

## **7 - EMPIRICAL RESULTS & FINDINGS**

This chapter discusses all the results and findings of the research. Here we shall look first at the correlation between the returns of the Commodities and Nifty Futures Indices. Then we shall continue by analyzing the various portfolios that are constructed on the basis of MPT theory and subsequently commodities futures as inflation hedge is analyzed in detail. And lastly this chapter includes results of the testing of hypotheses formed in chapter six.

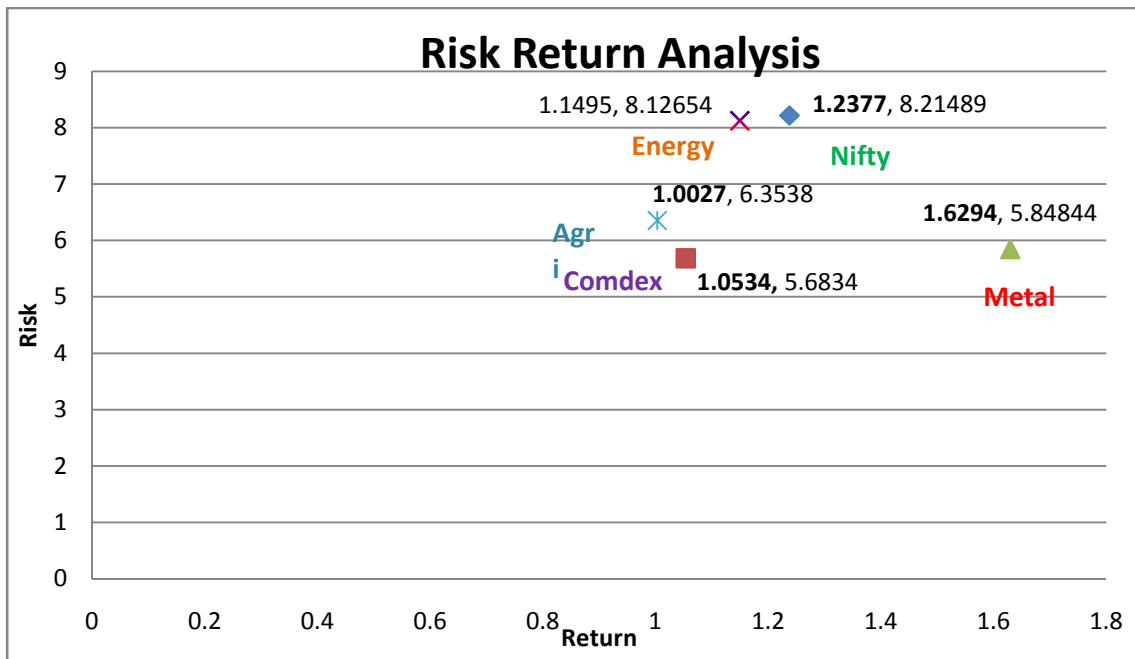
The empirical analysis of commodity investing is based on exactly 6 years and 6 months (or 79 months) that ranges from June 2005 to December 2012. We will apply all the techniques and methods that we have discussed so far to our data sets. The risk reduction benefit of diversification is an important ideology of finance. However, diversification studies typically employ only stocks and bonds. Though a few studies have been carried out in the context of international markets on the benefits of adding commodities to enhance equity diversification (typically only using a commodity index or one category of futures to reduce risks), a meticulous study is conspicuously missing in the context of Indian markets. As commodity futures prices are influenced by factors different from those that affect financial assets, commodity futures are perceived to be excellent candidates for the diversification of equity and bond portfolios.

### **7.1 - RISK – RETURN CHARACTERISTICS**

We hereby conduct empirical analyses of data series and on the basic idea that risk should be rewarded. As the academic literature provides limited studies into the statistical characteristics of asset classes, this study uses ex-anate analysis to get the risk return characteristics of assets.

With the purpose of providing an overview of the data, the analysis of our results begins with a glance at the performance of the indices. The focus is on monthly data and the chart below provides an overview of performance of all the indices Nifty, Comdex, Energy, Metal and Agri from June 2005 to Dec 2011. Appendix B provides the graphs showing monthly return from the indices.

**Chart 7.1 – Risk Return Analysis of commodity & stock indices**



The chart shows the average return and the variance (risk) of all the five indices under the study. The main objective here is to show the performance of indices in terms of risk and return. This is the starting point as we are seeing the standalone performance of the indices without combining it in a portfolio to achieve diversification benefit. After this we can move forward to develop portfolios by combining stocks and commodities.

**Some observations** - The first look at the chart clearly shows that Metal has totally dominated the other indices in terms of providing highest return (1.62%) with the risk level of just 5.84 which is the second lowest one after Comdex (5.68) and thus, the difference in risk level of Comdex and Metal is very minor and can be considered totally negligible. Further in terms of return Metal is an outperformer, next comes Nifty but the difference in risk level of Nifty and Metal is quiet significant (i.e. 5.68 of Metal vs. 8.21 of Nifty).

Thus, to conclude we can say that out of all indices the performance of Metal has been best during the time period in terms of risk and return characteristics. However this does not show which index will be the best performer in terms of portfolio. As the

objective of this study is to see that whether there is any advantage in combining stock with commodities we shall move further and develop optimal portfolios.

Now, in the Table 7.1, we shall see the descriptive statistics for the month for the monthly returns for the equity, commodity indices. In addition to minimum, maximum, mean and standard deviation, also skewness and kurtosis are taken into consideration.

<b>Table 7.1 - Monthly Descriptive Statistics 2005-2011</b>								
	<b>Min</b>	<b>Max</b>	<b>Mean</b>	<b>Std. Dev</b>	<b>Skewness</b>	<b>Std. Error</b>	<b>Kurtosis</b>	<b>Std. Error</b>
<b>Nifty</b>	-35.14	24.52	1.2377	8.21489	-0.947	0.271	4.259	0.535
<b>Comdex</b>	-22.1	14.93	1.0534	5.6834	-0.827	0.271	2.852	0.535
<b>Metal</b>	-18.49	21.04	1.6294	5.84844	-0.079	0.271	2.152	0.535
<b>Energy</b>	-27.31	19.79	1.1495	8.12654	-0.529	0.271	1.36	0.535
<b>Agri</b>	-20.8	27.81	1.0027	6.3538	0.236	0.271	5.168	0.535

Traditionally, we expect commodities to have high volatility and return than those of large capitalization stocks. Contrary to this expectation we here find that Nifty is having the highest volatility and looking at the table we find that Metal has been the best performer amongst all. Further energy is the worst performer as the volatility is as high as Nifty and the return is very low. On the other hand Agri is having return as good as Comdex but with high volatility. This is just comparison of the raw data and now we shall consider it in portfolio context.

We can clearly see that metal has dominated amongst all other indices, metal has outperformed totally. Next comes Nifty, the expectation was that Nifty would be having highest return, and much lower variance compared to other commodity indices but the scenario is different here. We can see that Nifty is having the highest volatility.

As pointed out by previous studies, Gorton and Rouwenhorst (2005) and Erb and Harvey (2006), the returns on financial instruments sometimes deviate from a normal distribution, i.e. displaying skewness and having so called fat tails. The distribution has positive skewness if, in some sense, the tail of high values longer than the tail of low values, and negative skewness if the reverse is true.

Table 7.1 shows that all the indices besides Agri is having negative skewness. Nifty and Comdex are having higher negative skewness whereas Agri is having a small positive skewness. In addition, Comdex, Metal and Energy indices display relatively low kurtosis, whereas Nifty and Agri is having a higher kurtosis.

If on the other hand we see the reward to variability ratio Metal is a best performer with highest excess return over unit of risk taken and Nifty and Comdex stand equally at second position and Agri is the worst performer according to the reward to variability ratio. The strong performance of Metal index is clearly visible, however, as previously discussed and stated by the theory, the attractiveness of these asset classes does not reside in their individual risk-return ratio, but in their hedging potential. This is discussed in detail later.

With the purpose of providing an overview of the data, the analysis of our results also includes a glance at the performance of the indices.

**Chart 7.2 – Historical performance of the indices**

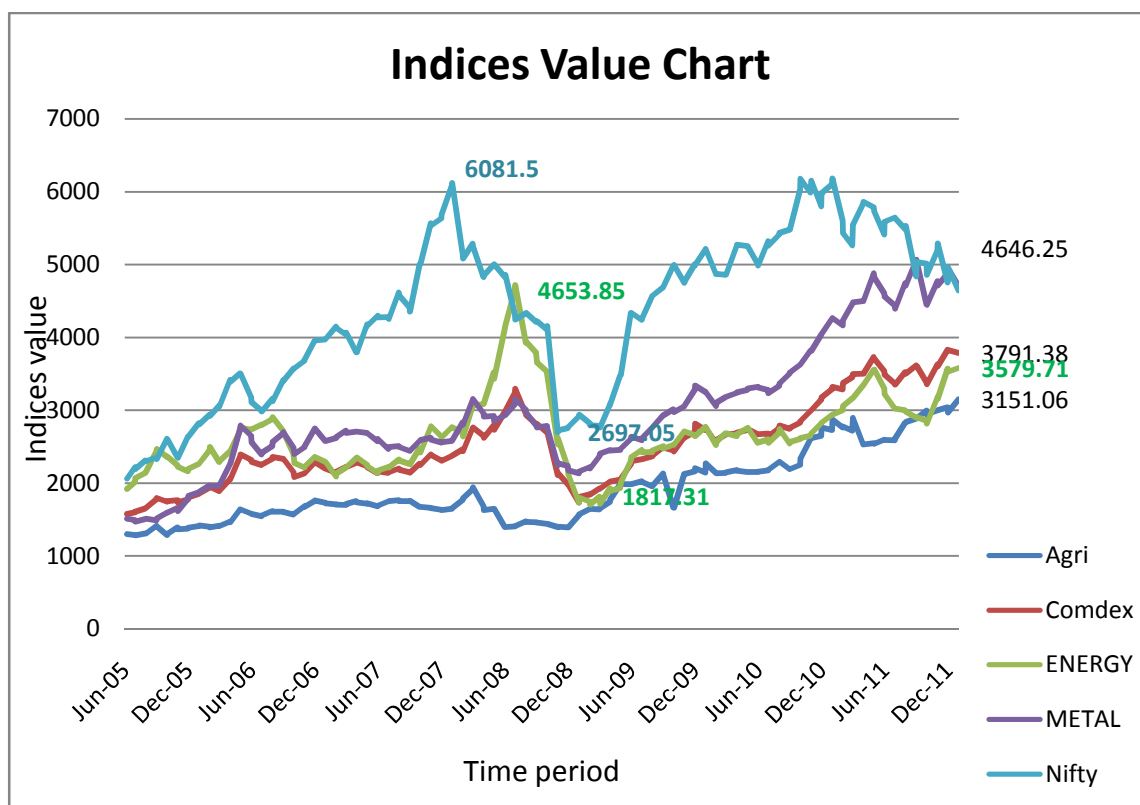


Chart 7.2 shows the performance of different asset classes namely Agri, Comdex, Energy, Metal and Nifty. The data set starts from June 2005 when commodities indices were introduced for the first time in India.

First we shall consider Nifty index. Nifty was on the level of around 2000 in June 2005 and it saw the highest peak when it crossed above 6000 by the end of 2007 and 2008 again saw the huge downfall cause of global recession and Nifty again reached its all time low of around 2500 by the end of 2008. We can also see that Nifty closed around a level of 4500 in Dec 2011.

Next, we analyze Metal index which has shown immense gain and outperformed every other index. Metal started with around 1500 in June 2005 and ended at a high note of 4700 in Dec 2011. From the above chart we can see that when Nifty crossed the level of 6000 Metal index was around 4600 only. Thus, Metal on one side failed to cross the 5000 mark; on the other hand it shows that Metal has a lot of stability in performance.

Lastly in terms of absolute value Comdex, Energy and Agri level by Dec 2011 was 3700, 3500 and 3100. And we can see from the above chart that Energy's performance has been worse in terms of absolute value as on June 2005 it was having higher value than Comdex and Agri.

### **7.1.1 - Normality Test - Jarque-Bera**

Since Markowitz assumes a normal distribution (also called Gaussian distribution) in the mean-variance framework, there is an importance to study the effects of non-normal distribution of the sample returns. There exist many normality tests, such as the Lilliefors test or Kolmogorov–Smirnov test. But the use of Jarque-Bera test is preferred in our research since it takes into account the higher central moments, skewness and kurtosis. The Jarque-Bera test is a test of goodness-to-fit measure of departure from normality. The test is a joint-hypothesis test that is based on skewness and kurtosis. The null hypothesis is that both of the higher central moments are equal to or close to zero at a specified significance level. In other words this means that if the skewness and/or

excess kurtosis deviate significantly from the value zero, there will be an increased Jarque-Bera statistics. The Jarque-Bera (1987) test is a goodness-of-fit (i.e. how well it fits a set of observations.) So the higher the statistic one gets, the less normally distributed the sample is. Formula for JB:

$$JB = \frac{n}{6} \left( S^2 + \frac{(K - 3)^2}{4} \right)$$

**Equation 7.1**

We will use the significance alpha level of 5% to test our data in gretl which returns the hypothesis result, JB-statistics and the significant p-value result. (Jarque and Bera, 1987)

**7.1.2 - Results of Normality Test**

<b>Table 7.2 – Results of Jarque-Bera Test</b>			
	<b>Jarque-Bera test statistics</b>	<b>p-value</b>	<b>Null hypothesis</b>
<b>Nifty</b>	61.9869	3.46516e <sup>-014</sup>	Rejected
<b>Energy</b>	8.29303	0.0158195	Rejected
<b>Comdex</b>	30.9026	1.94795e <sup>-007</sup>	Rejected
<b>Metal</b>	12.494	0.00193627	Rejected
<b>Agri</b>	75.7095	3.62981e <sup>-017</sup>	Rejected

So we can observe that for each p-value being below the critical value of 5% significance level we reject the null hypothesis that the data set is normally distributed. If the distribution has very high peak and have some skewness to the right than there is some exclusion of some negative returns that is not even within the curves. This is very crucial to notice, because as an investor you don't want to ignore any potential losses. The returns on financial returns often deviate from a normal distribution, display skewness, and have "fat tails"(Gorton & Rowenhorst 2004). There do exist other risk measurements that could be used in further studies such as semi-variance, modified value at risk and downside risk, but we have not gone any further into any of these as it is beyond the scope of the study.

