APPLICATION OF INFORMATION THEORY FOR STRUCTURE-BASED RATING OF EPICYCLIC GEAR TRAINS
5.1 INTRODUCTION

Distinct kinematic chains do have some similar properties like equal number of links and joints. Yet they differ in some aspects and some linkages have different kinematic properties due to difference in the type of joints and their arrangement. Work reported on the synthesis of mechanisms so far is based either on closed form solutions or on mathematical programming. Mathematical programming covers formulating an objective function and impose the requirements as the Constraints. The objective function thus formulated is then minimized using any one of the established methods. The objective function can be formulated following the least squares method. The concept of entropy in mechanisms can be used to formulate the objective function. Information theory developed in 1960 utilizes the entropy concept in communication engineering. Information theory has great potential for application to mechanism synthesis.

5.2 INFORMATION THEORY.

The theory of probability helps to provide a quantitative measure to the information received in a system. Considering a finite number of mutually exclusive events \( E_k \) with corresponding probabilities of occurrence \( P_k \), the set of all events can be designated as a row matrix \( E \) and the set of corresponding
probabilities as another row matrix $P$. Such a situation can be represented by the matrix subject to the condition

$$\sum_{k=1}^{n} P_k = 1 \quad \text{.....(1)}$$

$$[E] = \begin{bmatrix} E_1 & E_2 & \cdots & E_n \end{bmatrix}$$

$$[P] = \begin{bmatrix} P_1 & P_2 & \cdots & P_n \end{bmatrix} \quad \text{.....(2)}$$

Where $n$ is the total number of events. Equation (2) may be called as an uncertainty scheme. The basic problem of interest is to associate a measure of uncertainty, $H(P_1, P_2, \ldots, P_n)$ with the probability scheme. Measure of uncertainty is termed as "Entropy". Measure of uncertainty or entropy associated with the sample space of a finite scheme is given by

$$H(x) = H(P_1, P_2, \ldots, P_n) = -\sum_{i=1}^{n} P_i \log P_i \quad \text{.....(3)}$$

Since any $P_k < 1$, $H(x)$ is always positive. A scheme is considered as a better one in which $H(x)$, the entropy, is minimum so that the occurrence of the events can be predicated with more certainty. Shannon’s fundamental theorem states that it is possible to communicate information at an ideal rate with utmost reliability in the presence of "noise". This can be extended to linkages (in the study of mechanisms) i.e., the output from a mechanism can be obtained with utmost reliability in the presence of structural errors.
5.2.1 ANALOGY OF MECHANISMS TO CHANNEL IN INFORMATION THEORY

A mechanism can be interpreted as "channel" in the basic model of a communication system for which information theory is developed. Channel is an interface between transmitter and receiver. On the same lines a mechanism is used to transmit or transform motion or power from input end to output end. The analogy is described by the fig.5.1.

Fig. 5.1- The analogy of a channel in information theory to a mechanism

5.3 APPLICATION OF INFORMATION THEORY

In mechanisms for example such as a four-bar function generator, the generated output is, different from the desired output. The difference of these two, the error known as structural error can be considered as a random variable and the same can be expressed in terms of design parameters, i.e. link lengths and the input and output angles. If the function generated \( y = f(x) \) is continuous within the range of operation \( x_s < x < x_f \), there is error at every
point in the design space and the scheme can be understood as one of continuous probability and Eq.(3) does not hold.

In case of continuous functions, the entropy $H(x)$ can be expressed as

$$H(x) = \int f(x) \log f(x) dx$$

Where $f(x)$ is the probability density function which depends on the distribution function.

In case of discrete probability schemes, the maximum entropy occurs when all the individual properties in the scheme are equal i.e. $P_1 = P_2 = \ldots P_x = \ldots P_n$; while in schemes with continuous probability the maximum of entropy depends on the nature of probability density distributions. In the study of mechanisms one can assume the errors, random variables as normally distributed.

For continuous functions with normal distribution maximum entropy is given by

$$H(x) = \ln \sqrt{2\pi\sigma}$$

Where $\sigma$ is the standard deviation.

