CHAPTER IV

LOT-SIZE MODEL FOR EXPONENTIALLY DETERIORATING INVENTORY WITH FINITE PRODUCTION RATE

For the models developed in the last two Chapters it was assumed that replenishment rate is infinite so that replenishments in any desired lot-size was permissible. We now consider a model for deteriorating items in which the production rate is finite and hence only a finite number of units can be replenished per unit of time. The replenishment cost occurs only at the commencement of the production process. We consider only exponential distribution for the time to deterioration of an item, because for the case of Weibull distribution for the time to deterioration, no explicit solution of the model is possible. First a model without shortages is developed and then it is extended to situations where shortages are permissible.

It is to be noted that a similar model without shortages has already been developed by Misra [11]; but we have developed the model in a more elegant form and also the model is extended to allow shortages.
4.1: **The mathematical model without shortages:**

The mathematical model is developed with the following assumptions:

(i) The demand rate of \( R \) units per unit of time is known and constant.

(ii) The system has a uniform finite replenishment rate \( p \); the replenishment size during a cycle is constant; the lot-size is of \( q \) units per replenishment. Lead-time is zero.

(iii) The system is in steady state i.e. the production rate is greater than the demand rate.

(iv) Shortages are not allowed.

(v) There is no repair or replacement of the deteriorated inventory during a given cycle.

(vi) The unit cost \( C \), the inventory holding cost \( C_1 \) per unit per time unit and the replenishment cost \( C_3 \) per production run are known and constant during the period under considerations.

(vii) The time to deterioration of an item follows exponential distribution with p.d.f.

\[
4.4.1 \quad g(t) = \varnothing \exp(-\varnothing t), \quad t \geq 0, \quad \varnothing > 0
\]

\[
= 0, \quad \text{otherwise}
\]

where

\[
e^{-1} = E(t) = \int_{0}^{\infty} t \, g(t) \, dt
\]
is the mean life of a unit. The c.d.f. of (4.1.1) is

\[ G(t) = 1 - \exp(-\theta t), \quad t \geq 0 \]

so that, following Cox [5], the age specific failure-rate function of an item is

\[ \Phi(t) = \frac{g(t)}{1-G(t)} = \theta \quad \text{(constant)} \]

The inventory fluctuations of the system can be described graphically as follows:

Let \( T \) denote the total period of a cycle then, in this case, the inventory cycle consists of two phases. During the Phase I period \((0, t_1)\), units are produced at a constant rate of \( p \) units per time unit and demands occur at a constant rate of \( R \) units per time unit, leaving a balance of \((p-R)\) units per unit of time to enter the inventory system. During the Phase II period \((t_1, T)\) there is no production and demands occurring at a
constant rate $R$ are met from the inventory accumulated during phase I period. Let $t_2 = T - t_1$ denote the duration of phase II period. During both the phases a constant fraction of inventory get deteriorated because of the deteriorating nature of the items, and hence in determining the optimum schedule this factor should also be taken into considerations.

Let $Q(t)$ denote the inventory level of the system at time $t$ $(0 \leq t \leq T)$. The rate of change of inventory level $dQ(t)/dt$ during a small interval of time $dt$ is a function of the demand rate $R$, production rate $p$, rate of deterioration $\theta$ and the instantaneous inventory level $Q(t)$. Thus the differential equation governing the system is

\begin{equation}
\frac{dQ(t)}{dt} + \theta Q(t) = p - R, \quad 0 \leq t \leq t_1
\end{equation}

\begin{equation}
= -R, \quad t_1 < t \leq T.
\end{equation}

Solution of the first order linear differential eqn. (4.1.4) is

\begin{equation}
Q(t) = \left[ K_1 + \frac{(p-R)}{\theta} \{ \exp(\theta t) - 1 \} \right] \exp(-\theta t), \quad 0 \leq t \leq t_1
\end{equation}

\begin{equation}
= \left[ K_2 - \frac{R}{\theta} \{ \exp(\theta(t-t_1)) - 1 \} \right] \exp\{-\theta(t-t_1)\}
\end{equation}