## 7.2 - THE CORRELATIONS OF COMMODITIES WITH OTHER ASSET CLASSES

Correlation is no doubt the single most important parameter in portfolio theory, as it is used to measure the dependence between the returns on different assets or asset classes (Kat and Oomen 2006). The rule is simple: lower correlation provides for good diversification and highly correlated assets or asset classes are to be avoided. As the literature reveals that one of the most attractive features of commodity futures as an investment class is its assumed low correlation with stocks and bonds. Previous studies like Gorton and Rouwenhorst (2005) find a very low or even negative correlation between the returns of commodity futures and stocks and bonds. Here, we examine the correlation, measured as Pearson's linear correlation coefficient, of Nifty futures returns with MCX commodity indices. In the table below the monthly correlations between the selected asset classes are displayed.

		Nifty	Comdex	Metal	Energy	Agri	Inflation
Nifty	Pearson Correlation	1	<b>.333(**)</b>	<b>.345(**)</b>	<b>.317(**)</b>	<b>-.008</b>	<b>-.059</b>
	Sig. (2-tailed)		.003	.002	.004	.943	.603
Comdex	Pearson Correlation	<b>.333(**)</b>	1	<b>.799(**)</b>	<b>.875(**)</b>	<b>.266(*)</b>	<b>.279(*)</b>
	Sig. (2-tailed)	.003		.000	.000	.018	.013
Metal	Pearson Correlation	<b>.345(**)</b>	<b>.799(**)</b>	1	<b>.484(**)</b>	<b>.241(*)</b>	<b>.137</b>
	Sig. (2-tailed)	.002	.000		.000	.032	.228
Energy	Pearson Correlation	<b>.317(**)</b>	<b>.875(**)</b>	<b>.484(**)</b>	1	<b>.028</b>	<b>.281(*)</b>
	Sig. (2-tailed)	.004	.000	.000		.804	.012
Agri	Pearson Correlation	<b>-.008</b>	<b>.266(*)</b>	<b>.241(*)</b>	<b>.028</b>	1	<b>-.089</b>
	Sig. (2-tailed)	.943	.018	.032	.804		.434
Inflation	Pearson Correlation	<b>-.059</b>	<b>.279(*)</b>	<b>.137</b>	<b>.281(*)</b>	<b>-.089</b>	1
	Sig. (2-tailed)	.603	.013	.228	.012	.434	

\*\*Correlation is significant at the 0.01 level (2-tailed).

\* Correlation is significant at the 0.05 level (2-tailed).

The above table provides insight into the diversification potential of each asset class. MCX AGRI is having a high diversification potential when combined with Nifty; as it is the only one having negative correlation and thus maximum diversification benefit. Previously we saw that Agri was the worst performer but now as we have the correlation data, we can see in terms of portfolio context the maximum diversification benefit is from Agri. Thus, on one side in standalone context Agri is worst performer and on the other hand in portfolio context it's the best, in terms of increases return and reduced risk. Comdex on the other hand is having very high correlation with Metal and Energy and thus there is no use of combining them.

Further as our study is based on combining Equity Portfolio with Commodities we shall move forward to test our hypothesis that is constructed in Chapter 6. Here, our dilemma is whether commodity can be considered as an asset class in Indian context or not.

#### ***HYPOTHESIS TESTING –***

##### **Hypothesis 1: Correlation between commodities and stocks is larger than zero.**

This hypothesis is constructed to find out that whether commodity can be considered as an asset class in Indian scenario or not? US research papers find a negative or zero correlation between stocks and commodities, e.g. Gorton and Rouwenhorst (2005) and Buyuksahin, Haigh and Robe (2007) and other international literature have revealed that GSCI and DJAIG have significant negative correlation with S&P 500. But, the result of this study is completely different and we can see that a significant positive correlation exist between stock and commodities. Nifty and Comdex is having a correlation of 0.333, Nifty vs. Metal is 0.345 and Nifty vs. Energy is 0.317; all these correlation are significant at 1% level which means that we are 99% confident about our result. The result is disappointing if we compare it with international literature but that doesn't mean that commodity investment in Indian scenario has no diversification benefit. MPT theory states that if the correlation is less than perfect positive between the asset classes then diversification benefits can be achieved. So, even though commodity investment in India is not having as much advantage as shown in international literature but still it has some advantage from the view point of



diversification of portfolio. Thus, portfolio optimization can be achieved by combining Nifty with MCX indices, and with this result we will move forward and develop optimal portfolios by combining Nifty and MCX indices in the next section.

Well the reason that I can be considered for such positive relationship between stock and commodity in India is that; Indian economy is more or less dependent on commodity markets. For example oil and gas & steel etc. which make Indian markets more exposed to commodities than for example USA. Therefore, in contrast to previous research that has found a zero or negative correlation between commodities and US stocks, we suppose there to be a positive correlation, though not very large, between commodities and Indian stocks.

Lastly Agri is analyzed and we find that it has a negative correlation of  $-0.008$  almost very near to zero so we can conclude that there is no correlation between Agri and Nifty and this makes Agri to give the maximum diversification benefit. But, this result is not significant so we cannot confidently say that Agri will give maximum advantage in terms of diversification of the portfolio.

### **7.3 - FINDINGS OF PORTFOLIOS ON EFFICIENT FRONTIERS**

The return patterns analyzed above are used to create mean-variance efficient portfolios according to the Markowitz framework, described in the Methodology. At a given level of expected return, the optimal asset allocation in order to minimize the standard deviation is calculated. We consider how commodity futures perform when considered in the portfolio context, as from the above analysis we have shown that commodity investment is having diversification benefit, we shall now move forward with developing optimal portfolios out of it.

First, an optimal mean-variance portfolio which gives minimum variance is established. Then an efficient frontier is developed by using the Matlab software which shows all Portfolio that are long only portfolio, i.e. it is only allowed to take positive positions. The minimum individual position is consequently 0 percent and the maximum is 100

percent, while the sums of all positions, of course, always add up to 100 percent. We have first estimated expected return and covariances, this is the input which is fed into the optimization program and we derive optimal portfolios. As the solution to the optimization problem includes the portfolio proportions (weights) and expected return and standard deviation, our solution starts from below. We have then identified the optimal Sharpe portfolio. Once this was done we introduce the riskless asset T-bill (and create a new investment horizon by connecting it to the Sharpe portfolio) and then identify the Complete portfolio, given the risk preference.

Lastly, Tables 7.4 to 7.23 demonstrate different combinations of weights of equity future index and commodity futures indices in a portfolio and the corresponding portfolio return, risk and Sharpe ratio. Figures 7.1 to 7.8 show the corresponding efficient frontiers and the optimum risk portfolio and optimum complete portfolio.

### ***7.3.1 - Portfolio optimization with Nifty and MCX Comdex***

The analysis starts with combining Nifty with Comdex. Table 7.4 shows all possible portfolios formed out of monthly return data of Nifty and Comdex, the correlation between them is 0.33 (Table 7.2) thus because of this less than perfect positive correlation we can find that there is diversification benefit out of combining both portfolios. The return of optimal risky portfolio is 1.1312% which is a reduction of just 8.77 % (0.0877 basis point) and on the other hand the risk has been reduced to the extent of 32.70% (0.3277 basis point). So we can see that by combining Nifty & Comdex, diversification benefit can be achieved.

From the view point of Strategic Asset Allocation the optimum allocation to Nifty = 43% and Comdex = 57%.

<b>Table 7.4 - Analysis of the Nifty and Comdex Portfolios</b>				
<b>Return</b>	<b>Risk</b>	<b>Portfolio Weights</b>		<b>Sharpe Ratio</b>
		<b>Nifty</b>	<b>Comdex</b>	
1.0963	5.2786	0.2437	0.7563	0.0808
1.1039	5.2888	0.2835	0.7165	0.0820
1.1114	5.3191	0.3233	0.6767	0.0830
1.119	5.3694	0.3631	0.6369	0.0836
1.1266	5.4389	0.4029	0.5971	0.0840
1.1341	5.527	0.4427	0.5573	0.0840
1.1417	5.6329	0.4826	0.5174	0.0837
1.1492	5.7555	0.5224	0.4776	0.0833
1.1568	5.8938	0.5622	0.4378	0.0826
1.1644	6.0466	0.6020	0.3980	0.0818
1.1719	6.2131	0.6418	0.3582	0.0808
1.1795	6.392	0.6816	0.3184	0.0797
1.1871	6.5823	0.7214	0.2786	0.0786
1.1946	6.7832	0.7612	0.2388	0.0773
1.2022	6.9937	0.8010	0.1990	0.0761
1.2097	7.2129	0.8408	0.1592	0.0748
1.2173	7.4401	0.8806	0.1194	0.0736
1.2249	7.6745	0.9204	0.0796	0.0723
1.2324	7.9156	0.9602	0.0398	0.0710
1.24	8.1627	1.0000	0.0000	0.0698
<b>MINIMUM VARIANCE</b>				
<b>1.0964%</b>	<b>5.2801</b>	<b>0.2443</b>	<b>0.7557</b>	<b>0.0808%</b>
<b>OPTIMAL RISKY PORTFOLIO</b>				
<b>1.1312%</b>	<b>5.4915</b>	<b>0.4276</b>	<b>0.5724</b>	<b>0.0840%</b>

The Table 7.4 shows the number of portfolios that have been generated by the matlab software, as we know that the theory states that “n” number of portfolios can be constructed by varying proportion of investment in a particular asset. But, the important portfolios out of all are minimum variance portfolio and the optimal risky portfolio.

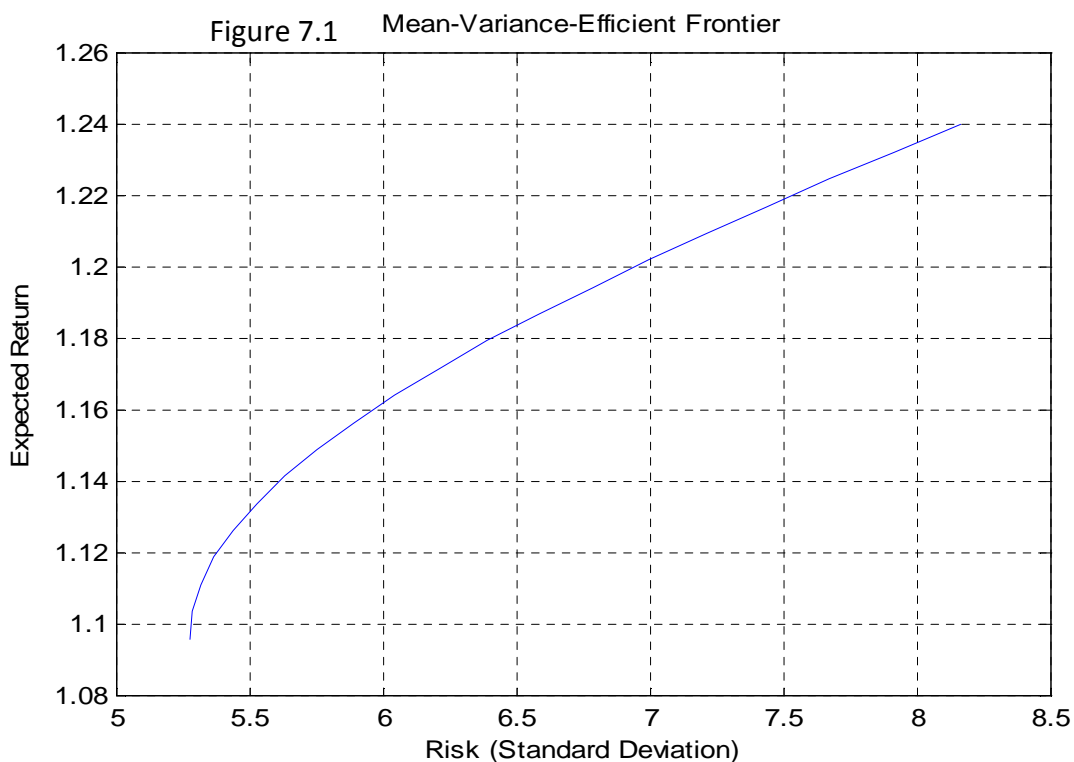


Figure 7.1 shows the asset allocation on the efficient frontier in an all asset portfolio starting from a minimum variance allocation towards a risky portfolio. It maximizes the expected return at a given level of volatility. The frontier starts with the minimum variance portfolio (MVP) also called as Global minimum variance portfolio and further moves on upward direction from there. Details of that portfolio are given in the table 7.5 below. The principal idea behind the frontier set of risky portfolios is that, for any risk level (SD) we are interested in only that portfolio which gives highest expected return.

Table 7.5 shows the minimum variance portfolio and we can see that by combining Nifty with Comdex a minimum variance of 5.28% can be achieved, the risk reduction is not very high if we compare with Comdex only but if we compare the risk of Nifty (i.e. 8.16%) the risk reduction is quite significant. Further, one more thing need to be highlighted here i.e. Comdex return is 1.05% but the minimum variance portfolio is giving a return of 1.09% which shows real good diversification benefit that has been achieved.

<b>Table 7.5 - Minimum Variance Portfolio</b>			
<b>Nifty &amp; Comdex</b>			
	Standard Deviation	Variance	Expected Return
Nifty	8.16	66.59	1.24%
Comdex	5.65	31.92	1.05%
Risk Free Return	0.67%	Covariance 15.37	
		Correlation 0.33	
$W_{\min \text{ nft}} = \frac{\sigma_{\text{Com}}^2 - \text{COV}_{\text{nft Com}}}{\sigma_{\text{nft}}^2 + \sigma_{\text{Com}}^2 - 2 [\text{COV}_{\text{nft Com}}]}$			
<b>Weight Nifty</b>		<b>0.2443</b>	
<b>Weight Comdex</b>		<b>0.7557</b>	
$E(R_p) = W_{\text{nft}} \times ER_{\text{nft}} + W_{\text{com}} \times ER_{\text{com}}$			
$\sigma_p^2 = (W_{\text{nft}}^2 \times \sigma_{\text{nft}}^2) + (W_{\text{com}}^2 \times \sigma_{\text{com}}^2) + 2(W_{\text{nft}} \times W_{\text{com}} \times \text{COV}_{\text{nft Com}})$			
<b>Expected return</b>		<b>1.0964%</b>	
<b>Standard deviation</b>		<b>5.2801</b>	
<b>Minimum Variance</b>		<b>27.8795</b>	

Now, we shall move forward with the optimal risky portfolio that gives us highest reward to variability ratio. After knowing the minimum variance portfolio, an investor is interested in getting the portfolio which will give the maximum return, thus, we construct the optimal risky portfolio. This portfolio is the one which the all risky investor shall choose to invest in.