A mechanism is termed as an ideal one if its maximum entropy is minimum. In terms of Eq. (5), for minimum values of maximum entropy $\sigma$ must be minimum. Hence the problem reduced to expressing $\sigma$ in terms of linkage parameters and then minimizing it. Also it is convenient to minimize
\[ \sigma^2, \text{ variance instead of } \sigma \text{ without losing accuracy as } \sigma^2 \text{ can be expressed in the form of least squares and the solution becomes linear.} \]

Further properties such as flexibility or mobility, i.e. the capacity to transmit quantified motion reflect kinematic property. It is desirable to know how the flexibility and efficiency are affected by the structural layout of links, type of links like binary, ternary, quaternary etc and their number in planar linkages and the type of joints, their sequences etc. Graph theory and information theory can be combined to formulate simple equations to reveal the relative merits of kinematic chains.

A kinematic chain can be represented by a graph with links as vertices and joints as edges. Further a graph can be represented by a zero-one adjacency matrix \([A]\) given by

\[
[A] = [a_{ij}]
\]

Where \(a_{ij} = 1\) if link \(i\) is connected to link \(j\)

\[= 0\text{ if link } i \text{ is not connected to link } j\]

\(a_{ii} = 0\) also \(a_{ij} = a_{ji}\)

The above method of matrix representation enables to develop formulae revealing the relative merits of chains with the same number of links and joints or otherwise. To accomplish this, Information theory is used as described below. Entropy or uncertainty can be interpreted to represent flexibility or mobility. Greater the entropy the greater is the flexibility. It is
known by intuition that four bar chain is more flexible or uncertain compared to a three bar frame. Flexibility in chain increases with increase in the number of links and joints in the chain. A six bar chain with six links, seven joints and one DOF more flexible than 4 link, 4 joint and one DOF four bar chain. Designer's interest is to know and measure this sort of property. Thus entropy can be used as a tool to reveal some properties of interest in kinematic chains.

Returning to linkages and their adjacency matrices, the entry one in matrix denotes the certainty of motion transmission between two of links under consideration while zero entry indicates certainty that no motion is transmitted. Also the entry one may be understood as the DOF of revolute joint of planar four-bar chain. For spatial linkages the entries in the matrix are either zero or the number 1,2,3 etc depending on the DOF of the joint. Consider an RSSR spatial linkage shown in figure 5.2. The adjacency matrix of the RSSR chain is

\[
\begin{pmatrix}
0 & 1 & 0 & 1 \\
1 & 0 & 3 & 0 \\
0 & 3 & 0 & 3 \\
1 & 0 & 3 & 0
\end{pmatrix}
\]
Sum of the rows in order are 2, 4, 6 & 4. Entropies of the links are

\[
H(x) = H(y) = 1(2 \log 2) + 2(4 \log 4) + 1(1 \log 6) = 10.089 \quad (6)
\]

Since Adjacency Matrix is symmetric, \(H(x) = H(y)\).

And \(H(x, y) = 4(1 \log 1) + 4(3 \log 3) = 5.725\) \quad (7)

5.3.1 PHYSICAL INTERPRETATION OF ENTROPIES

In the entropy \(H(x)\), terms \(2 \log 2\), \(4 \log 4\) etc in equation (6) correspond to total connectivity of a link and the coefficients 1, 2 & 1 indicate the number of links having the same connectivity.

For example, \(2(4 \log 4)\) reveals that there are two links each with a total connectivity 4. The terms \((1 \log 1)\) \((3 \log 3)\) in the entropy \(H(x, y)\) in equation (7) correspond to the connectivity of the each joint while their coefficients indicate the number of links participating in such joints. Some links may enter more than once in counting. For example term \(4(1 \log 1)\) means that there are four links 1, 2, 4 and 1 participating in forming revolute joint with DOF 1. With the above theory the general expression can be written as

\[
H(x) = \sum n_j (j \log j)
\]

Where \(j\) is the total connectivity of a link and \(n_j\) is the number of links having a total connectivity of \(j\).
And \( H(x, y) = \sum n_k (k \log^k) \)

where \( k \) is the connectivity of a joint, i.e. degree of freedom and \( n_k \) is the number of links participating in forming joints with a connectivity \( k \).
Fig. 5.2 RSSR SPATIAL MECHANISM
5.4 APPLICATION OF INFORMATION THEORY TO EGTs:

Almost all the study reported on structural aspects of linkages and gear trains pertain to generation of distinct kinematic chains and gear trains [59-73] and to know the type of freedom. All the relevant studies reported so far would not have much significance, if quantitative methods are not developed to compare all the distinct gear trains with the same number of elements and degree-freedom (DOF) for different aspects. It is always desirable to know the anticipated behaviour of gear trains without having to actually design and test them.