, $t_1 < t \leq T$

where $K_1$ and $K_2$ are the constants of integration. Using the boundary condition $Q(0) = 0 = Q(T)$, we have from (4.1.5)
(4.1.6) \[ K_1 = 0 \text{ and } K_2 = \frac{R}{\theta} \left[ \exp\{\theta(T-t_1)\} - 1 \right] \]

Substituting for \( K_1 \) and \( K_2 \) from (4.1.6) in (4.1.5) we have

(4.1.7) \[ Q(t) = \frac{(p-R)}{\theta} \left[ 1 - \exp(-\theta t) \right], \quad 0 \leq t \leq t_1 \]

\[ = \frac{R}{\theta} \left[ \exp\{\theta(T-t)\} - 1 \right], \quad t_1 \leq t \leq T. \]

Since the inventory level at the termination of phase I is equal to that at the commencement of phase II, we have from (4.1.7)

\[ Q(t_1) = \frac{(p-R)}{\theta} \left[ 1 - \exp(-\theta t_1) \right] = \frac{R}{\theta} \left[ \exp\{\theta(T-t_1)\} - 1 \right] \]

which in turn gives

(4.1.8) \[ t_1 = \frac{1}{\theta} \log\left[ 1 + \frac{R}{p} \{\exp(\theta T) - 1\} \right] \]

\[ = \frac{1}{\theta} \log \frac{Y}{p} \]

where

(4.1.9) \[ Y = p + R \{\exp(\theta T) - 1\}. \]

Now the total demand during the cycle of \( T \) time units is \( RT \) and the lot-size for a cycle is clearly
(4.1.10) \[ q = pt_1 = \frac{P}{\theta} \log \left[ 1 + \frac{R}{\theta} \left\{ \exp(\theta T) - 1 \right\} \right] \]

\[ = \frac{P}{\theta} \log \frac{Y}{p} \]

so that the number of units, \( D(T) \), that deteriorate during the cycle time \( T \) is

(4.1.11) \[ D(T) = q - RT = \frac{P}{\theta} \log \frac{Y}{p} - RT \]

In determining the average number of units in inventory per unit of time, \( I_1(T) \), during a cycle we once again assume that \( Q(t) \) is approximately linear in \( t \) over the cycle period \((0, T)\); so that \( I_1(T) \) is given by

(4.1.12) \[ I_1(T) = \frac{1}{2} Q(t_1) = \frac{R(p-R)}{2\theta} \left[ \frac{\exp(\theta T) - 1}{p+R} \right] \]

\[ = \frac{(p-R)(Y-p)}{2\theta Y} \]

The number of replenishment per unit of time is \( 1/T \) and hence, using (4.1.11) and (4.1.12), the average total cost of the system per unit of time is

(4.4.13) \[ K(T) = \frac{C}{T} D(T) + C_1 I_1(T) + \frac{C_3}{T} \]

\[ = \frac{CP}{\theta T} \log \frac{Y}{p} - CR + \frac{C_1(p-R)(Y-p)}{2\theta Y} + \frac{C_3}{T} \]

where \( Y \) is given by (4.1.9).
For optimum value of $T$, differentiating $K(T)$ given by (4.1.13) w.r.t. $T$ and equating the resulting expression to zero, we have

\[
(4.1.14) \quad \frac{C_p R}{Y T} \exp(\theta T) - \frac{C_p}{\theta T^2} \log \frac{Y}{P} + \frac{C_1 p R}{2 Y^2} (p - R) \exp(\theta T) - \frac{C_3}{T^2} = 0
\]

Eqn. (4.4.14) when solved for $T$, will yield the optimum cycle time $T = T_0$ (say). Substituting $T = T_0$, (4.1.10) will give the optimum lot-size $q = q_0$ for a cycle and (4.1.13) will give the minimum average total cost $K(T_0)$ of the system per unit of time.

When $p = \infty$, the replenishment rate is infinite. Noting that

\[
\lim_{p \to \infty} p \log \left[ 1 + \frac{R}{p} (e^{\theta T} - 1) \right] = R \{ \exp(\theta T) - 1 \}
\]

and

\[
\lim_{p \to \infty} \frac{(p - R)(Y - p)}{Y} = R \{ \exp(\theta T) - 1 \}
\]

we find that eqns. (4.1.13) and (4.1.14) reduce to eqns. (2.2.12) and (2.2.13) of the EOQ model for exponentially decaying inventory.