The above Table 7.6 shows the optimal risky portfolio with highest reward to variability ratio. The objective was to find the weights that result in the highest slope of CAL (i.e. weights that result in the risky portfolio with highest reward to variability ratio). The highest Sharpe Ratio of 0.0840% can be achieved by allocation 42.76% in Nifty and 57.24% in Comdex. We call this portfolio **P<sub>1</sub>**. Thus, the Optimal Sharpe Portfolio of Nifty & Comdex is represented as **P<sub>1</sub>**

<b>Table 7.6 - Optimal Risky Portfolio With Highest Reward To Variability Ratio</b>			
<b>Nifty &amp; Comdex</b>			
	Standard Deviation	Variance	Expected Return
Nifty	8.16	66.59	1.24%
Comdex	5.65	31.92	1.05%
Risk Free Return	0.67%		
Covariance	15.37		
Correlation	0.33		
$W_{nft} = \frac{[E(r_{nft}) - r_f] \sigma_{com}^2 - [E(r_{com}) - r_f] Cov(r_{nft}, r_{com})}{[E(r_{nft}) - r_f] \sigma_{com}^2 + [E(r_{com}) - r_f] \sigma_{nft}^2 - [E(r_{nft}) - r_f + E(r_{com}) - r_f] Cov(r_{nft}, r_{com})}$			
<b>Weight Nifty</b>	<b>0.4276</b>		
<b>Weight Comdex</b>	<b>0.5724</b>		
E (Rp) = $W_{nft} \times ER_{nft} + W_{com} \times ER_{com}$			
$\sigma^2 p = (W_{nft}^2 \times \sigma_{nft}^2) + (W_{com}^2 \times \sigma_{com}^2) + 2(W_{nft} \times W_{com} \times COV_{nft Com})$			
<b>Expected return</b>	<b>1.1312%</b>		
<b>Standard deviation</b>	<b>5.4915</b>		
<b>Variance</b>	<b>30.1567</b>		
<b>Sharpe Ratio (reward to variability ratio)</b>	<b>0.0840%</b>		

**P<sub>1</sub>** maximizes reward to variability ratio and at this point the portfolio manager is done. This portfolio is an unconstrained portfolio and the most striking conclusion is that a portfolio manager will offer the same risky portfolio to all clients regardless of their degree of risk aversion. Thus the only difference between client's choices is that the more risk averse-client will invest more in risk free asset. Thus, now we will introduce risky-free asset.

As we have the efficient frontier and optimum portfolio **P<sub>1</sub>**, we proceed to the next step and introduce risk free asset. As we have seen that all the clients will invest in portfolio

$P_1$  as their optimal risky investment vehicle. This is called “separation property” as it tells us that the portfolio choice is having 2 tasks, first task is to determine optimal risky portfolio (and we have seen this is purely technical). This is the best risky portfolio for all clients regardless of risk aversion.

Now, Table 7.7 has focused on second task of separation property is allocation of complete portfolio to Risk-free asset (T-Bills) versus the risky portfolio, this depends on personal preference. Here the investor is a decision maker. Now, as we have constructed optimal portfolio  $P_1$ , we use investor’s degree of risk aversion “A” to calculate the optimal proportion of the complete portfolio to invest in the risky component.

In below Table 7.7 an investor with a coefficient of risk aversion  $A=3$  (a moderate risk taker) would invest 51% in  $P_1$  portfolio and rest 49% in risk free asset.

<b>Table 7.7 - Optimal Complete Portfolio With Risk Level (A=3)</b>	
$E(R_C) = W_P \times ER_P + W_F \times ER_F$	
$\sigma^2_C = y^* \sigma^2_P$	
$y^* = \frac{[E(rp) - rf]}{0.01A\sigma^2_P}$	
<b>Risky portfolio fraction</b>	<b>51%</b>
<b>Complete portfolio return</b>	<b>0.9052</b>
<b>Complete portfolio risk</b>	<b>3.922</b>

Table 7.7 shows us that the Portfolio Risk has dramatically reduced by introduction of the risk free asset. Thus, if the proportion of investment in risky portfolio reduces we will be able to reduce the risk more. Here for moderate investor; optimal level of investment in risky portfolio shown at level of 51%, thus more than fifty percent can be considered as a significant level from the view point of theory of asset allocation.

Now, below is the representation of a figure which shows the efficient frontier and CAL of optimal risky portfolio.

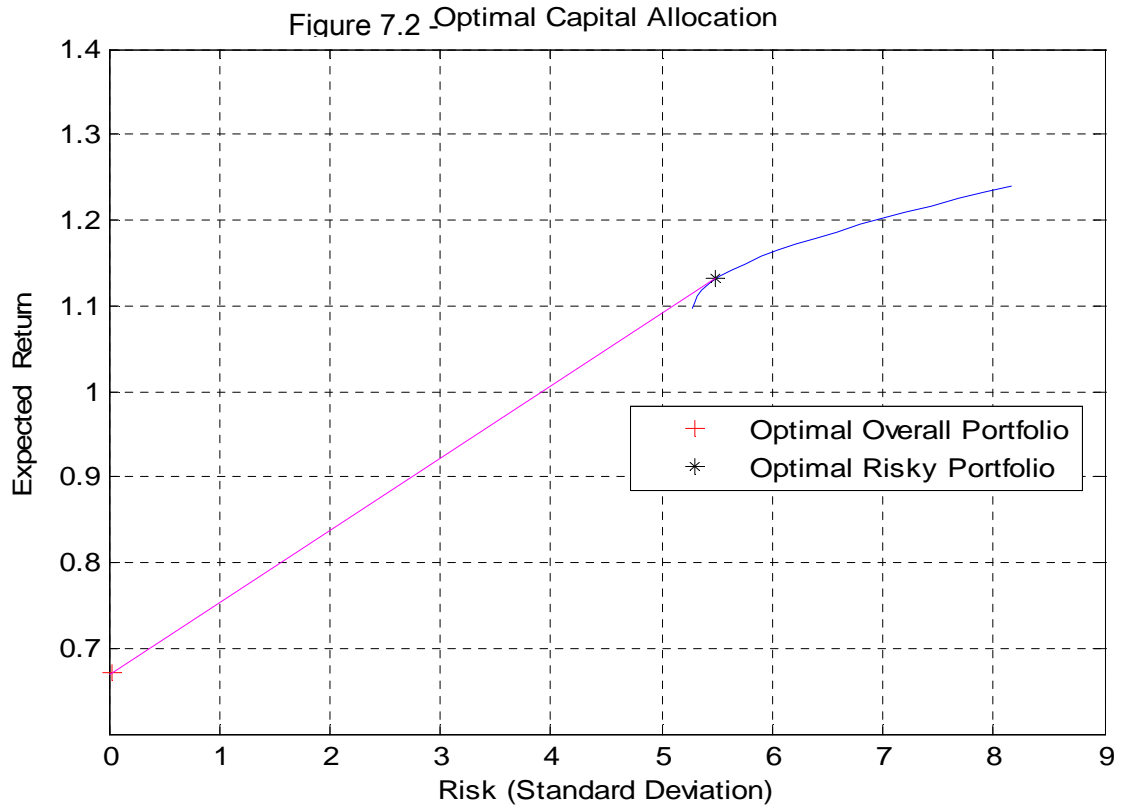


Figure 7.2 shows the CAL with the tangency portfolio (i.e the optimal risky portfolio) and optimal complete portfolio. This figure is generated by the Matlab software.

Before moving on, we must recall that our two risky assets Nifty and Comdex are already diversified portfolios. And the diversification within each of these portfolios must be credited for a good deal of risk reduction compared to undiversified single securities. An example from Meir Statman 1987 can highlight this. He states that when a standard deviation of the rate of return on an average stock is about 50% and in contrast the standard deviation of stock index fund is only 20%, about equal to historical standard deviation of the S&P 500 portfolio. This also signifies the importance of using Nifty as a proxy for equity asset class in this thesis.

Table 7.8 represents the portfolios with a given level of risk aversion, one can determine the portfolio that provides the highest level of Utility. (Utility is discussed Chapter 3) according to the relationship  $U = E(r) - 0.005 * A * \sigma^2$



Here, we consider investment in risky portfolio only and no investment in risk free asset. So if the investor wants optimal portfolio from the opportunity set, it will depend on their risk aversion. The best trade-off among these choices is a matter of personal preference. Investors with greater risk aversion will prefer portfolios with lower expected return and lower risk and vice-versa.

<b>Table 7.8 - Optimal Portfolio According to Risk Level</b>			
<b>Nifty &amp; Comdex</b>			
	Standard Deviation	Variance	Expected Return
NIFTY	8.16	66.59	1.24%
Comdex	5.65	31.92	1.05%
Risk Free Return	0.67%		
Covariance	15.37		
Correlation	0.33		
$W_{nft} = ER_{nft} - ER_{Eg} + 0.01 A (\sigma_{Eg}^2 - COV_{nft Eg}) / 0.01 \times A (\sigma_{nft}^2 + \sigma_{Eg}^2 - 2 [COV_{nft Eg}])$			
$E (Rp) = W_{nft} \times ER_{nft} + W_{com} \times ER_{com}$			
$\sigma^2 p = (W_{nft}^2 \times \sigma_{nft}^2) + (W_{com}^2 \times \sigma_{com}^2) + 2(W_{nft} \times W_{com} \times COV_{nft Com})$			
FOR INVESTOR WITH <b><u>A = 2</u></b> IS A HIGH RISK TAKER			
Optimal weight for Nifty	<b>0.3844</b>	Expected return	<b>1.1230%</b>
Optimal weight for Comdex	<b>0.6156</b>	Variance	<b>29.2106</b>
		Standard deviation	<b>5.4047</b>
		Sharpe Ratio	<b>0.0838%</b>
FOR INVESTOR WITH <b><u>A = 3</u></b> IS A MODERATE RISK TAKER			
Optimal weight for Nifty	<b>0.3377</b>	Expected return	<b>1.1142%</b>
Optimal weight for Comdex	<b>0.6623</b>	Variance	<b>28.4713</b>
		Standard deviation	<b>5.3359</b>
		Sharpe Ratio	<b>0.0832%</b>
FOR INVESTOR WITH <b><u>A = 4</u></b> IS A LOW RISK TAKER			
Optimal weight for Nifty	<b>0.3143</b>	Expected return	<b>1.1097%</b>
Optimal weight for Comdex	<b>0.6857</b>	Variance	<b>28.2120</b>
		Standard deviation	<b>5.3115</b>
		Sharpe Ratio	<b>0.0828%</b>

From the Table 7.8 following observations can be made –

- If the investor is a high risk taker then he will invest more in Nifty as we have seen that Nifty is having high risk high return compared to Comdex.
- If the investor is a low risk taker then his proportion of investment in Nifty is reduced by 1.22% and the impact will be that the return and risk both will also reduce as well as the sharpe ratio will also reduce.

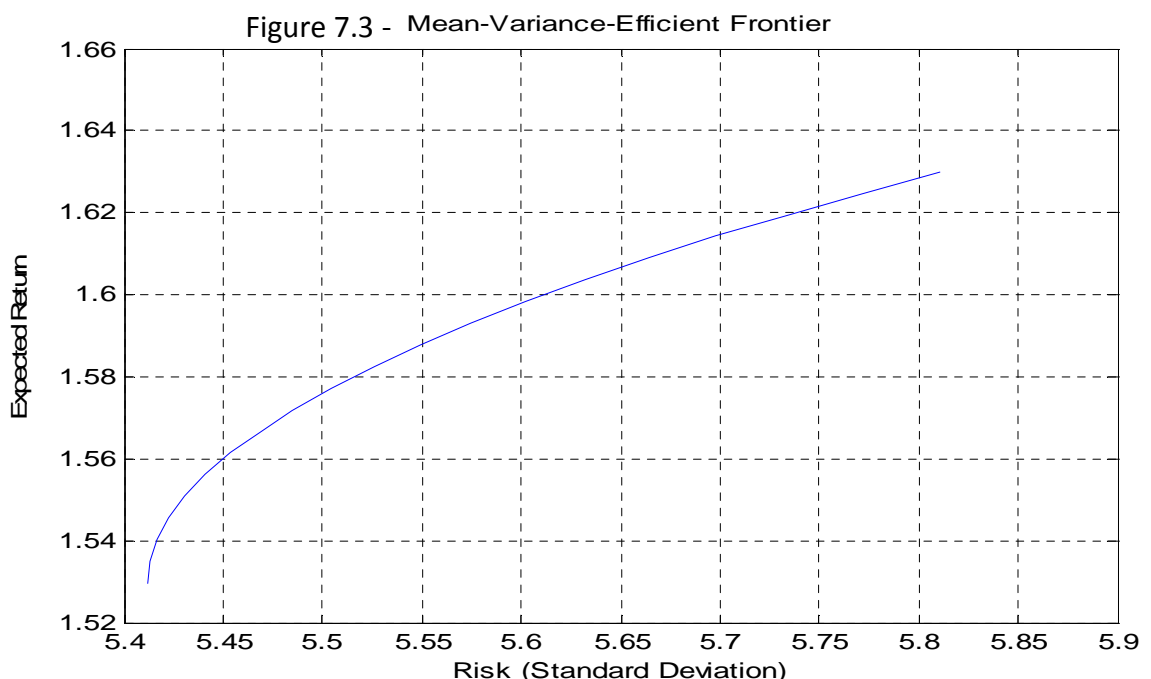
### 7.3.2 - Portfolio optimization with Nifty and MCX Metal

<b>Table 7.9 - Analysis of the Nifty and Metal Portfolios</b>				
<b>Return</b>	<b>Risk</b>	<b>Portfolio Weights</b>		<b>Sharpe Ratio</b>
		<b>Nifty</b>	<b>Metal</b>	
1.5297	5.4122	0.2572	0.7428	0.1588
1.5350	5.4133	0.2437	0.7563	0.1598
1.5402	5.4167	0.2302	0.7698	0.1607
1.5455	5.4225	0.2166	0.7834	0.1615
1.5508	5.4305	0.2031	0.7969	0.1622
1.5561	5.4407	0.1895	0.8105	0.1629
1.5614	5.4533	0.1760	0.8240	0.1635
1.5666	5.4680	0.1625	0.8375	0.1640
1.5719	5.4850	0.1489	0.8511	0.1644
1.5772	5.5042	0.1354	0.8646	0.1648
1.5825	5.5256	0.1219	0.8781	0.1651
1.5878	5.5491	0.1083	0.8917	0.1654
1.5930	5.5748	0.0948	0.9052	0.1656
1.5983	5.6025	0.0812	0.9188	0.1657
1.6036	5.6323	0.0677	0.9323	0.1658
1.6089	5.6642	0.0542	0.9458	0.1658
1.6142	5.6980	0.0406	0.9594	0.1657
1.6194	5.7338	0.0271	0.9729	0.1656
1.6247	5.7716	0.0135	0.9865	0.1654
1.6300	5.8112	0.0000	1.0000	0.1652
<b>MINIMUM VARIANCE</b>				
<b>1.5297%</b>	<b>5.4112</b>	<b>0.2573</b>	<b>0.7427</b>	<b>0.1589%</b>
<b>OPTIMAL RISKY PORTFOLIO</b>				
<b>1.6063%</b>	<b>5.6471</b>	<b>0.0608</b>	<b>0.9392</b>	<b>0.16580%</b>

Table 7.9 is generated from Matlab software and it shows all possible portfolios formed out of monthly return data of Nifty and Metal, the correlation between them is 0.34 (Table 7.2). Because of this less than perfect positive correlation we can find that there is diversification benefit out of combining both portfolios. The return of optimal risky portfolio is 1.6063% which has investment of 93.94% in metal.