At present, the designer has to depend on intuition to select the best possible gear train and this may not always lead to optimum results. Hence, quantitative methods—simple and less time consuming—are needed to compare the planetary gear kinematic chains and trains at the conceptual stage of design for characteristics like capacity for power transmission, power loss and circulation. This chapter deals with these aspects.

Every gear train is represented by a graph, vertices of which correspond to elements and edges correspond to joints of the gear train. The connectivity of vertices, edges and their values and fundamental circuits pertaining to a train or its graph are related to design parameters and their deployment. An attempt is made to logically correlate the vertices, edges and
fundamental circuits with capacity to transmit, loss of power and circulation. Numerical measures to compare different chains are developed using the concepts of information theory, used in communications and described earlier. The concepts and the numerical measures developed for comparison of gear trains are illustrated.

In the process of graph/chain generation, the pseudo-isomorphic graphs are sometimes encountered. These are graphs, mathematically distinct but are kinematically identical. Out of them usually only one is retained when graphs with more number of elements/links are generated by adding elements and pairs to existing ones. However, a judicious choice in rejecting can be made by observing and comparing their characteristics like efficiencies, capacity, etc described in this chapter.
5.5 CONSTITUENTS OF A GRAPH OF PGT

Distinct planetary gear trains (PGT) differ among themselves in one or more of the aspects like Vertex Assortment, Type of joints, edges and their Number.

5.5.1 ELEMENT OR VERTEX ASSORTMENT

Every element of a PGT is represented by a vertex in its graph. The Vertex assortment can be understood as the groups of vertices with different connectives. In the graph of PGT, edge corresponding to a turning pair is assigned a numerical value of 2 while the edge corresponding to a gear pair is assigned a numerical value of 1. The connectivity of a vertex is defined as the sum of the numerical values of all the edges coincidence on it. A vertex with a connectivity of 2 is called a binary vertex and a vertex with a connectivity of 3 is called a ternary and so on similar to binary, ternary and quaternary links, etc in linkages. Every link in linkages is associated with a definite number of design parameters. For example, a binary link has one design parameter, a ternary link has three design parameters and quaternary link has five design parameters etc.

Every gear train is represented by an equivalent linkage. With this analogy, it is easy to understand that the elements of a gear train or vertices of its graph are associated with design parameters. Exact relationships between the
connectivity and the number of design parameters, not being very important, it is sufficient to say that the number of design parameters will increase with connectivity. Hence the number of design parameters associated with a vertex is taken equal to its connectivity.

5.5.2 TYPE OF JOINTS, EDGES AND THEIR NUMBER

Every edge in a graph represents either a turning pair or a gear pair. The role of an edge between two vertices in a graph in relation to the graph as a whole is signified by the number and type of other edges to which it is directly connected. Thus contribution of every edge in a graph is quantified by a numerical value, which is the sum of the numerical values of all other edges that are directly connected to it. This sum is called joint or edge value. The edge value is related to the vertex connectivity in the following manner. The value of an edge \((ij)\), that is, the edge connecting the vertices \(i\) and \(j\) connectivity can be expressed as

\[
(\text{Connectivity of vertex } i) + (\text{connectivity of vertex } j) - (2e_{ij})
\]

where \(e_{ij}\) is equal to 2 if the edge \(e_{ij}\) corresponds to a turning pair and \(e_{ij}\) is equal to 1, if the edge \(e_{ij}\) corresponds to a gear pair. Thus, it is easy to note that the edge or joint value is associated with a number of design parameters.
5.5.3 FUNDAMENTAL CIRCUIT

Each gear pair is associated with a fundamental circuit or loop. All the other edges in a fundamental circuit on a gear pair are turning pair edges. Each circuit will have a transfer vertex, the edges on either side of which are of different levels. It should be noted that all the edges on the same side of transfer vertex are at the same level.

5.6 SIGNIFICANCE OF GRAPH CONSTITUENTS

There are a number of distinct gear trains available with the same number of elements and DOF. Hence, it is necessary to compare all of them and select the best. The criteria on which gear trains should be compared are: (i) power transmission capacity, (ii) transmission efficiency or power loss, and (iii) power circulation.
Fig. 5.3 (a)

Fig. 5.3 (b)  Fig. 5.4
An attempt has been made in this work to relate the graph constituents of a gear train to different criteria or requirements. Numerical measures, for the purpose of comparison, are proposed in a later section.