**First order approximation:**

We reduce the complexity of (4.1.13) and (4.1.14) by
using series form of the exponentials and logarithms and ignoring
the terms with second and higher order powers of $e$ under the
assumption that $e << T$. Noting that to the first order approxi-
mation in $e$ we have

$$\frac{1}{e} \log \frac{Y}{p} = \frac{RT}{p} (1 + \frac{1}{2} eT) - \frac{2eT^2}{2p^2}$$

$$\frac{Y-p}{eY} = \frac{RT}{p} (1 + \frac{1}{2} eT - \frac{ReT}{p})$$

$$\frac{1}{Y} \exp(eT) = \frac{1}{p} (1 + eT - \frac{ReT}{p})$$

and

$$\frac{\exp(eT)}{Y^2} = \frac{1}{p} (1 + eT - \frac{2ReT}{p})$$

Then, to the first order approximation in $e$, the cost function (4.1.13) becomes

$$K(T) = \frac{C_2 eRT}{2} \left(1 - \frac{R}{p}\right) + \frac{C_1 eRT}{2} \left(1 - \frac{R}{p}\right) \left(1 + \frac{1}{2} eT - \frac{ReT}{p}\right) + \frac{C_3}{T}$$

and eqn. (4.1.14) now becomes

$$\frac{C_2 eR}{2} \left(1 - \frac{R}{p}\right) + \frac{C_1 eR}{2} \left(1 - \frac{R}{p}\right) \left(1 + eT - \frac{2ReT}{p}\right) - \frac{C_3}{T^2} = 0$$

Solving (4.1.16) for $T$ we get the optimum cycle time $T = T_o$ to first order approximation. Substituting $T = T_o$, 

(4.1.15) will yield the minimum total average cost per unit of time of the system.

When $\theta = 0$ there is no deterioration and the cost function (4.1.15) becomes

\[(4.1.17) \quad K(T) = \frac{C_1RT}{2} (1 - \frac{R}{p}) + \frac{C_3}{T}\]

and in this case eqn. (4.1.16) gives

\[(4.1.18) \quad T^{(o)} = \left[ \frac{2C_3}{C_1R(1 - R/p)} \right]^{1/2}\]

which is a standard formula for obtaining the optimum cycle time $T = T^{(o)}$ for a finite production rate lot-size model for non-deteriorating items.

When $p = \infty$, the replenishment rate is infinite and eqns. (4.1.15) and (4.1.16) reduce respectively to eqns. (2.2.15) and (2.2.16) of the EOQ model.

To obtain the numerical solution:

Eqn. (4.1.14) can be solved by using the Newton-Raphson iterative procedure described in Section 3.2 by taking

\[h(T) = C_p R T \exp(\theta T) - \frac{C_p Y^2 \log Y}{p} + \frac{1}{2} C_1 p (p-R) T^2 \exp(\theta T) - C_3 Y^2,\]

\[h'(T) = C_p \left[ R (p-R) \exp(\theta T) + Y \exp(\theta T) - 2 Y \log \frac{Y}{p} \right] \exp(\theta T) - 2 C_3 Y R \exp(\theta T) + \frac{1}{2} C_1 p (p-R) T (2+\theta T) \exp(\theta T)\]
and for the initial value to the iterative procedure we use the optimum value \( T = T^{(0)} \) given by (4.1.18).

**A numerical example:**

In order to illustrate the model just developed, a hypothetical system is considered with the following parameter values:

\[
\begin{align*}
C &= 3.00 \text{ per unit}, \quad C_1 = 0.05 \text{ per unit per month}, \\
C_3 &= 50.00 \text{ per order}, \quad p = 625 \text{ units per month}, \\
R &= 200 \text{ units per month and } \theta = 0.02.
\end{align*}
\]

A computer program was written to solve eqn. (4.1.14) iteratively until \( T^{(m+1)} - T^{(m)} < 0.0001 T^{(m+1)} \). The optimum value of \( T \) was found to be \( T_0 = 2.566 \) months with the corresponding minimum total cost \( K(T_0) = 38.77 \) per month. The optimum lot-size per cycle is

\[
q_0 = \frac{p}{\theta} \log \left[ 1 + \frac{R}{p} \{ \exp(\theta T) - 1 \} \right] = 522.23 \text{ units},
\]

and the number of units that deteriorate during a cycle is

\[
D(T_0) = q_0 - R T_0 = 9.03 \text{ units}
\]