Metal over here has outperformed Nifty in all aspects it has a return of 1.63% with a volatility of just 5.81 vs. Nifty 8.16. The minimum variance that can be achieved by combining two portfolios is 5.41. And we can also see the optimum allocation is giving a variance of 5.64 vs. 5.81 a reduction of 0.0293 basis points and return is reduced from 1.63 to 1.6063 which is a reduction of mere 0.0145 basis points so we can see that the optimal portfolio is giving higher risk reduction compared with reduction in return. From the view point of Strategic Asset Allocation the optimum allocation to Nifty = 6% and Metal = 94%.

Figure 7.3 below shows the asset allocation on the efficient frontier in an all asset portfolio starting from a minimum variance allocation towards a risky portfolio. It maximizes the expected return at a given level of volatility. The frontier starts with the minimum variance portfolio. The details of that portfolio are given in the Table 7.10 below.



<b>Table 7.10 – Minimum Variance Portfolio</b>			
<b>Nifty &amp; Metal</b>			
	Standard Deviation	Variance	Expected Return
Nifty	8.16	66.59	1.24%
Metal	5.81	33.76	1.63%
Risk Free Return	0.67%	Covariance	16.36
		Correlation	0.34
$W_{\min \text{ nft}} = \frac{\sigma_{\text{Met}}^2 - \text{COV}_{\text{nft Met}}}{\sigma_{\text{nft}}^2 + \sigma_{\text{Met}}^2 - 2 [\text{COV}_{\text{nft Met}}]}$			
<b>Weight Nifty</b>		<b>0.2573</b>	
<b>Weight Metal</b>		<b>0.7427</b>	
$E(R_p) = W_{\text{nft}} \times ER_{\text{nft}} + W_{\text{com}} \times ER_{\text{com}}$			
$\sigma^2 p = (W_{\text{nft}}^2 \times \sigma_{\text{nft}}^2) + (W_{\text{com}}^2 \times \sigma_{\text{com}}^2) + 2(W_{\text{nft}} \times W_{\text{com}} \times \text{COV}_{\text{nft Com}})$			
<b>Expected return</b>		<b>1.5297%</b>	
<b>Standard deviation</b>		<b>5.4112</b>	
<b>Minimum Variance</b>		<b>29.2808</b>	

The findings here are very unique and no other research in the literature has such findings as we can see Metal has totally outperformed Nifty in terms of risk and return, and also the correlation between them is very less just 0.34, so from the view point of diversification also it is very much advantageous to invest in Metal.

Now in Table 7.11 optimal risky portfolio with highest reward to variability ratio is shown. The highest Sharpe Ratio of 0.16580% can be achieved by allocation 6% in Nifty and 94% in Comdex. In case if we invest 100% in metal than we get a Sharpe ratio of 0.1652% which is lower than the optimal portfolio.

We call this portfolio **P<sub>2</sub>**. This portfolio maximizes reward to variability ratio and at this point the portfolio manager is done. This portfolio is an unconstrained portfolio and the most striking conclusion is that a portfolio manager will offer the same risky portfolio to all clients regardless of their degree of risk aversion. Thus the only difference between client's choices is that the more risk averse-client will invest more in risk free asset. Thus, now we will introduce risky-free asset in our Nifty and Metal portfolios.

<b>Table 7.11 – Optimal Risky Portfolio with Highest Reward to Variability Ratio</b>			
<b>Nifty &amp; Metal</b>			
	Standard Deviation	Variance	Expected Return
Nifty	8.16	66.59	1.24%
Metal	5.81	33.76	1.63%
Risk Free Return	0.67%		
Covariance	16.36		
Correlation	0.34		
$W_{nft} = \frac{[E(r_{nft}) - r_f] \sigma_{com}^2 - [E(r_{com}) - r_f] \text{Cov}(r_{nft}, r_{com})}{[E(r_{nft}) - r_f] \sigma_{com}^2 + [E(r_{com}) - r_f] \sigma_{nft}^2 - [E(r_{nft}) - r_f + E(r_{com}) - r_f] \text{Cov}(r_{nft}, r_{com})}$			
<b>Weight Nifty</b>	<b>0.0608</b>		
<b>Weight Metal</b>	<b>0.9392</b>		
$E(R_p) = W_{nft} \times ER_{nft} + W_{Met} \times ER_{Met}$			
$\sigma^2 p = (W_{nft}^2 \times \sigma_{nft}^2) + (W_{Met}^2 \times \sigma_{Met}^2) + 2(W_{nft} \times W_{Met} \times \text{COV}_{nft Met})$			
<b>Expected return</b>	<b>1.6063%</b>		
<b>Standard deviation</b>	<b>5.6471</b>		
<b>Variance</b>	<b>31.8903</b>		
<b>Sharpe Ratio (Reward to variability ratio)</b>	<b>0.1658%</b>		

As we have the efficient frontier and optimum portfolio  $P_2$ , we proceed to the next step and introduce risk free asset. All the clients will invest in portfolio  $P_2$  as their optimal risky investment vehicle and the first task of separation property is finished here.

Table 7.12 we focus on second task of separation property is allocation of complete portfolio between Risk-free asset (T-Bills) versus the risky portfolio. This depends on personal preference. Here the investor is a decision maker. Now, as we have constructed optimal portfolio  $P_2$ , we use investor's degree of risk aversion "A" to calculate the optimal proportion of the complete portfolio to invest in the risky component.

Table 7.12 an investor with a coefficient of risk aversion  $A=3$  (a moderate risk taker) would invest 98% in  $P_2$  portfolio and rest 02% in risk free asset. This finding is also very unique as because of metal outperforming in every aspect, for a moderate risk taker the

allocation is coming to 98% that is too much and such result is not found in any other study.

<b>Table 7.12 - Optimal Complete Portfolio With Risk Level (A=3)</b>	
$E(R_C) = W_p \times ER_p + W_F \times ER_F$	
$\sigma^2_C = y^* \sigma^2_p$	
$y^* = \frac{[E(rp) - rf]}{0.01A\sigma^2_p}$	
<b>Risky portfolio fraction</b>	<b>98%</b>
<b>Complete portfolio return</b>	<b>1.5876</b>
<b>Complete portfolio risk</b>	<b>5.5904</b>

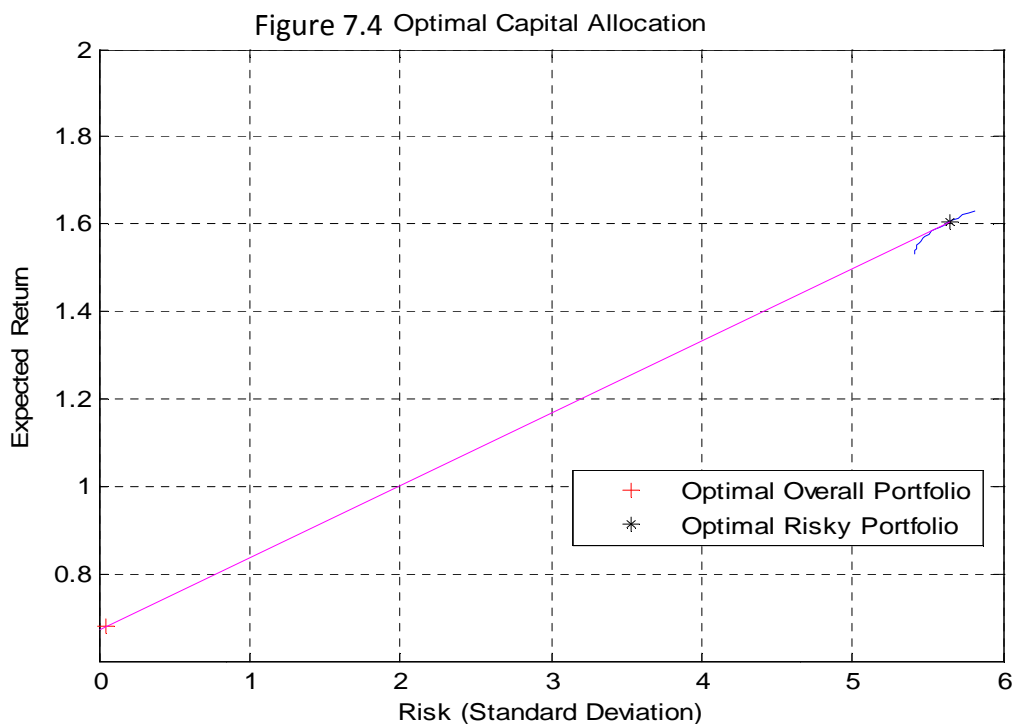


Figure 7.4 above shows the CAL with the tangency portfolio (i.e the optimal risky portfolio) and optimal complete portfolio.

As we know that the two risky assets Nifty and Metal are already diversified portfolios and the diversification within each of these portfolios is credited for a good deal of risk reduction compared to undiversified single securities.

In Table 7.13 we will find out that a given level of risk aversion, one can determine the portfolio that provides the highest level of Utility.  $U = E(r) - 0.005 * A * \sigma^2$

Here, we consider investment in risky portfolio only and no investment in risk free asset. So if the investor wants optimal portfolio from the opportunity set, it will depend on their risk aversion. The best trade-off among these choices is a matter of personal preference. Investors with greater risk aversion will prefer portfolios with lower expected return and lower risk and vice-versa.

<b>Table 7.13 - Optimal Portfolio According to Risk Level</b>			
<b>Nifty &amp; Metal</b>			
	Standard Deviation	Variance	Expected Return
NIFTY	8.16	66.59	1.24%
Metal	5.81	33.76	1.63%
Risk Free Return	0.67%	Covariance	16.36
		Correlation	0.34
$W_{nft} = \frac{ER_{nft} - ER_{Met} + 0.01 A (\sigma_{Met}^2 - COV_{nft Met})}{0.01 \times A (\sigma_{nft}^2 + \sigma_{Met}^2 - 2[COV_{nftMet}])}$			
$E(R_p) = W_{nft} \times ER_{nft} + W_{Met} \times ER_{Met}$			
$\sigma^2 p = (W_{nft}^2 \times \sigma_{nft}^2) + (W_{Met}^2 \times \sigma_{Met}^2) + 2(W_{nft} \times W_{Met} \times COV_{nft Met})$			
FOR INVESTOR WITH <b><u>A = 2</u></b> IS A HIGH RISK TAKER			
Optimal weight for Nifty	<b>0.0000</b>	Expected return	<b>1.6300%</b>
Optimal weight for Metal	<b>1.0000</b>	Variance	<b>33.7561</b>
		Standard deviation	<b>5.8100</b>
		Sharpe Ratio	<b>0.1652%</b>
FOR INVESTOR WITH <b><u>A = 3</u></b> IS A MODERATE RISK TAKER			
Optimal weight for Nifty	<b>0.0651</b>	Expected return	<b>1.6046%</b>
Optimal weight for Metal	<b>0.9349</b>	Variance	<b>31.7777</b>
		Standard deviation	<b>5.6372</b>
		Sharpe Ratio	<b>0.16579%</b>
FOR INVESTOR WITH <b><u>A = 4</u></b> IS A LOW RISK TAKER			
Optimal weight for Nifty	<b>0.1131</b>	Expected return	<b>1.5859%</b>
Optimal weight for Metal	<b>0.8869</b>	Variance	<b>30.6861</b>
		Standard deviation	<b>5.5395</b>
		Sharpe Ratio	<b>0.1653%</b>

Some unique findings can be noted from the above Table –

- The highest risk taker with A=2 got more than 100% investment in Metal, but as we are having our limits of allocation to maximum 100%, the table shows 0% allocation in Nifty and 100% in Metal. As we are considering Utility over here on one side and no short position on the other side, if we combine it the maximum allocation in an long only portfolio of Nifty and Metal will be 100% investment in Metal.

