CHAPTER 6

RELIABILITY BASED TOTAL COST OF OWNERSHIP
MODEL FOR OPTIMAL REDUNDANCY ALLOCATION

6.1 INTRODUCTION

The system engineering principles suggest that the best design is the design which maximizes the system operational effectiveness and at the same time minimizes the Total cost of Ownership ‘TCO’ (Dinesh Kumar et al 2007b). TCO models are found in various applications (Pasqua 1996; Sohn and Moon 2003; Castellani et al 2005). DineshKumar et al (2007b) pointed out that in systems such as commercial aircraft, the operation and maintenance cost can be 12 times as much as the procurement cost. Since the in-service cost accounts for the vast majority of the TCO, it is necessary to include in-service cost in product design and development decisions. For instance, Hitachi Construction Truck Manufacturing Ltd, a multi-international manufacturer of rigid haulers (30 to over 300 ton) reported that the operating costs can be over three to four times of the initial cost (Chen and Keys 2008). The above discussions disclose that

- TCO based models in product design and development can offer substantial cost savings.

- TCO based design strategy is emerging for the development of newer products and services.
- Manufacturers can apply TCO models to take decision on the product guarantee period.

- Procurement, replacement and down time costs have direct relationship with reliability of the product (Figure 1.1).

On the above consideration, a Reliability Based Total Cost of Ownership (RBTCO) model is proposed for optimal redundancy allocation decisions. The rest of the chapter presents product configuration, assumptions, objective criterion, problem statement and mathematical formulation of the RBTCO model developed for optimal redundancy allocation decisions.

### 6.2 PRODUCT CONFIGURATION

The product comprises of $n$ elements and every element is critical for the functioning of the product and hence the product is considered as a system of $n$ elements arranged in series. Failure of any one element $i$ would lead to failure of the product. Each element $i$ is composed with one or more number of identical components $x_i$ that are arranged in parallel (i.e. $x_i$ number of redundant identical components becomes an element). The product requires at least one component in each element (i.e. $x_i > 1, \forall i$). It is understood that the product fails to perform its function when all the components of any one of its elements fails. Besides, the design specification warrants (i) a certain minimum required reliability for the product $R_s$ and (ii) restriction on the maximum weight $W_s$. The intended life of the product is $t$ periods. For the components of each element $i$ (\(\forall i, i=1\ to \ n\)), the reliability $r_i$ for the intended period $t$, the initial procurement cost $c_i$ and the weight $w_i$ are known. Figure 6.1 shows the schematic representation of the product configuration.
6.3 ASSUMPTIONS

- Whenever an element fails, the entire set of its components are replaced and the maintenance of the product is also carried out during such replacement instances only.

- The disposal cost is much lower than the initial procurement cost and hence considered as negligible.

- No preventive maintenance is performed on the elements.

- All redundancy is active. If all redundant components operate simultaneously from time zero, even though the system needs only one of them at any time, the arrangement is called active redundancy.

6.4 OBJECTIVE CRITERION

The redundancy allocation in the product development considers reliability and availability as the principal design strategy. Dineshkumar et al (2007b) mentioned that the best design is the design that maximizes the system operational effectiveness and at the same time minimizes the TCO.
Taking account of the above points, in this thesis a TCO equation is formed with reliability related cost elements and the minimization of the developed TCO is considered as the objective criterion for the redundancy allocation problem.

The TCO equation for the proposed RBTCO model is developed based on the concepts discussed by Dineshkumar et al (2007b). However, the problem environment and the TCO equation of the proposed RBTCO model differ from those of Dineshkumar et al (2007b).

Dineshkumar et al (2007b) formulated the model with

- the decision variables of number of redundant components from different potential suppliers of each subsystem (element) of the product,
- an objective criterion of minimization of TCO, which is developed involving initial procurement cost, operational cost, maintenance cost and disposal cost and
- a constraint on minimum availability developed by taking the failure rate and repair rate.

On the other hand, the proposed RBTCO model is structured with

- the decision variables of number of identical redundant components for each subsystem (element) of the product,
- an objective criterion of minimization of TCO, which is developed involving procurement cost, replacement cost and downtime cost that have direct relationship with reliability of the product and
two constraints to meet the design specifications of minimum reliability for the product $R_s$ and restriction on the maximum weight $W_s$.

