Chapter 1

Introduction

Noticing that the Newton's law of gravitation is irreconcilable with the special theory of relativity, Einstein postulated a new theory of space, time, and gravitation which is known as the general theory of relativity (GTR). Unlike the case of the special theory of relativity, there have been some objections by few physicists as well as Einstein himself against the new theory of gravitation. Nonetheless, it has witnessed better experimental evidences as compared to the Newtonian gravity. Therefore, one has a predilection for the Einstein's theory of gravitation over the Newtonian theory. However, it remained almost in quiescent stage for a long time, partly due to the reason that it presents an entirely new standpoint which was difficult to understand, and partly because the high energy compact objects were not discovered and the Newtonian theory was sufficient to deal with the physical systems in hand. However, the discovery of the quasars and compact X-ray sources encouraged many researchers to study
general relativity to investigate about various esoteric compact objects. Another striking reason for growing interest in this subject is that a deeper understanding of the classical theory of gravitation could be helpful in the way of achieving a viable theory of quantum gravity. Many researchers have put painstaking efforts for investigating into the various aspects of this subject.

The dynamical features of any field is best understood through the trajectories of the test particles in the representative spacetime. In general theory of relativity, as the gravitational field is represented by the spacetime curvature, the spacetime structure itself dictates the orbit of the test particles which when not subjected to any other interaction, move along geodesics. The trajectories of the test particles in various spacetimes have been studied by many authors which are cited in a paper by Sharp (1979). However, in many of the physical situations there are other fields and interactions, amongst which the electromagnetic field is prevalent one. Charged particles in curved spacetime in the presence of the electromagnetic fields do not follow geodesics and similarly the test particles with spin also deviate from geodesics as shown by Papapetrou (1951). Further Corinaldesi and Papapetrou (1951) discussed the equations of motion for spinning test particles satisfying the condition $S^{0} = 0$ (where $S^{ik}$ is the spin tensor). One of the important features related to the dynamics of the spin is its precession induced by the interaction with the field. Schiff (1960) studied the geodetic spin precession of a test particle in a free fall about a mas-
sive sphere and proposed the result for a plausible test of general relativity. Unfortunately, due to some technical problems, the gyroscope experiment could not be accomplished as yet. However, the study of the spin precession of charged particles in curved spacetime background endowed with electromagnetic fields has not been paid proper attention. A charged spinning particle has its magnetic dipole moment \( \delta \), proportional to its spin angular momentum \( S \), through the relation \( \delta = g e S/2 m_e \), where \( g \) the Lande factor has values for example, for electron and proton, \( g_e = 2.0023 \) and \( g_p = 5.59 \), respectively. In the presence of a magnetic field, the spin angular momentum vector suffers a precession due to the torque \( \delta \times B \) acting on it. Anderson (1967) gave a relativistic generalization for a torque acting on a spinning charged particle due to the electromagnetic field. Prasanna and Kumar (1973), using the Anderson's generalization of the torque and Papapetrou's equations of spin and orbit with a Lorentz force term, studied the spin precession of charged particles in Melvin's magnetic universe.

We (Prasanna and Virbhadra 1989; Virbhadra and Prasanna 1989, 1990, 1991) expressed these generalized equations in a rather convenient form and have investigated the same in the following physical situations: (a) a magnetic field dipolar at infinity superposed on Schwarzschild background (Ginzburg-Ozernoi solution), (b) a uniform magnetic field superposed on Schwarzschild background (Wald solution), (c) a Reissner-Nordström source, and (d) a Schwarzschild object embedded in a uniform magnetic field (Ernst solution). Unlike the case of the Wald solution which
is the solution of the Maxwell equations on Schwarzschild background, the Ernst solution being the solution of the Einstein-Maxwell equations incorporates the effects of curvature due to the magnetic field. We confined our attention to the particles in circular motion on the equatorial plane and found the spin precession frequencies for the aforesaid cases. The flat space limit to the expressions obtained for the spin precession frequency yields interesting results. We have found that cases (a) and (b) yield special relativistic contribution to the well known Larmor frequency, whereas case (c) puts a bound \( 0 > g, g > 2 \) on the Lande g-factor. Apart from the above there are other interesting outcomes of these investigations. Though the calculations have been accomplished in a fully covariant prescription, the purely geodetic terms have not appeared in the result. However, when we (Virbhadra and Prasanna 1991) have introduced local Lorentz frames the purely geodetic terms have appeared explicitly along with the other terms.

