeds greater than 1 m/s. These rays are very penetrating and allows non-invasive tracking in actual engineering structures.

The PEPT technique has been used by a number of authors to investigate granular flow and mixing in various kinds of processing equipment including: (a) Lodige mixer (Broadbent et al., [1993]), (b) partially filled horizontal cylinder slowly rotated about its axis, (Parker et al., [1997]), (c) planetary mixer, consisting of a vertical cylinder stirred by a vertical blade, (Hiseman et al., [1997]), (d) ploughshare mixer, (Jones and Bridgwater [1998]) and (e) horizontal cylinder stirred by a single flat blade, (Laurent et al. [2000]).

2.8.3 Magnetic Resonance Imaging:

Magnetic Resonance Imaging commencing with the work of Nakagawa et al. [1993], a number of authors have reported the results of Magnetic Resonance Imaging (MRI) measurements of granular flows. The technique allows non-invasive measurement of the particle velocity, concentration and velocity fluctuations. The technique works well at all positions in the sample and does not require mechanical markers and has no preferred orientation of measurement.
2.8.4 Gamma Ray Tomography:

In Gamma Ray Tomography, gamma rays or X-rays are scattered back to a detector array for producing an image. Since the X-ray response to scattering differs from that of absorption and transmission, reflected images can reveal details that shadow images miss. In addition, reflection mode only requires access to one surface. This can have advantages where the high-density bulk of an object obliterates all transmission shadows but a volume of interest is near one surface. The gamma ray tomography system for the measurement of radial voidage consists of a 67.5 microcurie 137Cs gamma source (disc source of 2 cm diameter), sodium iodide (NaI) with thallium (TI) activated scintillation detectors (BICRON, 5 in number), a photomultiplier tube, a preamplifier, a multi-channel (5 channels) analyser, data acquisition systems (para electronics) and related hardware and software. In the multi-channel counter, each channel generates an amplified energy pulse for a corresponding input pulse. These amplified generated pulses lie between the base line energy ($E$) and the base line window energy ($E \pm \Delta E$) in window mode operation.

2.9 Conclusion

This Chapter provides a thorough literature survey on the transport phenomena of the granular bed motion in a horizontal rotating cylinder. Literature on the axial solid bed movement is surveyed first, followed by a detailed description of the different modes of motion of the granular solid bed in the transverse plane. The transition behaviour as predicted by Henein et al. [1983a, 1983b], Ding et al. [2002] and Mellmann [2001] is discussed at length. From the discussions it has been observed that transition behaviour depends mainly on the rotational speed, fill fraction, particle size and cylinder diameter. There is no unified empirical relation of these variables, which predicts the transition behaviour exactly.
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Literature on particle mixing and segregation during the rolling mode of the granular solid bed is presented next followed by the description of the mechanism of particle segregation. The works of Henein et al. [1983a, 1983b], Nityanand et al. [1986], Metcalfe et al. [1995] are discussed in detail. It has been observed that the effect of particle shape on radial segregation and the segregation in a granular solid made up of many different particle sizes has not been studied yet.

This is followed by the modelling studies related to the flow of granular materials both on macroscopic scale and on microscopic scale. Several models have been proposed in the literature, but each model is validated only with the corresponding individual experiments, and has not been validated with other set of experiments or with other models.

Finally the experimental techniques used to measure the particle positions, velocities, accelerations etc. of granular particles is presented. Hence based on the literature it has been concluded that a unified model without any empirical relations of the process variables is necessary for granular dynamics in a horizontal rotating cylinder, to predict the dynamics in a more realistic way and to study the effect of various process variables like rotational speed, fill fraction etc.