5.6.1 POWER TRANSMISSION CAPACITY

The study reported here is essentially structure-based and hence, dimensions, strength, etc are not considered. Consider a gear train and its graph shown in Figure 5.3 (a) and 5.3 (b). If gear wheel 2 is chosen as the input element, it will transmit power (motion) to wheel 3, which in turn transmits to wheel 4. The velocity ratios between wheel 2 and wheel 3 and between wheel 3 and wheel 4 depend upon their sizes not considered here. Thus, it is clear that the same power flows through all the moving elements. It is also noted from Figure 5.3 (b) that the connectivity of the vertex-2 corresponding to the wheel 2 is three. On the other hand wheel 3, if taken as the input element, can transmit power to wheel 2 and wheel 4 simultaneously, maintaining the same velocity ratio between the wheels as in the earlier case. It can be seen from the graph that the vertex-3 has a connectivity of five.

It can thus be generalized saying that a vertex with greater connectivity can transmit more power. It is, therefore, obvious that a gear train with greater connectivity that is, sum of the connectivities of all vertices of its graph can transmit more power. The problem, however, arises when
comparison has to be made between the gear trains with the same number of
elements and DOF, which have equal connectivity. For example consider
graphs of two distinct gear trains Figure 5.3(b) and Figure 5.4 with four
elements. The total connectivity of all vertices in both the graphs is equal to
fourteen but the distribution of connectivity among vertices is not the same. In
case of Figure 5.3(b) the distribution among vertices 1,2,3 and 4 is respectively
3,3,5 and 3 while in Figure 5.4 it is 4,4,3 and 3.

From that has been explained earlier, it is clear that the capacity of
an element (vertex) to transmit power depends upon its connectivity. Since the
graph that is, gear trains consists of vertices with different connectivities, it is
necessary to know which of the combinations or distributions, with the same
total connectivity will lead to greater capacity to transmit power. Hence, a
numerical measure becomes necessary to compare such graphs or trains and the
same is proposed in a later section.
5.6.2 TRANSMISSION EFFICIENCY

Efficiency or loss of power is associated with every joint that is, edge and the quantum of power transmission as well as loss depends upon the type of the joint. Two gear trains with the same vertex assortment can behave differently performance wise due to difference in edge values. It will be evident from the rules of graph formation for a gear train that the number of edges and their type will remain the same for all the gear trains with the same number of elements and DOF. For example, in the Figure 5.3(b) and Figure 5.4 the edges are numbered (1),(2),..., etc, and for the Figure 5.3(b) their edge values are respectively 4,6,4,4 and 4 while for the Figure 5.4 they are respectively 5,6,5,3 and 3.

Transfer efficiency or the loss of energy at a joint depends on the number of other joints it is directly connected to. In fact, the edge value is related to number of design parameters involved by equation (8) and hence to the efficiency of power transmission. Thus, the distribution of edge values influences the transmission efficiency while the edge value of a graph is indicative of the power loss. A numerical measure for comparing the transmission efficiency of distinct gear train is also proposed.
5.6.3 POWER CIRCULATION

The power circulation among various gear pairs depends upon their fundamental circuits and their adjacency. Fundamental circuit of each gear pair can easily be identified from the graph following the explanation given earlier. For example for the gear train and its graph shown in Figure 5.3 (a) and Figure 5.3(b), it is easy to see that there are two fundamental circuits, that is, vertices 2-3 and vertices 3-4 corresponding to both gear pairs, vertex-1 is the transfer vertex. Fundamental circuit $F_{23}$ consists of vertices 1,2 and 3 while the circuit $F_{34}$ consists of vertices 1,3 and 4. A matrix called the circuit matrix 'C' can be formulated to indicate the design parameters involved in each circuit.

The value of an element $C_{ij}$ is considered to be the sum of the connectives of all vertices common to the fundamental circuits $i$ and $j$, and $C_{ii}$ is the sum of the connectivities of all the vertices that take part in the fundamental circuit $i$.

For the graph shown in Figure 5.3(b) one can write

\[
C = \begin{pmatrix}
11 & 8 \\
8 & 11
\end{pmatrix}
\]
The elements of both the rows of matrix $C$ being identical, identical power flow can be expected in both the circuits.