### 7.3.3 - Portfolio optimization with Nifty and MCX Energy

<b>Table 7.14 Analysis of the Nifty and Energy Portfolios</b>				
<b>Return</b>	<b>Risk</b>	<b>Portfolio Weights</b>		<b>Sharpe Ratio</b>
		<b>Nifty</b>	<b>Energy</b>	
1.1943	6.5880	0.4921	0.5079	0.0796
1.1967	6.5929	0.5188	0.4812	0.0799
1.1991	6.6075	0.5455	0.4545	0.0801
1.2015	6.6318	0.5723	0.4277	0.0801
1.2039	6.6657	0.5990	0.4010	0.0801
1.2063	6.7090	0.6257	0.3743	0.0799
1.2087	6.7615	0.6525	0.3475	0.0797
1.2111	6.8231	0.6792	0.3208	0.0793
1.2135	6.8935	0.7059	0.2941	0.0788
1.2159	6.9724	0.7327	0.2673	0.0783
1.2183	7.0595	0.7594	0.2406	0.0777
1.2208	7.1545	0.7861	0.2139	0.0770
1.2232	7.2572	0.8129	0.1871	0.0762
1.2256	7.3672	0.8396	0.1604	0.0754
1.2280	7.4842	0.8663	0.1337	0.0746
1.2304	7.6078	0.8931	0.1069	0.0737
1.2328	7.7378	0.9198	0.0802	0.0727
1.2352	7.8738	0.9465	0.0535	0.0718
1.2376	8.0156	0.9733	0.0267	0.0708
1.2400	8.1627	1.0000	0.0000	0.0698
<b>MINIMUM VARIANCE</b>				
<b>1.1943</b>	<b>6.5880</b>	<b>0.4921</b>	<b>0.5079</b>	<b>0.0796</b>
<b>OPTIMAL RISKY PORTFOLIO</b>				
<b>1.2017</b>	<b>6.6346</b>	<b>0.5748</b>	<b>0.4252</b>	<b>0.0801</b>

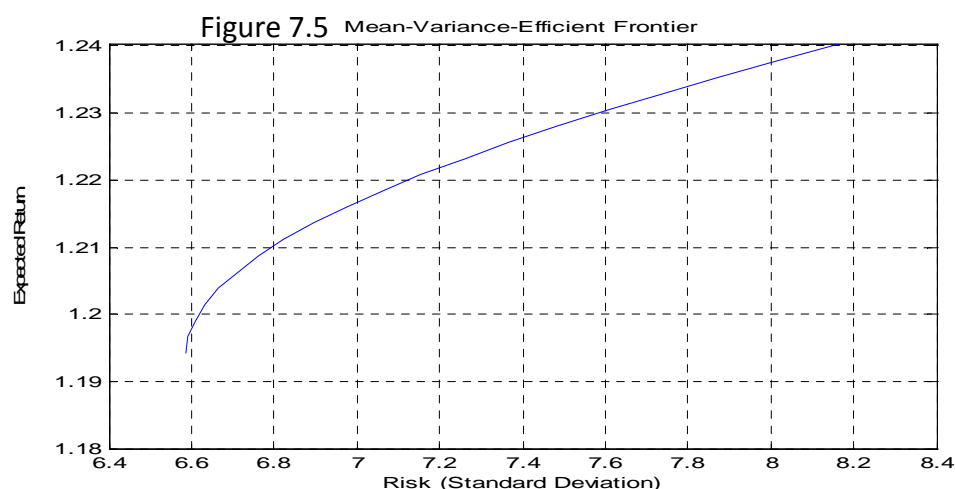


Table 7.14 shows all possible portfolios formed out of monthly return data of Nifty and Energy, the correlation between them is 0.32 (Table 7.3) as we have seen that Nifty is having almost same correlation with Comdex Metal and Energy, thus because of this less than perfect positive correlation we can find that there is diversification benefit out of combining both portfolios. The return of optimal risky portfolio is 1.2017% which has investment of 42.5% % in energy.

Energy is the worst performer amongst all, as its risk is as high as Nifty but the return is lower. But excellent diversification advantage can be seen by combining Nifty with Energy as the correlation is low the risk has been reduced to 6.59 from 8.16 which is a reduction of about **19%**. And on the other hand the return of optimal portfolio has increased by **4.35%** compared with the return of standalone energy portfolio. So we can conclude that investment in energy is beneficial only if combined with Nifty. Standalone investment in Energy is not considered to be beneficial.

From the view point of Strategic Asset Allocation the optimum allocation to Nifty = 57% and Energy = 43%.

Figure 7.5 below shows the shows the asset allocation on the efficient frontier in an all asset portfolio starting from a minimum variance allocation towards a risky portfolio. It maximizes the expected return at a given level of volatility. The frontier starts with the minimum variance portfolio the details of that portfolio are given in the Table 7.15 below.



<b>Table 7.15 - Minimum Variance Portfolio</b>			
<b>Nifty &amp; Energy</b>			
	Standard deviation	Variance	Expected Return
Nifty	8.16	66.63	1.24%
Energy	8.07	65.20	1.15%
Risk Free Return	0.67%		
Covariance	20.90		
Correlation	0.32		
$W_{\min \text{ nft}} = \frac{\sigma_{\text{Eg}}^2 - \text{COV}_{\text{nft Eg}}}{\sigma_{\text{nft}}^2 + \sigma_{\text{Eg}}^2 - 2 [\text{COV}_{\text{nft Eg}}]}$			
<b>Weight Nifty</b>		0.4921	
<b>Weight Energy</b>		0.5079	
$E(R_p) = W_{\text{nft}} \times ER_{\text{nft}} + W_{\text{Eg}} \times ER_{\text{Eg}}$			
$\sigma^2 p = (W_{\text{nft}}^2 \times \sigma_{\text{nft}}^2) + (W_{\text{Eg}}^2 \times \sigma_{\text{Eg}}^2) + 2(W_{\text{nft}} \times W_{\text{Eg}} \times \text{COV}_{\text{nft Eg}})$			
<b>Expected return</b>		<b>1.1943%</b>	
<b>Standard deviation</b>		<b>6.5880</b>	
<b>Minimum Variance</b>		<b>43.4018</b>	

We can see from the above table that minimum variance that can be achieved by combining Nifty and energy is 43.40 and on the other hand Comdex is giving minimum variance of 27.88 and Metal 29.29 the risk reduction advantage given by Energy is not at all significant. Thus, it can be concluded that when compared to Comdex and Metal one would never want to invest in Energy.

Thus finally we can conclude that the performance of energy is very low compared to others. But that doesn't mean that Nifty when combined with energy has no benefit, it is definitely having diversification properties as its better to invest in combination of both rather than only Nifty or Energy. As the Table 7.16 below shows that the highest Sharpe ratio that can be achieved by the optimal combination of Nifty and Energy is just 0.0801% which is the lowest ratio amongst all other portfolios.

<b>Table 7.16 - Optimal Risky Portfolio With Highest Reward To Variability Ratio</b>			
<b>Nifty &amp; Energy</b>			
	Standard deviation	Variance	Expected Return
NIFTY	8.16	66.63	1.24%
Energy	8.07	65.20	1.15%
Risk Free Return	0.67%	Covariance	20.90
		Correlation	0.32
$W_{nft} = \frac{[E(r_{nft}) - r_f] \sigma_{com}^2 - [E(r_{com}) - r_f] Co(r_{nft}, r_{com})}{[E(r_{nft}) - r_f] \sigma_{com}^2 + [E(r_{com}) - r_f] \sigma_{nft}^2 - [E(r_{nft}) - r_f + E(r_{com}) - r_f] Co(r_{nft}, r_{com})}$			
<b>Weight Nifty</b>		<b>0.5748</b>	
<b>Weight Energy</b>		<b>0.4252</b>	
E (Rp) = W <sub>nft</sub> × ER <sub>nft</sub> + W <sub>Eg</sub> × ER <sub>Eg</sub>			
$\sigma^2 p = (W_{nft}^2 \times \sigma_{nft}^2) + (W_{Eg}^2 \times \sigma_{Eg}^2) + 2(W_{nft} \times W_{Eg} \times COV_{nft Eg})$			
<b>Expected return</b>		<b>1.2017%</b>	
<b>Standard deviation</b>		<b>6.6346</b>	
<b>Variance</b>		<b>44.0179</b>	
<b>Sharpe Ratio (reward to variability ratio)</b>		<b>0.0801%</b>	

The optimal allocation here is Nifty 57.48% and Energy 42.52%. We call this portfolio **P<sub>3</sub>**. This portfolio maximizes reward to variability ratio. Now next step is to introduce risk free asset in our risky portfolio of Nifty and Energy.

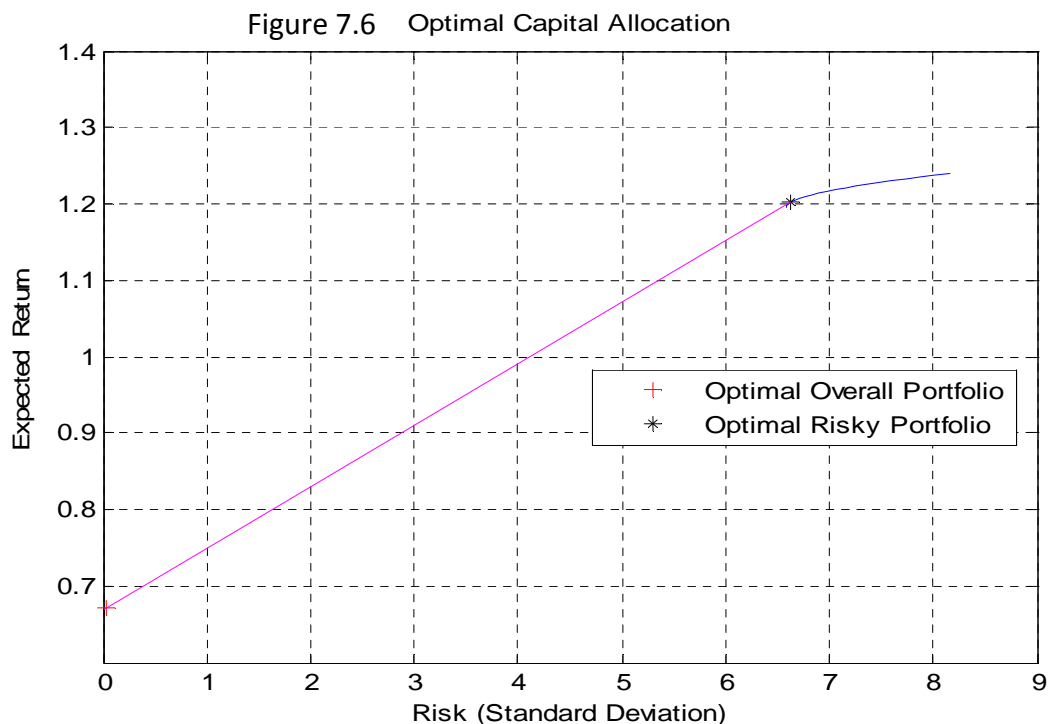
The main observation from the above table is that investment in Energy as a standalone portfolio is not beneficial at all. And thus it is advised that not to invest in energy alone but combine it with Nifty to achieve optimal returns.

In table 7.17 we focus on second task of separation property is allocation of complete portfolio to Risk-free asset (T-Bills) versus the risky portfolio, as we already know that this depends on personal preference and investor is the decision maker. We use investor's degree of risk aversion "A=3" to calculate the optimal proportion of the complete portfolio to invest in the risky component.

With A=3 (a moderate risk taker) would invest 40% in P<sub>3</sub> portfolio and rest 60% in risk free asset. This finding is showing as P<sub>3</sub> is having a very high risk, it will be more advantageous for the investor to invest in Risk free asset rather than P<sub>3</sub>. Thus the allocation to risk free asset has been increased dramatically in case of Nifty and Energy Complete Portfolio.

<b>Table 7.17 - Optimal Complete Portfolio With Risk Level (A=3)</b>	
$E(R_C) = W_P \times ER_P + W_F \times ER_F$	
$\sigma^2_C = y^* \sigma^2_P$	
$y^* = \frac{[E(rp) - rf]}{0.01A\sigma^2_P}$	
<b>Risky portfolio fraction</b>	<b>40%</b>
<b>Complete portfolio return</b>	<b>0.8827</b>
<b>Complete portfolio risk</b>	<b>4.1961</b>

The main conclusion out of the above Table 7.17 is that for an investor it is highly risky to invest in Nifty & Energy so the investment in risk-free asset is coming to 60% which is the highest amongst all other asset portfolios constructed in this study.



The chart of optimal complete portfolio as generated by Matlab is shown in Figure 7.6 above. It represents the optimal overall portfolio which is the optimal complete portfolio with A=3 and optimal risky portfolio. Here the complete portfolio return is 0.8827 which is very less compared to other portfolios generated in this study. And as we know that the risk-free rate is 0.67% so there is no use taking so much risk for generating very minimal high return than risk-free return.

<b>Table 7.18 - Optimal Portfolio According to Risk Level</b>			
<b>Nifty &amp; Energy</b>			
	Standard deviation	Variance	Expected Return
NIFTY	8.16	66.63	1.24%
Energy	8.07	65.20	1.15%
Risk Free Return	0.67%	Covariance	20.90
		Correlation	0.32
$W_{nft} = ER_{nft} - ER_{Eg} + 0.01 A (\sigma_{Eg}^2 - COV_{nft Eg}) / 0.01 \times A (\sigma_{nft}^2 + \sigma_{Eg}^2 - 2 [COV_{nft Eg}])$			
$E(Rp) = W_{nft} \times ER_{nft} + W_{Eg} \times ER_{Eg}$			
$\sigma^2 p = (W_{nft}^2 \times \sigma_{nft}^2) + (W_{Eg}^2 \times \sigma_{Eg}^2) + 2(W_{nft} \times W_{Eg} \times COV_{nft Eg})$			
<b>FOR INVESTOR WITH <u>A = 2</u> IS A HIGH RISK TAKER</b>			
<b>Optimal weight for Nifty</b>	<b>54.20%</b>	<b>Expected return</b>	<b>1.1988%</b>
<b>Optimal weight for Energy</b>	<b>45.80%</b>	<b>Variance</b>	<b>43.6264</b>
		<b>Standard deviation</b>	<b>6.6050</b>
		<b>Sharpe Ratio</b>	<b>0.0801%</b>
<b>FOR INVESTOR WITH <u>A = 3</u> IS A MODERATE RISK TAKER</b>			
<b>Optimal weight for Nifty</b>	<b>52.54%</b>	<b>Expected return</b>	<b>1.1973%</b>
<b>Optimal weight for Energy</b>	<b>47.46%</b>	<b>Variance</b>	<b>43.5019</b>
		<b>Standard deviation</b>	<b>6.5956</b>
		<b>Sharpe Ratio</b>	<b>0.0799%</b>
<b>FOR INVESTOR WITH <u>A = 4</u> IS A LOW RISK TAKER</b>			
<b>Optimal weight for Nifty</b>	<b>51.70%</b>	<b>Expected return</b>	<b>1.1965%</b>
<b>Optimal weight for Energy</b>	<b>48.30%</b>	<b>Variance</b>	<b>43.4578</b>
		<b>Standard deviation</b>	<b>6.5923</b>
		<b>Sharpe Ratio</b>	<b>0.0799%</b>

Now, the above table shows the combination of Nifty and Energy i.e. only risky portfolio allocation without considering risk free asset. Similar, to above we have divided investors into High, Moderate and Low risk takers.