Another subject which drew our attention is the energy and angular momentum in curved spacetimes. The energy-momentum localization in general relativity is a problematic issue and it has been a subject of extensive research since the outset of the general relativity. While an adequate localization of energy and momentum would have immense benefits, the status is that one does not have that so far. Since Einstein's original pseudotensor (Møller 1958), a fairly large number of expressions for energy and momentum in general relativistic systems have been suggested by many authors, e.g., Tolman (1930), Landau and Lifshitz (1985), Papapetrou (1948),
Gupta (1954), Möller (1958), Goldberg (1958), Bergmann (1958), Dirac (1959), Komar (1959, 1962, 1963), Arnowitt et al. (1961), Bondi et al. (1962), Hawking (1968), Weinberg (1972), Witten (1981), Penrose (1982), Lynden-Bell and Katz (1985), Nahmad-Achar and Schutz (1987a, 1987b), Kulkarni et al. (1988), Bartnik (1989), Katz and Ori (1990) etc. There are mutually opposing viewpoints that authors share regarding the physical importance of the energy-momentum pseudotensors as well as a possibility of successful localization of energy and momentum in curved spacetime. However, the total energy and momentum in asymptotically flat spacetimes have an unambiguous importance. Weinberg (1972), using his own prescription for energy and angular momentum in asymptotically flat spacetimes, found the total energy and angular momentum associated with the Kerr spacetime to be $M$ and $Ma$ respectively ($M$ stands for the mass parameter whereas $a$ stands for the rotation parameter in Kerr metric). Palmer (1980) discussed the significance of Einstein's energy-momentum pseudotensor in detail. It is well known that the pseudotensors of Einstein, Tolman, and Landau and Lifshitz (LL) can yield sensible result only if the calculations are carried out in quasi-Cartesian coordinates (that in which increase in spatial distance converges the components of the metric tensor to their values of special relativity) which are usually very lengthy to accomplish. Rosen (1956) evaluated energy and momentum of cylindrical gravitational waves in Cylindrical polar coordinates in LL prescription and found that the waves in empty space did not carry energy and momentum. Later he (Rosen 1958) repeated the calculation in quasi-Cartesian coordinates and got the energy
and momentum to be finite and reasonable. Møller (1958) constructed a new energy-momentum complex and claimed that one is not anymore bound to the use of the quasi-Cartesian coordinates. Only three years after this work, Møller (1961) realized that the total energy-momentum vector of a closed physical system is not a Lorentz four-vector in his formulation. However, the energy density component of the Møller’s complex transforms like a scalar density under purely spatial transformations.

Cohen and de Felice (1984) calculated the Komar energy in Kerr-Newman spacetime and interpreted that to be effective gravitational mass that a neutral test particle present in the field of the Kerr-Newman object experiences through the gravitational interaction. However, switching off the charge parameter gives no energy to the exterior of the Kerr black hole. Looking into the result of Cohen and de Felice, Kulkarni et al (1988) argued that a modification of the Komar integral was warranted since that did not yield the repulsive effect arising from the rotation. They proposed a new definition of the effective gravitational mass of the Kerr black hole that incorporated the contribution due to the rotation.

We believe that so long one does not have a successful localization of energy and momentum in general relativistic system, it is desirable that the relative merits and demerits of various definitions to give energy as well as angular momentum be investigated. The energy and momentum pseudotensors of Einstein, Tolman, Landau and Lifshitz (LL), and Møller are largely discussed in the literature and we (Virbhadra 1990a, 1990b, 1990c,
1991a, 1991b, 1991c) investigated these complexes for the Kerr-Newman as well as the Vaidya radiating spacetimes. We have evaluated all the components of the energy-momentum pseudotensors of Einstein, Tolman, Landau and Lifshitz, and Möller for the Kerr-Newman (upto the third power of the rotation parameter) as well as the Vaidya radiating spacetimes. We (Virbhadra 1990a, 1990c) have found that the pseudotensors of Einstein, Tolman, and LL give exactly same energy density in Kerr-Newman spacetime, whereas that of Möller gives twice the value obtained using these definitions. All these four pseudotensors yield no energy in the Kerr spacetime and the entire energy in the Kerr-Newman spacetime is due to the electromagnetic field present there. The pseudotensor of LL, being symmetric in indices, can be used to evaluate angular momentum in asymptotically flat spacetimes. We (Virbhadra 1990c) calculated the same for the K-N spacetime and got sensible result. Unlike the case of the K-N field, we (Virbhadra 1991b) have found that all these comlexes give the same energy density in the Vaidya spacetime. The pseudotensors of Einstein and Tolman gave same result (for all of their components) for the K-N as well as the Vaidya spacetimes. Despite their non-tensor character, the aforementioned pseudotensors are found to be traceless for both spacetimes.

Recently Cooperstock and Richardson (1991) have extended our result for energy in Kerr-Newman field upto the seventh order of the rotation parameter and have found the same relationship that the prescription of Einstein, Tolman, and Landau and Lifshitz give same result whereas that of
Møller yields twice the value. They have also pointed out that the Komar energy for the R-N metric calculated by Cohen and de Felice does not give the correct flat space limit whereas that of Einstein, Tolman, and Landau and Lifshitz do give.

The energy-momentum complexes discussed above give meaningful result if the calculations are carried out in quasi-Cartesian coordinates (Kerr-Schild Cartesian coordinates satisfy the condition of quasi-Cartesian coordinates). An asymptotically flat metric can always be expressed in quasi-Cartesian coordinates though it may not be in Kerr-Schild form. Using the pseudotensors of Einstein, Landau and Lifshitz, and Møller (Vaidya, 1952, calculated the same in Tolman's prescription), we (Virbhadra 1991a) have calculated energy in the Reissner-Nordström metric (in quasi-Cartesian coordinates though the line-element being not in the Kerr-Schild form) and have found that the pseudotensors of Einstein, Tolman, and Møller give respectively same result as we obtained for the same spacetime in Kerr-Schild Cartesian coordinates. However, the pseudotensor of Landau and Lifshitz does not give a consistent result.

The thesis is organised as follows: the chapters two and three contain study of spin dynamics, and the chapters four to six are devoted to the study of energy, momentum and angular momentum in curved spacetimes. We use the geometrized units ($G = c = 1$) and follow the convention that Latin and Greek indices run from 0 to 3 and 1 to 3 respectively. We adopt the Einstein summation convention.