The RBTCO model under consideration assumes identical components for each element. The operational cost that depends on the performance of the components is independent of the redundancy of the components. It is an acceptable practice to assume that the disposal cost is much lower than the initial procurement cost. Hence, operational cost and disposal cost terms are not included in the TCO of RBTCO model. Besides, cost for down time, which is considered as an availability constraint by Dinesh kumar et al (2007b), is included as a cost component of TCO in the present model. The three cost items of the objective function, Procurement Cost (PC), Replacement Cost (RC) and Down Time Cost (DTC), are explained in detail in the following subsections.

### 6.4.1 Procurement cost

It is the total cost of purchasing all the $n$ elements of the product. The cost of each element $i$ become the multiplication of component cost $c_i$ by the number of components $x_i$, which is given as:

$$C_i = c_i x_i \quad (6.1)$$

Then the procurement cost of the product is the sum of the purchase cost of all $n$ elements and is given as:

$$PC = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} (c_i x_i) \quad (6.2)$$
6.4.2 Replacement cost

It is the sum of the cost of replacing the failed elements during the intended product life time $t$. The replacements are carried out as elements, i.e., when element $i$ fails, its $x_i$ number of components are replaced. It is assumed that the failure time of each element $i$ follows negative exponential distribution with failure rate of $\lambda_i$ and the element is required to operate for a specified period of time $t$. Then the reliability of the $i^{th}$ element $R_i$ can be represented as follows (Misra 2001):

$$R_i = e^{-\lambda_i t}$$  \hspace{1cm} (6.3)

As element $i$ comprises of $x_i$ number of redundant identical components with reliability $r_i$ for the life $t$, which is same as the intended life of the element as well as the product, the reliability of element $R_i$ thus becomes (Misra 2001):

$$R_i = 1 - \prod_{1}^{x_i}(1 - r_i)$$  \hspace{1cm} (6.4)

i.e. \( R_i = 1 - (1 - r_i)^{x_i} \)  \hspace{1cm} (6.5)

Substituting the value of $R_i$ from equation (6.3), the equation (6.5) is rewritten as:

$$1 - (1 - r_i)^{x_i} = e^{-\lambda_i t}$$  \hspace{1cm} (6.6)

By taking logarithm on both sides and rearranging them, the equation (6.6) reduces to

$$\lambda_i t = \ln \left( \frac{1}{1 - (1 - r_i)^{x_i}} \right)$$  \hspace{1cm} (6.7)
The number of times an element \( i \) fails during the product life period \( N_i \) is the multiplication of its failure rate \( \lambda_i \) and the intended life period \( t \).

Hence

\[
N_i = \lambda_i t = \ln \left( \frac{1}{1 - (1 - r_i)^{x_i}} \right)
\]

(6.8)

The other point, which is to be taken into account, is the money value and individual replacement. Under the change in money value and replacements at various time periods, the cost of the elements would be different from the procurement cost. A factor \( \beta_i \) is introduced here to account for the decrease in money value and the extra cost for replacement. Hence, the average cost of replacing an element can be expressed as:

\[
RC_i = \beta_i C_i = \beta_i c_i x_i
\]

(6.9)

The replacement cost due to the failure of element \( i \) during the product life \( t \) is

\[
TRC_i = (\text{Number of times the element } i \text{ fails during the life period } t) \times \left( \text{Average replacement cost of an element } i \right)
\]

\[
TRC_i = \ln \left( \frac{1}{1 - (1 - r_i)^{x_i}} \right) \beta_i c_i x_i
\]

(6.10)

Hence, the total replacement cost of all \( n \) elements of the product

\[
RC = \sum_{i=1}^{n} TRC_i
\]

(6.11)

\[
RC = \sum_{i=1}^{n} \left\{ \ln \left( \frac{1}{1 - (1 - r_i)^{x_i}} \right) \right\} \beta_i c_i x_i
\]