Observations noted from the table below are –

- We can see that as the risk taking ability of the investor decreases the allocation to Nifty decreases as well. On the other hand only in case of Metal if the risk taking ability of the investor goes down than the allocation to Nifty is increasing. The reason is in any case standalone asset class will have higher risk than a diversified portfolio of two asset classes.

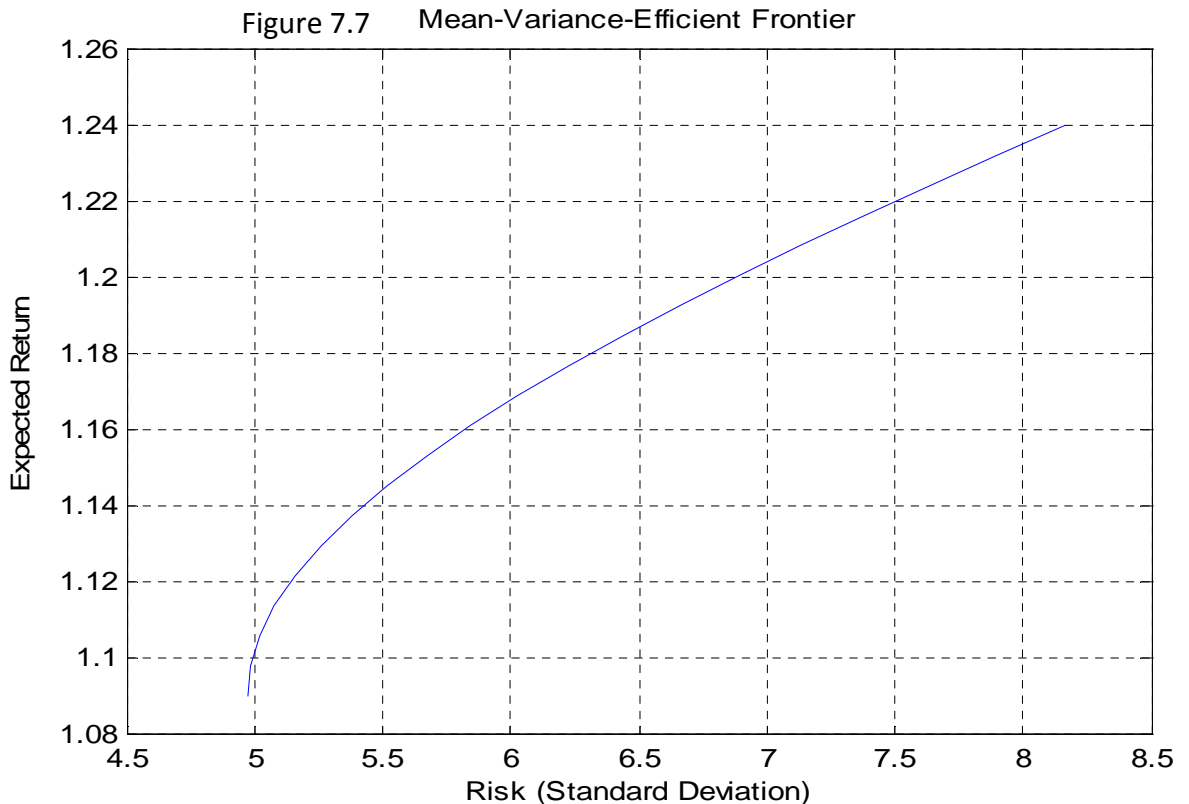
### 7.3.4 - Portfolio optimization with Nifty and MCX Agriculture

<b>Table 7.19 Analysis of the Nifty and Agri Portfolios</b>				
<b>Return</b>	<b>Risk</b>	<b>Portfolio Weights</b>		<b>Sharpe Ratio</b>
		<b>Nifty</b>	<b>Agri</b>	
1.0901	4.9743	0.3753	0.6247	<b>0.0845</b>
1.0980	4.9859	0.4082	0.5918	<b>0.0858</b>
1.1059	5.0207	0.4410	0.5590	<b>0.0868</b>
1.1137	5.0781	0.4739	0.5261	<b>0.0874</b>
1.1216	5.1575	0.5068	0.4932	<b>0.0876</b>
1.1295	5.2578	0.5397	0.4603	<b>0.0874</b>
1.1374	5.3778	0.5726	0.4274	<b>0.0869</b>
1.1453	5.5162	0.6054	0.3946	<b>0.0862</b>
1.1532	5.6718	0.6383	0.3617	<b>0.0852</b>
1.1611	5.8431	0.6712	0.3288	<b>0.0840</b>
1.1690	6.0288	0.7041	0.2959	<b>0.0828</b>
1.1769	6.2276	0.7370	0.2630	<b>0.0814</b>
1.1848	6.4383	0.7698	0.2302	<b>0.0800</b>
1.1927	6.6598	0.8027	0.1973	<b>0.0785</b>
1.2005	6.8909	0.8356	0.1644	<b>0.0770</b>
1.2084	7.1309	0.8685	0.1315	<b>0.0755</b>
1.2163	7.3788	0.9014	0.0986	<b>0.0740</b>
1.2242	7.6339	0.9342	0.0658	<b>0.0726</b>
1.2321	7.8954	0.9671	0.0329	<b>0.0712</b>
1.2400	8.1627	1.0000	0.0000	<b>0.0698</b>
<b>MINIMUM VARIANCE</b>				
<b>1.0900</b>	<b>4.9719</b>	<b>0.3752</b>	<b>0.6248</b>	<b>0.0845</b>
<b>OPTIMAL RISKY PORTFOLIO</b>				
<b>1.1217</b>	<b>5.1556</b>	<b>0.5070</b>	<b>0.4930</b>	<b>0.0876</b>

Table 7.19 shows all possible portfolios formed out of monthly return data of Nifty and Agri, the correlation between them is -0.01 (Table 7.3) and very uniquely it is having negative correlation with Nifty, this is very good in terms of achieving diversification benefit, as the assets with negative correlation are rarely found and they act as best diversification agents. This coincides with the international literature that has shown negative correlation between stock and commodities.

Thus maximum diversification benefit can be achieved by combining Nifty with Agri. The return of optimal risky portfolio is 1.1217% which is an increase in level of return to the extent of 10.85%. (10850 basis points) and on the other hand risk reduction of 36.82% which is the maximum amongst all other portfolios (Comdex, Metal and Energy). The minimum variance that can be achieved by combining these two risky assets is 4.97% which is again the lowest amongst all other portfolios.

From the view point of Strategic Asset Allocation the optimum allocation to Nifty = 50.7 % and AGRI = 49.3%.



The above figure 7.7 represents the efficient frontier of Nifty and Agri portfolios. And we can see that minimum risk level of less than 5% is achieved only in this case. And the allocation to Agri is very high (62%) compared to other asset classes.

<b>Table 7.20 - Minimum Variance Portfolio</b>			
<b>Nifty &amp; Agri</b>			
	Standard Deviation	Variance	Expected Return
NIFTY	8.16	66.59	1.24%
AGRI	6.31	39.82	1.00%
Risk Free Return	0.67%	Covariance	-0.42
		Correlation	-0.01
$W_{\min nft} = \frac{\sigma_{Ag}^2 - COV_{nft Ag}}{\sigma_{nft}^2 + \sigma_{Ag}^2 - 2 [COV_{nft Ag}]}$			
<b>Weight Nifty</b>		<b>0.3752</b>	
<b>Weight Agri</b>		<b>0.6248</b>	
$E(R_p) = W_{nft} \times ER_{nft} + W_{Ag} \times ER_{Ag}$			
$\sigma^2 p = (W_{nft}^2 \times \sigma_{nft}^2) + (W_{Ag}^2 \times \sigma_{Ag}^2) + 2(W_{nft} \times W_{Ag} \times COV_{nft Ag})$			
<b>Expected return</b>		<b>1.0900%</b>	
<b>Standard deviation</b>		<b>4.9719</b>	
<b>Minimum Variance</b>		<b>24.7199</b>	

The above table 7.20 shows that the minimum variance that can be achieved by combining Nifty and Agri is 24.7199 and when we compare that with Comdex, Metal and Energy we find that Agri is giving maximum diversification benefits.

Now in Table 7.21 optimal risky portfolio with highest reward to variability ratio is shown. The weights that result in the highest slope of CAL (i.e. weights that result in the risky portfolio with highest reward to variability ratio) is 50.7% in Nifty and 49.3% in Agri. The highest Sharpe Ratio of 0.0876% can be achieved by it. We call this portfolio **P<sub>4</sub>**. This is the last portfolio in our analysis. The optimal portfolio **P<sub>4</sub>** is giving the minimum risk amongst all other optimal risky portfolios. Thus again it is proved that Agri provides maximum diversification benefit.



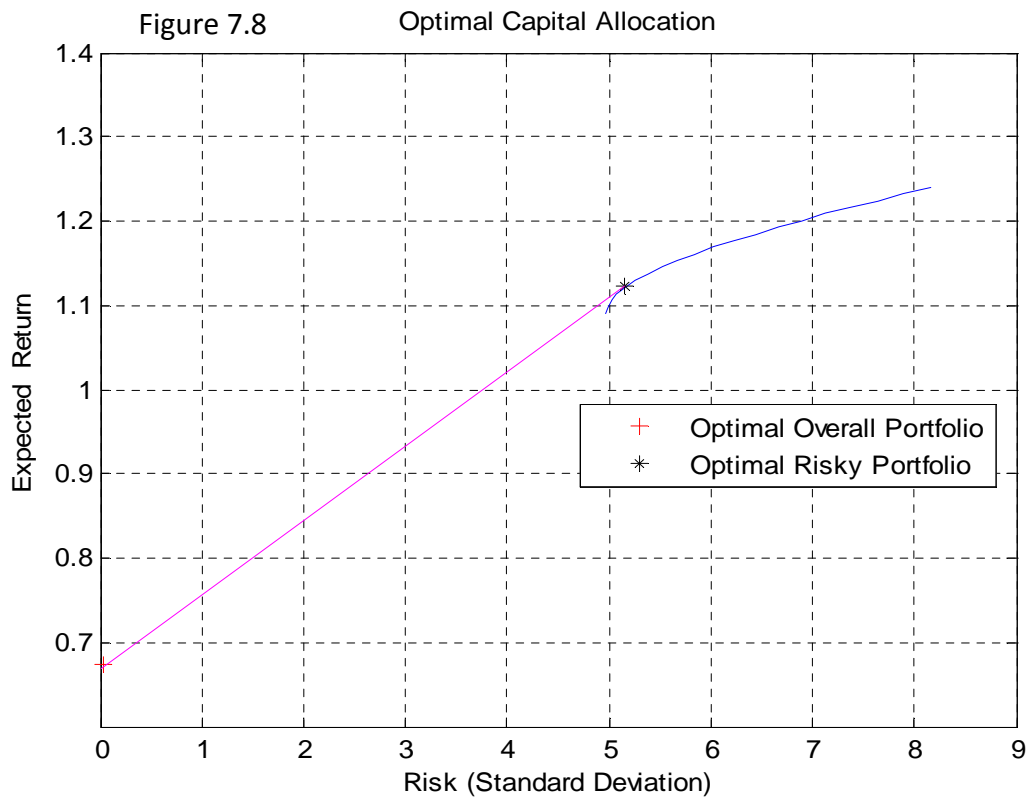
<b>Table 7.21 - Optimal Risky Portfolio With Highest Reward To Variability Ratio</b>			
<b>Nifty &amp; Agri</b>			
	Standard Deviation	Variance	Expected Return
NIFTY	8.16	66.59	1.24%
AGRI	6.31	39.82	1.00%
Risk Free Return	0.67%	Covariance	-0.42
		Correlation	-0.01
$W_{nft} = \frac{[E(r_{nft}) - r_f] \sigma_{com}^2 - [E(r_{com}) - r_f] \text{Co}(r_{nft}, r_{com})}{[E(r_{nft}) - r_f] \sigma_{com}^2 + [E(r_{com}) - r_f] \sigma_{nft}^2 - [E(r_{nft}) - r_f + E(r_{com}) - r_f] \text{Co}(r_{nft}, r_{com})}$			
<b>Weight Nifty</b>		<b>0.5070</b>	
<b>Weight Agri</b>		<b>0.4930</b>	
$E(R_p) = W_{nft} \times ER_{nft} + W_{Ag} \times ER_{Ag}$			
$\sigma^2 p = (W_{nft}^2 \times \sigma_{nft}^2) + (W_{Ag}^2 \times \sigma_{Ag}^2) + 2(W_{nft} \times W_{Ag} \times \text{COV}_{nft Ag})$			
<b>Expected return</b>		<b>1.1217%</b>	
<b>Standard deviation</b>		<b>5.1556</b>	
<b>Variance</b>		<b>26.5801</b>	
<b>Sharpe Ratio (reward to variability ratio)</b>		<b>0.0876%</b>	

Now moving towards introduction of risky free asset in our risky portfolio of Nifty and Agri we get the following results at an investors degree of risk aversion of "A=3"

<b>Table 7.22 - Optimal Complete Portfolio With Risk Level (A=3)</b>	
$E(R_c) = W_p \times ER_p + W_F \times ER_F$	
$\sigma^2 c = y^* \sigma^2 p$	
$y^* = \frac{[E(rp) - rf]}{0.01A\sigma^2 p}$	
<b>Risky portfolio fraction</b>	<b>57%</b>
<b>Complete portfolio return</b>	<b>0.9275</b>
<b>Complete portfolio risk</b>	<b>3.8924</b>

The table 7.22 shows that if investor wants to incorporate risk free asset in his portfolio then the optimal complete portfolio will give an allocation of 57% to our  $P_4$  portfolio and rest 43% in risk free asset. And the return that can be achieved will be 0.93% and maximum risk reduction is done as the risk is being reduced to less than 4%. As Nifty and Agri is having negative correlation we have achieved maximum risk reduction by combining them.

