Abstract

The concept of a fuzzy metric space is more suitable in studying the uncertainty due to fuzziness. This space was first introduced by Kramosil and Michalek [59] by generalizing the concept of probabilistic metric space to the fuzzy situation. Fuzzy metric space was defined in various ways by many, especially Erceg [26], Deng [22], Kaleva et.al. [54] and George and Veeramani [35].

Following this definition, a lot of research have been done on the existence of fixed points for the mappings under different conditions (see [14], [41], [58], [67], [68], [69], [90], [101] ). Fuzzy metric space has numerous generalizations such as fuzzy quasi metric space, fuzzy 2-metric space, fuzzy 3-metric space, fuzzy generalized metric space (Q-fuzzy metric space) etc. The interesting literature in this area are [2], [22], [40], [41], [55], [69], [79] and [98].

By the definition of fuzzy metric space, one can easily verify that $M(x, y, t)$ denotes the degree of nearness of the points $x$ and $y$ with respect to $t$, where $M$ is a fuzzy set in $X \times X \times [0, \infty)$. The fuzzy metric space has variety of applications. For example it has been applied to color image filtering improving some filters when replacing classical metrics. The main advantage of fuzzy met-
ric is that the values given by fuzzy metrics are in the interval $(0, 1]$ regardless the nature of the distance concept being measured.

In this thesis, we obtained a few fixed point theorems on different spaces and different mappings. The thesis is organized into four chapters. The first chapter is introductory in nature, it contains all the definitions and theorems that are necessary in the succeeding chapters.

The concept of fixed point theorems for compatible maps are obtained in the second chapter, which are the extensions of the theorem due to Bijendra Singh et.al. In literature we have noticed that most of the fixed points are obtained using the continuous t-norm $a \ast b = \min\{a, b\}$ or $a \ast b = ab$. But, we have considered $\ast$ as any arbitrary t-norm. We mainly obtained fixed points for four self maps using contractive conditions. We defined $W$-compatible maps of type (P) and $W^*$-compatible maps of type (P) in fuzzy metric space and studied common fixed points. Also, we illustrate our results with suitable examples.

The prime objective of the third Chapter is to investigate fixed point theorems in fuzzy 2-metric spaces. Sushil Sharma obtained fixed point theorem for compatible and continuous maps using contractive condition. Urmila Mishra et.al. obtained fixed point theorem for reciprocal continuous maps in fuzzy metric space. The concept of reciprocal continuity is introduced by us in fuzzy 2-metric space and we extend the result due to Urmila Mishra et.al.in
fuzzy 2-metric space. Moreover, we extend the results of Chapter 2 in the setting of fuzzy 2-metric space by defining W-compatible maps of type (P). Also, a few examples are illustrated here.

The fourth Chapter is devoted to the study of fixed point theorems in Q-fuzzy metric space. Here also we define reciprocal continuity of maps and W-compatible maps of type (P) and obtained a few theorems for these maps under varied conditions. Common fixed point theorems are also studied in this frame work. Moreover generalized fuzzy version of Banach Contraction Principle is also discussed here.