The $C$-matrix need not be symmetric in all cases and in such cases a numerical measure is necessary to have an idea about the power flow in various circuits. It must be noted that the numerical measure proposed in a later section does not give the actual values of power but only an idea regarding power sharing among the circuits.
5.7 NUMERICAL MEASURES

In the earlier section, factors influencing (i) power transmission, (ii) transmission efficiency, and (iii) power circulation among various fundamental circuits have been dealt with. In this section, formulae to estimate the capacity, efficiency and circulation are developed.

5.7.1 CAPACITY

It has been stated in the earlier section that the connectivity of a vertex (element) is related to a number of design parameters. It is also proved through the example of the train that is, Figure 1, that an element with greater connectivity can transmit more power. Obviously, it also means that an element with lesser connectivity will transmit less power. Since, the gear train is a combination of elements with different connectivities, its capacity to transmit maximum power is limited by element (vertex) with least connectivity. It is, therefore, evident that a train can transmit maximum power when all its elements are of equal connectivity, which in reality may not be possible. Hence, a relative estimate is necessary.

Let the total connectivity of the gear train (graph) is $D$ and let $d_i$ be the connectivity of the vertex $i$. Then the ratio of the design parameters of element $i$ to the total design parameters of the gear train will be $d_i/D$. 
Hence,

\[ n \sum_{i=1}^{n} \left( \frac{d_i}{D} \right) = l \] \hspace{1cm} (9)

Where \( n \) is the total number of elements (vertices)

Equation (9) holds good for all the gear trains and hence cannot be used as a measure to compare different gear trains. Any mathematical expression that can be considered for estimating the power transmitting capacity of a gear train must satisfy the following requirements.

- The quantum element of gear transmitted is maximum when all elements have equal Design parameters, that is \( d_i = d_j = \ldots = d_n \).
- No single element of the gear train can transmit the entire power.
- The quantum of power transmitted by a fixed link is zero.

A mathematical equation that satisfies the above requirements is expressed as

\[ P = \sum P_i \log P_i \] \hspace{1cm} (10)

It is chosen in such a way that \( 1 \log_{10} 1 = 0 \), and \( 0 \log_{10} 0 = 0 \) in order to satisfy the above-mentioned requirements: especially the last two conditions. Then, the power transmitting capacity \( P \) of the gear train is expressed as

\[ P = \sum P_i \log P_i \] \hspace{1cm} (11)
It may be noted that equation (11) satisfies all the three requirements. Equation (10) has strong resemblance to the expressions for entropy commonly used in information theory while equation (9) resembles the condition that sum of the probabilities of various events is equal to one. It is, therefore, evident that by proper interpretation the concepts developed in Information Theory can also be applied to gear trains. The above concept is illustrated by the following example problem.
Fig. 5.5 (a)

Fig. 5.5 (b)
5.8 EXAMPLE

Consider the two gear trains and their graphs shown in Figure 5.5(a) and Figure 5.5(b). Gear trains are compared for their ability to transmit power. For the gear trains, the connectivity of the vertices (elements) can be obtained directly from their graphs.

For the train in Figure 5.5(a), the connectivity values for the vertices 1, 2, ..., 6 are respectively 7, 2, 5, 2, 5, and 5. Therefore, the total connectivity is 26. Using equations (9), (10), and (11), one can write

\[ P = - \left[ \frac{2}{26} \log \left( \frac{2}{26} \right) + \frac{3}{26} \log \left( \frac{5}{26} \right) + \frac{7}{26} \log \left( \frac{7}{26} \right) \right] = 0.7379 \]

For the gear shown in Figure 5.5(b), the connectivities of the vertices 1, 2, ..., 6 are respectively 6, 2, 6, 2, 5, and 5. The sum of the connectivities is 26. Using equations (9), (10), and (11) one gets

\[ P = -2 \left[ \frac{2}{26} \log \left( \frac{2}{26} \right) + \frac{5}{26} \log \left( \frac{5}{26} \right) + \frac{6}{26} \log \left( \frac{6}{26} \right) \right] = 0.7406 \]

The comparison of P-values reveals that the train shown in Figure 5.5(b) has greater capacity to transmit power.

5.8.1 TRANSMISSION EFFICIENCY

Using equation (8), the numerical values all edges of a graph can be determined. Let \( f_i \) be the edge value with \( i^{th} \) edge. Then, the edge value of the graph or train can be represented as
\[ J = \sum_{i=1}^{k} j_i \quad \text{(12)} \]

Where \( k \) is the total number of edges in the graph.