Now, the figure below shows the chart plotting optimal complete portfolio and optimal risky portfolio of Nifty and Agri.



Lastly we find out how much allocation is required for getting maximum utility in the portfolio of risky assets of Nifty and Agri and this is according to the risk taking ability of the investors. Below table shows the details.

<b>Table 7.23 - Optimal Portfolio According to Risk Level</b>			
<b>Nifty &amp; Agri</b>			
	Standard deviation	Variance	Expected Return
NIFTY	8.16	66.59	1.24%
AGRI	6.31	39.82	1.00%
Risk Free Return	0.67%	Covariance	-0.42
		Correlation	-0.01
$W_{nft} = ER_{nft} - ER_{Ag} + 0.01 A (\sigma_{Ag}^2 - COV_{nft Ag}) / 0.01 \times A (\sigma_{nft}^2 + \sigma_{Ag}^2 - 2 [COV_{nft Ag}])$			
$E(R_p) = W_{nft} \times ER_{nft} + W_{Ag} \times ER_{Ag}$			
$\sigma^2 p = (W_{nft}^2 \times \sigma_{nft}^2) + (W_{Ag}^2 \times \sigma_{Ag}^2) + 2(W_{nft} \times W_{Ag} \times COV_{nft Ag})$			
FOR INVESTOR WITH <b>A = 2</b> IS A HIGH RISK TAKER			
<b>Optimal weight for Nifty</b>	<b>48.70%</b>	<b>Expected return</b>	<b>1.1169%</b>
<b>Optimal weight for Agri</b>	<b>51.30%</b>	<b>Variance</b>	<b>26.0605</b>
		<b>Standard deviation</b>	<b>5.1050</b>
		<b>Sharpe Ratio</b>	<b>0.0875%</b>
FOR INVESTOR WITH <b>A = 3</b> IS A MODERATE RISK TAKER			
<b>Optimal weight for Nifty</b>	<b>45.00%</b>	<b>Expected return</b>	<b>1.1080%</b>
<b>Optimal weight for Agri</b>	<b>55.00%</b>	<b>Variance</b>	<b>25.3201</b>
		<b>Standard deviation</b>	<b>5.0319</b>
		<b>Sharpe Ratio</b>	<b>0.0870%</b>
FOR INVESTOR WITH <b>A = 4</b> IS A LOW RISK TAKER			
<b>Optimal weight for Nifty</b>	<b>43.11%</b>	<b>Expected return</b>	<b>1.1035%</b>
<b>Optimal weight for Agri</b>	<b>56.89%</b>	<b>Variance</b>	<b>25.0551</b>
		<b>Standard deviation</b>	<b>5.0055</b>
		<b>Sharpe Ratio</b>	<b>0.0866%</b>

Observations –

- This observation is similar to Comdex and Energy, as the risk taking ability of the investor decreases the allocation to Agriculture increases. Thus, if an investor wants to put money only in the risky asset and not in risk free asset than his optimal allocation according to the risk level are given in the above table 7.23.

## 7.4 - PERFORMANCE MEASUREMENTS

Another aspect of analyzing the performance characteristics is to calculate the excess return to volatility; this can be arrived at by calculating the Sharpe ratio for each asset. The Sharpe ratio is often referred to as the reward-to-volatility ratio. It measures the excess return, or risk premium, per unit of risk in an investment asset, trading strategy or portfolio. The Sharpe ratio is defined as:

$$S = \frac{(R_p - R_f)}{\sigma_p}$$

Equation 7.2

The Sharpe ratio is used to characterize how well the return of the portfolio compensates the investor for the risk taken, the higher the ratio number, the better. Furthermore, if we compare the expected return of two portfolios given a fixed standard deviation for both, the portfolio with the highest Sharpe ratio provides the most excess return for the associated risk.

**Figure 7.9 - Sharpe Ratio for All Standalone Portfolios**

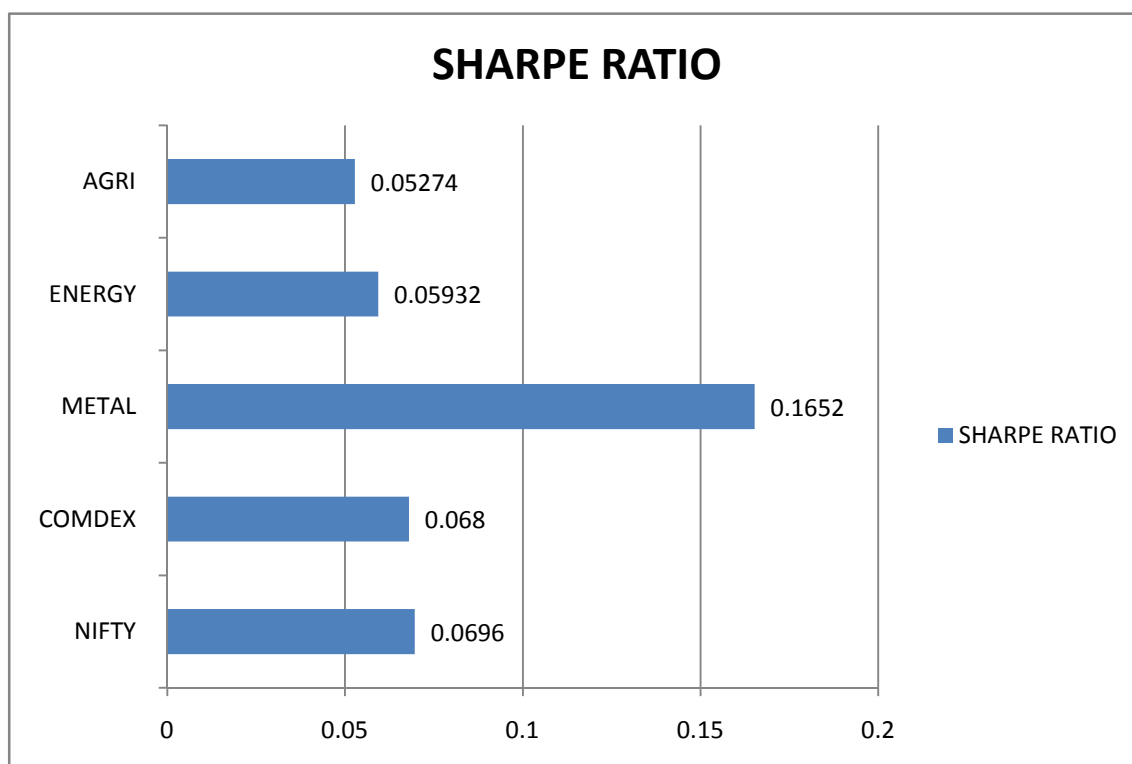
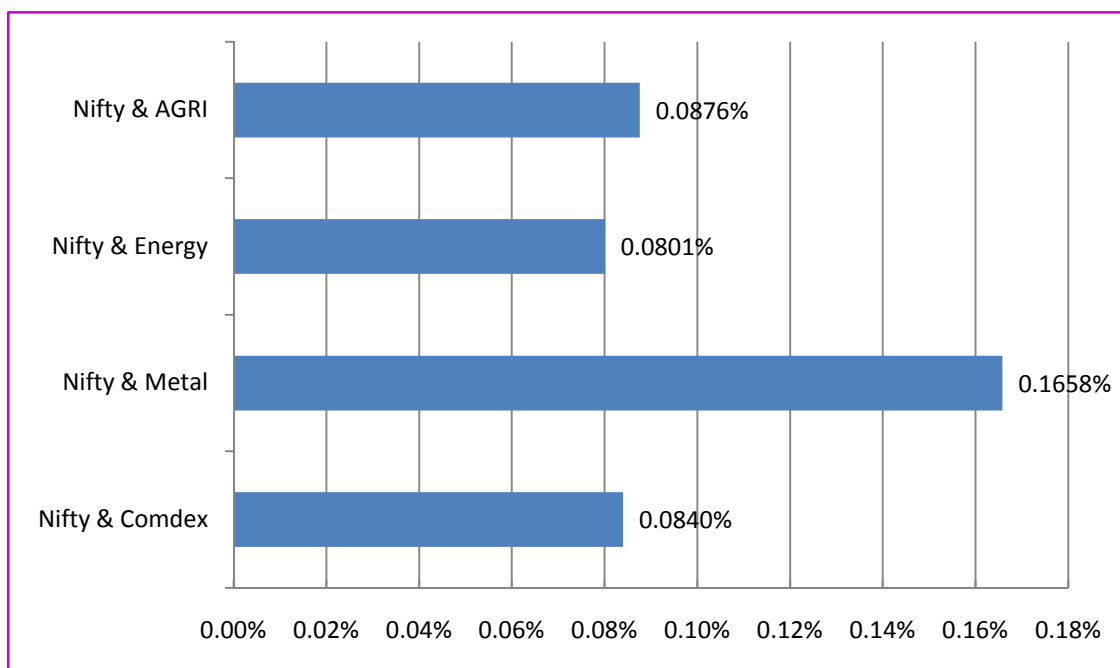


Figure 7.9 represents reward to variability of all standalone portfolios; it means if you invest in only stock or commodity indices only without combining it, you won't get the diversification benefit. The maximum Sharpe ratio is achieved by Metal index. Now we shall compare these ratios with optimal portfolio's Sharpe ratio, found by us by the Mean-Variance Analysis.

**Figure 7.10 - Sharpe Ratio for all Optimal Portfolios**



We can see here maximum risk adjusted return is obtained by Nifty & Metal optimal portfolio i.e. 0.1658%. Further, what drive these allocation changes are returns and correlations. We have seen that historically stock and commodity returns have had a low correlation and therefore a portfolio that invested in both stocks and commodity futures has a lower level of volatility than either stocks or commodity futures separately. Consequently, a mixed portfolio of stocks, bonds and commodity futures can be more efficient, have a higher ratio of return to risk, than a standalone stocks and bonds portfolio.

If we compare Figure 9 and Figure 10, we can see optimal portfolio dramatically improves the Sharpe ratio of standalone portfolios. The improvement in Sharpe ratio if we compare it with stock only portfolio i.e. Nifty, it ranges from 15% (when combine with Energy) to 138% (when combined with Metal).

## 7.5 - COMBINED FINDINGS AND RESULT FROM THE EMPIRICAL DATA

This section is showing a combined analysis of the all the portfolios that are constructed above, as we have seen above that what is the effect of combining stocks with commodities. Now, a further deep analysis is made that out of the four MCX indices chosen to find out which commodity index provides the maximum diversification advantage and gives the highest reward to variability ratio. This section is divided according to the portfolios that are formed starting from the minimum variance portfolio and then optimal risky portfolio and lastly optimal combined portfolio are analyzed.

### 7.5.1 - MINIMUM VARIANCE PORTFOLIOS

<b>Table 7.24 - Minimum Variance Portfolio</b>				
	Nifty/Comdex	Nifty/Metal	Nifty/Energy	Nifty/Agri
Expected return	1.10 %	1.53 %	1.19 %	1.09 %
Standard deviation	5.2801	5.4112	6.5880	4.9719
Minimum Variance	27.8795	29.2808	43.4018	24.7199
Sharpe Ratio	0.0814	0.1589	0.0789	0.0845
<u>WEIGHTS</u>	43% & 57%	26% & 74%	49% & 51%	37.5% & 62.5%

#### OBSERVATIONS –

- In portfolio context maximum risk reduction is obtained by combining **Nifty with Agri**. The risk reduction obtained is from 8.16 to 4.97 and in terms of percentage it comes out to **be 39% risk reduction**. And return has been reduced from 1.24% to 1.09% which turns out to be just a **reduction of 12% in return**.
- The maximum advantage in terms of return is available by combining **Nifty and Metal**. The return has been enhanced to 1.53% from 1.24% which turns out to be **increase in 23.39% in return** and the risk reduction obtained is from 8.16 to 5.41 which turn out to be **risk reduction of 33.70%**.

Thus we can see the advantage of combining stock portfolio with commodities and get advantage in terms of increase in return and reduction in risk.

### 7.5.2 - OPTIMAL RISKY PORTFOLIOS

Table 7.25 below shows the entire optimal risky portfolio's with highest Sharpe ratio that have been constructed in this thesis. One important aspect needs to be highlighted, these portfolios are optimal portfolios, but as we have taken four MCX commodities indices it would be of interest to see that out of all optimal portfolios which is the best one.

<b>Table 7.25 - Optimal Risky Portfolio With Highest Reward To Variability Ratio</b>				
	Nifty/Comdex	Nifty/Metal	Nifty/Energy	Nifty/Agri
Expected return	1.13%	1.61%	1.20%	1.12%
Standard deviation	5.4915	5.6471	6.6346	5.1556
Variance	30.1567	31.8903	44.0179	26.5801
Sharpe Ratio (reward to variability ratio)	0.0840%	0.1658%	0.0801%	0.0876%
Weights	43 % & 57%	06% & 94%	57% & 43%	51% & 49%

#### **OBSERVATIONS –**

- The highest reward to variability is achieved by combining Nifty and Metal. We can see that Sharpe ratio of stock only portfolio was 0.0696 (Table 7.1) which has been enhanced to 0.1658 and this is a massive increase in return of 138% per unit of risk taken.
- On the other hand the lowest Sharpe ratio is 0.0801 (i.e. between Nifty and Energy) that is also an increase from stock only portfolio by 15%.
- If we compare the return of optimal portfolio with risk then Nifty and Metal is giving a risk reduction of 34% and return has been increased by 31% these figures are showing massive portfolio optimization benefits of combining stock portfolio with commodity portfolio. Thus, Nifty & Metal is outperforming all others.
- On the other hand we find no advantage in combining Nifty with Energy in terms of risk and return.