(6.12)
6.4.3 Downtime cost

It is the cost for the non-availability of the product. Failure of any one of the elements of the product leads to the non-availability of the product. Under the assumption that the failure of the elements is independent (i.e. simultaneous failure of the elements does not occur) and the rectification/restoration time of element $i$ is known as $DT_i$, then the total downtime due to element $i$ becomes the multiplication of $N_i$ and $DT_i$. When the cost of non-availability of the product per unit time is given as $DC$, the total downtime cost ($DTC$) for all $n$ elements of the product becomes

$$DTC = DC \sum_{i=1}^{n} \left\{ \ln \left( \frac{1}{1-(1-r_i)^{w_i}} \right) \right\} DT_i$$

(6.13)

The TCO for the RBTCO model for optimal redundancy allocation decisions is obtained by adding the cost components given by equations (6.2), (6.12) and (6.13). Also the design specification warrants (i) a certain minimum required reliability for the product $R_s$ and (ii) restriction on the permissible maximum total weight $W_s$. They become the constraints in the number of redundant components for minimum TCO.

6.5 PROBLEM STATEMENT

Determination of optimal number of redundant components $x_{i}^{opt}$, ($\forall i, i = 1 \text{ to } n$) for the elements of a product for minimum TCO subjected to the constraints of minimum required product reliability $R_s$ and limitation on weight $W_s$ for the given input values of:

- Cost of component $c_i$ ($\forall i, i = 1 \text{ to } n$),
- Reliability of component $r_i$ for the indented life time $t$ ($\forall i, i = 1 \text{ to } n$),
- Weight of component $w_i$ ($\forall i, i = 1 \text{ to } n$),
User defined value of individual replacement $\beta_i (\forall i \in 1 \text{ to } n)$,

Time required for rectifying the defect $DT_i (\forall i \in 1 \text{ to } n)$,

User defined maximum number of components in parallel $x_{i,\text{max}} (\forall i \in 1 \text{ to } n)$ and

Down time cost of the product per unit time $DC$.

### 6.6 MATHEMATICAL MODEL

Minimize \[ TCO = PC + RC + DTC \]

\[
\sum_{i=1}^{n} (c_i x_i) + \sum_{i=1}^{n} \left\{ \ln \left( \frac{1}{1 - (1 - r_i)^{x_i}} \right) \right\} \beta_i c_i x_i + \\
DC \sum_{i=1}^{n} \left\{ \ln \left( \frac{1}{1 - (1 - r_i)^{x_i}} \right) \right\} DT_i
\]  

(6.14)

Subject to:

\[
\prod_{i=1}^{n} \left(1 - (1 - r_i)^{x_i}\right) \geq R_s \text{ (Product reliability constraint)} 
\]  

(6.15)

\[
\sum_{i=1}^{n} x_i w_i \leq W_s \text{ (Maximum weight constraint)}
\]  

(6.16)

\[
1 \leq x_i \leq x_{i,\text{max}} \quad x_i \text{ integer (} \forall i \in 1 \text{ to } n) 
\]  

(6.17)

The first constraint (equation 6.15) assures the minimum required reliability of the product. The second constraint (equation 6.16) imposes the limitation on the permissible maximum total weight of the product. The third set of constraint (equation 6.17) ensures at least one component in each element $i$ for the functioning of the product.
6.7 SUMMARY

A new RBTCO model is developed for optimal product configuration based on the costs, which are directly related with the product reliability. The RBTCO model presented in this thesis would be useful to evolve best design with respect to the performance during the intended life of the product. This model is expected to meet the design considerations of electronic products such as TV, PCs, Controllers, Mobile phones, Home appliances etc., The mathematical formulation of RBTCO model for redundancy allocation is similar to the well-known Redundancy Allocation Problem (RAP). The RAP is a classical nonlinear integer programming problem with separable constraints that has attracted considerable attention from the research community. The solution methodologies found in the literature (Kuo and Prasad 2000) for RAP are: Dynamic programming, integer programming, mixed-integer nonlinear programming, heuristics and meta-heuristics. Two meta-heuristics, SAA and GA, are proposed as the possible solution methodologies for this RBTCO model developed for redundancy allocation decisions. They are presented in chapters 7 and 8.