The ratio of edge-value of the edge \( i \) to the edge value of the graph is \((j_i / J)\)

And
\[ \sum_{i=1}^{k} (j_i / J) = 1 \quad \text{(13)} \]

Equation (12) holds good for all the gear trains and hence cannot be used as a measure to compare different gear trains. Any mathematical expression that can be considered for estimating the transmission efficiency must satisfy the following requirements.

(i) Energy transmitted is maximum when all the edges/joints are of equal value.

(ii) No single joint of a gear train will transmit the entire edge value.

(iii) Fixed elements, if any, in a joint will not contribute to the edge value.

A mathematical expression that satisfies the requirements, especially the second and third is as mentioned below:
\[ (-J_i / J) \log_{10}(J_i / J) \quad \text{(14)} \]

This expression has all the traits of the earlier expression (10).

Therefore, the energy transfer \( E \) through all the joints of a gear train can be expressed as
\[ E = \sum_{i=1}^{k} (-J/I) \log_{10} (J/I) \]  

This expression (15) assumes a maximum or ideal value \( E_m \) when \( j_i = j_2 = \ldots = j_k \), which satisfies the first of the above three requirements. Equations (13) and (15) are analogous, once again, to the equation of entropy in the Information theory.

The transmission efficiency (\( T_e \)) is defined as the ratio of the actual energy (\( E \)) to the maximum energy (\( E_m \)). The transmission efficiency (\( T_e \)) of a gear train can be expressed as

\[ T_e = \frac{E}{E_m} \]  

For the illustration, the two gear trains along with their graphs shown in Figure 5.5(a) and Figure 5.5(b) are considered.

For the graph in Figure 5.5(a), the edge values are 8, 8, 6, 6, 5, 5, 7, 7, 10. The total edge value is 62.

Then, \( E_m = -9 \left[ \frac{1}{9} \log_{10} \left( \frac{1}{9} \right) \right] = 0.95424 \)

And \( E = -\sum 2\left[ \frac{5}{62} \log \left( \frac{5}{62} \right) \right] + 2\left[ \frac{6}{62} \log \left( \frac{6}{62} \right) \right] + 2\left[ \frac{7}{62} \log \left( \frac{7}{62} \right) \right] + 2\left[ \frac{8}{62} \log \left( \frac{8}{62} \right) \right] + 2\left[ \frac{10}{62} \log \left( \frac{10}{62} \right) \right] = 0.9438 \)

Then using equation (16), one gets efficiency as 0.9891.

For the gear train shown in Figure 5.5(b), the edge values are 7, 7, 7, 5, 5, 6, 6, and 10. The total edge value is 60. Then,
\[ E = - \sum 2 \left[ \frac{5}{60} \log \left( \frac{5}{60} \right) \right] + 2 \left[ \frac{6}{60} \log \left( \frac{6}{60} \right) \right] + 4 \left[ \frac{7}{60} \log \left( \frac{7}{60} \right) \right] + \left[ \frac{10}{60} \log \left( \frac{10}{60} \right) \right] = 0.9449 \]

Then using equation (16), the transmission efficiency becomes 0.9903

The comparison of the efficiencies shows that the gear train shown in figure 5.5(b) is better.

### 5.8.2 POWER CIRCULATION

It has been explained earlier that every gear pair has a fundamental circuit and an idea regarding power distribution among different gear pairs can be obtained by knowing the number of design parameters, that is, vertices, connectivity and their adjacency. In order to know this, circuit matrix \( C \) is suggested.

The sum of the elements of each row in the \( C \) matrix is representative of the design parameters influencing power distribution in that fundamental circuit. Sum of all elements in the \( C \) matrix correspond to the design parameters representing all the fundamental circuits in the gear train. Let the sum of the elements in \( i \)th row or its circuit value be \( C_i \) and the sum of all the elements in the matrix or circuit value of the train be \( C_r \).

Then the ratio of circuit value of \( i \)th circuit-to-circuit value of the train is \( \frac{C_i}{C_r} \).
Therefore, \[ \sum_{i=1}^{l} \left( \frac{C_i}{C_1} \right) \] (17)

Where \( l \) is the number of fundamental circuits or gear pairs in the train. Equation (17) holds good for all the gear trains and hence cannot be used to compare different trains from the viewpoint of power distribution. Any mathematical expression proposed for the purpose. That is, to know how uniformly the power is circulated among various circuits must satisfy the following requirements.