### 7.5.3 - OPTIMAL COMBINED PORTFOLIOS WITH RISK FREE INVESTMENT

Table 7.26 - Optimal Complete Portfolio With Risk Level (A=3)				
	Nifty/Comdex	Nifty/Metal	Nifty/Energy	Nifty/Agri
<b>Risky portfolio fraction</b>	51%	98%	40%	57%
<b>Complete portfolio return</b>	0.9052	1.5876	0.8827	0.9275
<b>Complete portfolio risk</b>	3.922	5.5904	4.1961	3.8924

The main aim here is to get further risk reduction by introducing risk free asset in the optimal portfolio. After we have got Optimal Risky Portfolios now we add to it risk free asset and find out optimal complete portfolio. Here, all the investor will hold the same risky portfolio but if they want to decrease the risk than according to their risk aversion they will make some % investment in Risk Free Asset. This research has considered the view point of a moderate investor at risk level of A=3 i.e. moderate risk taker (which is by default used by Matlab). The following observations can be made.

#### **OBSERVATIONS –**

- The maximum return is obtained by combining Nifty with Metal. As we have already seen that the optimal risky portfolio allocates 94% in Metal and 6% in Nifty, so now the combined portfolio will allocate 98% in the optimal risky portfolio and rest 2% in risk free asset. The advantage of it would be a risk reduction of almost 1%.
- Further, we can see maximum risk reduction is obtained by combining Nifty with Agri as the risk is low from the Minimum variance portfolio to the **extent of 22%** from 4.9719 to 3.8924 by adding 43% of investment in risk free asset. On the other hand 15% return is reduced from the minimum variance portfolio.
- Further, all Nifty with Comdex is also giving a risk **reduction of 26%** from its minimum variance and return has been reduced by 17%.
- Well we can see here that the clear advantage of reducing the risk by addition of risk free asset in all portfolios except Nifty with Energy is not giving that much advantage although if anyone wants to invest in Energy for getting diversification benefits then they can get a risk reduction of 36% by making 60% investment in risk-free asset.



In totality the final conclusion can be that if the investor is a high risk taker he can get maximum out of combining Nifty with Metal, high return with low risk as compared to stock only portfolio. Another point is that as we have seen from above analysis diversification benefit out of investment in stock and commodity is there for all portfolios, as the correlation between them is low. And maximum benefit is out of combining Nifty and Agri as they are having a negative correlation. Finally, Nifty & Comdex optimal portfolio here is dominated by Nifty & Agri, that is a very surprise finding as the Nifty/Comdex portfolio has turned out to be inefficient.

Lastly, these are remarkable outcomes which are consistent with findings in the international literature that commodity futures can add diversification benefits to a traditional portfolio [Ankrim and Hensel (1993), Anson (1999) and Georgiev (2001), Scheeweis and Spurgin (1997), Gorton and Rouwenhorst (2004) and Doyle, Hill and Jack (2007)].

## **7.6 - COMMODITY PORTFOLIOS AS AN INFLATION HEDGE**

As we know that what matters in the long run is to gain purchasing power i.e. to outperform inflation. In contrast to stocks and bonds, commodity prices are not discounted future cash flows. Thus, it is valid to assume that the relation between commodity futures returns and inflation is fundamentally different as well. According to Kat and Oomen 2006, in times of strong economic growth, there will be an upward pressure on commodities producer and consumer prices, as well as interest rates. The increasing commodity prices and higher interest rates will reduce growth potential of company profits and also reduce the present value of future cash flows, i.e. the stock and bond returns will fall. However, commodities will still perform well. Given this view, higher inflation is likely to have a negative impact on stock and bond returns but a positive impact on commodities.

Literature has shown that actual inflation can be decomposed in two components: expected inflation and unexpected inflation. Expected future inflation will already be incorporated in today's asset prices. Therefore, what really matters is not how commodity returns respond to actual inflation, but to unexpected inflation (Gorton and

Rouwenhorst 2005). However, it is not easy to measure the unexpected inflation. There are two principal methods of doing this. Firstly, Fama and Schwert 1977 shows that unexpected inflation can be calculated as the difference between actual inflation and the T-Bill rate, serving as proxy for expected inflation. Secondly, Kat and Omen 2006 provide that the change in inflation rate can be used as a proxy for unexpected inflation, this totally assumes that today's inflation is the best predictor of future inflation. In this thesis, the first method is used. Here inflation is measured by WPI inflation percentage monthly change.

### **7.6.1 - Hedge against Inflation**

The results of this study show positive correlation between three commodity indices (Comdex, Metal and Energy) and inflation; whereas Nifty and Agri is showing negative correlation with inflation. Agri is surprisingly having negative correlation with inflation, its -0.089 so Agri cannot play a part in inflation hedge, but it's not statistically significant. If we compare the results with what have been witnessed in the past studies using GSCI and DJAIG indices, then we find similar kind of relationship i.e. positive correlation between inflation and commodities in India. Further, it is statistically significant at 5% level so we are confident about our result. Thus, we can conclude here that commodity investment in India can be considered as an inflation hedge.

		Nifty	Comdex	Metal	Energy	Agri	Inflation
Inflation	Pearson Correlation	<b>-.059</b>	<b>.279(*)</b>	<b>.137</b>	<b>.281(*)</b>	<b>-.089</b>	1
	Sig. (2-tailed)	.603	.013	.228	.012	.434	

\*\*Correlation is significant at the 0.01 level (2-tailed).

\* Correlation is significant at the 0.05 level (2-tailed).

### **HYPOTHESIS TESTING –**

The following hypothesis has been formulated in chapter 6. The results are discussed below. Hypothesis was to know that whether commodities can be considered as a better hedge against inflation than stocks. To test this correlation between the selected indices and unexpected inflation has been calculated. The result is shown in the above Table 7.27. The interpretation and discussion of each hypothesis is done below.

## **Hypothesis 2: Commodities is a better hedge against inflation than stocks –**

The literature has revealed that commodity has shown positive correlation with inflation and stock has been showing negative correlation to inflation. The present analysis gives a similar outcome, first we are considering Comdex index and we can see that a significant positive correlation is there between Comdex and inflation for which  $r = 0.279$ . From this it is totally evident that Comdex is having inflation hedging properties and can be used as a hedge against inflation.

Next comes Metal and we can see that Metal is also having positive correlation of 0.137 with inflation but it is not statistically significant so we cannot confidently say that Metal can be considered as an inflation hedge in Indian scenario.

Now, we shall consider Energy index. Although, in our thesis we have concluded that Energy is giving minimum diversification benefit but from the view point of inflation hedge properties it is showing significant positive correlation of 0.281; which makes it a good investment to hedge against inflation.

Lastly, we consider Nifty and Agri together as both are having negative correlation of -0.059 and -0.089 with inflation which is not statistically significant, so we can conclude that Nifty and Agri cannot be considered as inflation hedge as our results are not significant.

### **Comparison with other studies -**

Table 7.27 shows that during the period of analysis (June 2005 – December 2011), stocks demonstrate a negative correlation with that of inflation. We can see that Nifty is having a correlation of -0.059 with Inflation and this is in line with the observations made from the economies worldwide. However, commodity futures indices, such as MCX COMDEX, MCX Metal and MCX Energy demonstrate positive correlation with inflation. This also corresponds to the empirical findings in most of the economies that the commodities are real assets and can be used as a hedge against inflation as they are positively correlated with inflation. Investors ultimately care about the real purchasing power of their returns, which means that the threat of inflation is a concern for every

investor. Gorton & Rowenhorst 2004 has concluded that many traditional asset classes are a poor hedge against inflation – at least over short and medium-term horizons and as commodity prices increases, it leads to fears concerning inflation. Since one part of inflation is caused by rising commodity prices, it is not unreasonable to assume that commodities may provide a good hedge against inflation.

Gorton and Rouwenhorst (2004) demonstrated that over a 5-year rolling period their passive commodity futures portfolio demonstrated a correlation to inflation of just over 0.40. This relationship is clearly more meaningful (and beneficial) than the negative correlation between stocks or bonds and inflation. The present study on the other hand does not show that Agri is having negative correlation with inflation and Comdex and Energy has a positive correlation of 0.28 which is statistically significant but compare to other Gorton's study it is very less.

Based on empirical research Kaul (2007) shows that inflation rates have been much higher in several commodities which are not at all traded on the exchange. Further, price volatility in several commodities has in fact come down post-introduction of futures in those commodities. Thus there is no one to one correspondence between futures trading and commodity price inflation in India as yet. It is also shown that hedgers make up a significant part of trades in the market to invalidate fears related to excessive speculative activity.

Lastly we conclude that most assets do not benefit from rising inflation, particularly unexpected inflation, but commodities usually do; as demand for goods and services increases, the price of those goods and services usually rises as well, so does the price of the commodities used to produce those goods and services. Because commodities prices usually rise when inflation is accelerating, investing in commodities may provide portfolios with a hedge against inflation. On the other hand, stocks and bonds tend to perform better when the rate of inflation is stable or slowing. Faster inflation lowers the value of future cash flows paid by stocks and bonds because those future rupee will be able to buy fewer goods and services than they would today. For example, during the 1980s and 1990s, inflation fell and stocks and bonds experienced bull markets

(Campbell & Viceira, 2002). Also a higher rate of inflation could raise interest rates and corporate borrowing costs, weighing on both stocks and bonds, while increasing demand for commodities as a hedge against inflation (Greer, 2000).

Commodities may also react differently from stocks and bonds to other changes in economic or market conditions. For example, if OPEC unexpectedly reduced the supply of oil by a significant amount, the price of oil, gasoline and heating oil would likely rise. Natural gas prices might rise as well if industrial consumers switched from oil to gas. Rising energy costs could lead to higher commodity prices overall, which would most likely weigh on corporations' bottom lines and possibly result in inflation that would weigh on bonds (Halpern & Warsager, 1998).

Almost all past researches that have been done in US have shown that commodities futures have positive correlation with inflation and they made the principal argument for investing in commodities is that investing in assets that rise in price with inflation provides a natural hedge against losses in equity and debt holdings that typically lose value during periods of unexpected inflation. On the other hand this research finds that commodities is having positive correlation but it's not as high as it has been shown by previous researches and they are also not statistically significant and Agri is remarkably having negative correlation with inflation. This might be because the composition of products measuring inflation is different than that of Agri futures.

## **7.7 - RESULTS OF HYPOTHESIS TESTING**

Above we have discussed the result of hypothesis 1, here we shall start with hypothesis 2 which is to examine that whether commodities return are similar to equity return. To test this hypothesis t-test is used and the results are as under.

### **Hypothesis 3: Volatility There is no significant difference in the Volatility of Commodities and equities –**

This hypothesis has been divided into four parts as we have four commodity indices to compare with the one stock index. The result of testing the hypothesis that if there is

any significant difference between the volatility of stock and commodity is represented in the table below.

<b>Table 7.28 - Result of testing Hypothesis 2</b>		
Asset class under study	p –value	Result
Nifty & Comdex	0.000668729	as p value is less than 0.05 than null hypothesis is rejected it means that variance are <b>not equal</b> and there is significant difference bet the variance
Nifty & Metal	0.001521058	as p value is less than 0.05 than null hypothesis is rejected it means that variance are <b>not equal</b> and there is significant difference bet the variance
Nifty & Energy	0.461949648	as p value is more than 0.05 than null hypothesis is accepted it means that variance are <b>equal</b> and there is no significant difference bet the variance
Nifty & Agri	0.012233101	as p value is less than 0.05 than null hypothesis is rejected it means that variance are <b>not equal</b> and there is significant difference bet the variance

#### **Observations –**

1. We can see here that the null hypothesis is rejected in case of Nifty vs. Comdex, Metal and Agri, thus we conclude that there is significant difference between the volatility of these indices.
2. This finding has very much relevance on our study as the main objective of this study is to achieve diversification benefits by combining stock and commodities so it was necessary to check if the volatility is significantly different or not. And the answer is positive and that the volatility is significantly different.
3. Thus, we have already seen that the volatility of Nifty is the highest amongst all other indices and thus finding out that it is significantly high concludes that stocks are more volatile than commodity in the Indian markets.
4. Lastly, Energy and Nifty volatility is not significant; energy is little less volatile than Nifty, (its 8.07 vs. 8.16). We have already seen above that energy has been the

worse performer amongst all other indices. The fact that its volatility is not significantly different with Nifty doesn't have much impact on the result.

**Hypothesis 4: Commodities yield equity like returns –**

As we have measured the volatility significance we shall move forward and apply t-test for unequal variance and equal variance to check whether return of commodity and stock is having any significant difference or not. Previous research suggests that long-only portfolios of commodity futures have had average returns similar to stocks, e.g. Bodie and Rosansky's (1980) and Gorton and Rouwenhorst (2005). The result of our thesis is as under –

<b>Table 7.29 - Result of testing Hypothesis 3</b>		
Asset class under study	p –value	Result
Nifty & Comdex	0.435096803	as p value is more than 0.05 than null hypothesis is accepted it means that means of 2 variable are equal no significant difference bet the Arithmetic mean
Nifty & Metal	0.36503251	as p value is more than 0.05 than null hypothesis is accepted it means that means of 2 variable are equal no significant difference bet the Arithmetic mean
Nifty & Energy	0.472840148	as p value is more than 0.05 than null hypothesis is accepted it means that means of 2 variable are equal no significant difference bet the Arithmetic mean
Nifty & Agri	0.420507411	as p value is more than 0.05 than null hypothesis is accepted it means that means of 2 variable are equal no significant difference bet the Arithmetic mean

**Observations –**

1. The main objective behind testing this hypothesis was to find out whether commodity gives equity like returns or not? And it's very interesting to find out that there is no significant difference between the returns of commodity and stock. This, result coincides with the literature as all previous studies done have

concluded that commodity gives equity like returns and our thesis gives the same result in Indian scenario.

2. This is good news for commodity investors as one side Nifty is having significantly high volatility compared to commodity and on the other side the difference in return is not significant.
3. Further we have seen that Nifty is having high return compared to Comdex, Energy and Agri, but the return of Nifty is lower than Metal. But, the results being not significant it can be concluded that there is no difference between the returns of Nifty and other indices. Thus investment in either will give similar returns.
4. Lastly we can observe the result found with Nifty & Metal & can see that Metal is having lower volatility than Nifty which is statistically significant and that Metal is having higher return than Nifty which is not statistically significant thus in terms of return both are equal but in terms of risk Metal is significant having low risk than Nifty.