- The power is circulated most uniformly when all the circuit values are the same.
- The entire will not be circulated through one circuit only making other circuits ineffective.
- Fixed elements if any in a circuit, will not take part in power circulation and hence they should not be included in deciding the circuit value.

One of the expression that satisfy the first two requirements mentioned above is

\[-(C_i/C_1) \log \left( \frac{C_i}{C_1} \right)\]

Since \( 1 \log 1 = 0 \) and \( 0 \log 0 = 0 \)

Also the measure of uniformity \( U \) is expressed by

\[ U = \sum_{i=1}^{l} \left( \frac{C_i}{C_1} \right) \log \left( \frac{C_i}{C_1} \right) \] (18)
It is easy note that the uniformity will be maximum when

\[ C_1 = C_2 = \ldots = C_t, \] satisfying the first of requirements above.

For illustration, consider the two gear trains shown in Figures 5.5(a) and 5.5(b). The constituent vertices of the fundamental Circuits are given in Table 5.1.
<table>
<thead>
<tr>
<th>Gear Train</th>
<th>Gear Pair</th>
<th>Fundamental Circuit Number</th>
<th>Vertices in the fundamental circuit</th>
</tr>
</thead>
<tbody>
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<td>Figure 5.5(a)</td>
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<tr>
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<td>3-5</td>
<td>2</td>
<td>1-2-3-5</td>
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<tr>
<td></td>
<td>3-6</td>
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<td>1-3-4-6</td>
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<tr>
<td></td>
<td>1-6</td>
<td>4</td>
<td>1-4-6</td>
</tr>
<tr>
<td>Figure 5.5(b)</td>
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<td>1-2-3-5</td>
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<td></td>
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<td>4</td>
<td>1-4-6</td>
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The corresponding circuit matrix for Figure 5.5(a) is given below

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<td>7</td>
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<td>12</td>
<td>19</td>
<td>14</td>
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<tr>
<td>7</td>
<td>7</td>
<td>14</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

Hence, the circuit values are 42, 52, 52 and 42. Circuit value of graph is 188. Using Equation (18) for the train shown in Figure 5.5(a) the uniformity can be measured as
\[ U = -2 \left( \frac{52}{188} \right) \log \left( \frac{52}{188} \right) - 2 \left( \frac{42}{188} \right) \log \left( \frac{42}{188} \right) = 0.5995 \]

The circuit matrix for Figure 5.5(b) can be written as

\[
C = \begin{bmatrix}
19 & 13 & 12 & 6 \\
13 & 13 & 6 & 0 \\
12 & 6 & 19 & 13 \\
6 & 0 & 13 & 13
\end{bmatrix}
\]

Hence, the circuit values are 50, 32, 50, and 32, and the circuit value of the graph is 164.

For the Figure 5.5(b), the uniformity can be measured as

\[ U = -2 \left( \frac{50}{164} \right) \log \left( \frac{50}{164} \right) - 2 \left( \frac{32}{164} \right) \log \left( \frac{32}{164} \right) = 0.5915109 \]

Comparison of \( U \) values reveals that the power circulation is more uniform in the gear train shown in Figure 5.5(a). The numerical value \( \frac{C_i}{C_j} \) \( \log \left( \frac{C_i}{C_j} \right) \) obtained for a given circuit can be as a measure of power circulation in that fundamental circuit.
5.9 CONCLUSIONS

The following inferences are drawn based on the study discussed above.

- Every gear train can be represented by a graph.
- The constitutes of the graphs, that is, vertices, edges and the fundamental circuits can be related to the design parameters.
- Each of the graph constituents will reveal the anticipated behaviour of the train. Vertices and their connectivity will indicate the capacity for power transmission.
- Edges and their values reveal the efficiency of transmission. The above facts are analogous to two engines developing different power with different efficiencies.
- Fundamental circuits of the graphs and their adjacency will reveal the uniformity of power circulation among various circuits.
- Suggested numerical measures have strong resemblance to properties and the equations commonly used in Information Theory. Hence, there is every possibility that various other properties developed in Information Theory may be used with proper interpretation for the study of linkage, gear trains, etc.
- These example problems figures 5.5(a) & 5.5(b) are in fact pseudo isomorphic and are deliberately chosen for illustration of their
characteristics which are different. The better one (for example Figure 5.5(b)) can be retained for the further generations of chains.