To check the optimality of the solution obtained, the total variable cost \( K(T) \) given by (4.1.13) was computed for \( T = 1 \ (1) \ 10 \); they are shown as follows:
<table>
<thead>
<tr>
<th>Cycle time T (in months)</th>
<th>Total variable cost K(T) per month (in $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>57.509</td>
</tr>
<tr>
<td>2</td>
<td>40.047</td>
</tr>
<tr>
<td>2.5661</td>
<td>38.772</td>
</tr>
<tr>
<td>3</td>
<td>39.300</td>
</tr>
<tr>
<td>4</td>
<td>42.758</td>
</tr>
<tr>
<td>5</td>
<td>47.931</td>
</tr>
<tr>
<td>6</td>
<td>53.977</td>
</tr>
<tr>
<td>7</td>
<td>60.538</td>
</tr>
<tr>
<td>8</td>
<td>67.438</td>
</tr>
<tr>
<td>9</td>
<td>74.569</td>
</tr>
<tr>
<td>10</td>
<td>81.888</td>
</tr>
</tbody>
</table>

4.2: **The mathematical model with shortages:**

For the system described in Section 4.1 the reorder point was prescribed at zero, because shortages were not allowed. We now consider an extension of that system with no restriction on the reorder point. For this purpose we relax assumption (iv) of the previous model, i.e. we now assume that shortages are permissible and that the shortage cost is $C_2$ per unit per unit of time. The inventory fluctuations of the system can be described graphically as follows:
Let $T$ denote the total time period of a cycle then, in the present system, an inventory cycle can be thought as composed of four phases. During the time-interval $(0, t_1)$ of phase I, units are replenished at a constant rate of $p$ units per time unit and demands occur at $R$ units per time unit. During the period $(t_1, T')$ of phase II, there is no replenishment and demands occurring at the rate of $R$ units per time unit are satisfied from the inventory accumulated during phase I period. During the first two phase periods of the cycle units are carried in inventory and hence a fraction of units held in inventory gets deteriorated during $(0, T')$. At the time $t = T'$, the inventory level $Q(T') = 0$. Further demands during phase III period $(T', t_2)$ are backlogged and replenishments again start from $t = t_2$. During the final phase period $(t_2, T)$ of the cycle, backorders are first cleared from the production and then the cycle repeats at $t = T$. As there are
no units in inventory during \((T', T)\) question of deterioration of items does not arise.

As before, if \(Q(t)\) denotes the number of units in inventory at time \(t\) \((0 \leq t \leq T)\), the differential equation that describes instantaneous states of \(Q(t)\) during the cycle is given by

\[
\begin{align*}
\frac{d Q(t)}{dt} + \theta Q(t) &= p - R, & 0 \leq t \leq t_1 \\
\frac{d Q(t)}{dt} + \theta Q(t) &= -R, & t_1 \leq t \leq T' \\
\frac{d Q(t)}{dt} &= -R, & T' \leq t \leq t_2 \\
\frac{d Q(t)}{dt} &= p - R, & t_2 \leq t \leq T.
\end{align*}
\]  

(4.2.1)

Solution of (4.2.1), after adjusting for constants of integration, is

\[
(4.2.2) \quad Q(t) = \frac{(p-R)}{\theta} \left[ 1 - \exp(-\theta t) \right], \quad 0 \leq t \leq t_1
\]

\[
= \frac{R}{\theta} \left[ \exp \{ -\theta(T'-t) \} - 1 \right], \quad t_1 \leq t \leq T'
\]

\[
= R(T'-t), \quad T' \leq t \leq t_2
\]

\[
= (p-R)(t-T), \quad t_2 \leq t \leq T
\]

From (4.2.2) we have as before
\[ Q(t_1) = \frac{(p-R)}{\theta} \left[ 1 - \exp(-\theta t_1) \right] = \frac{R}{\theta} \left[ \exp\{\theta(T'-t_1)\} - 1 \right] \]

and

\[ Q(t_2) = R(T'-t_2) = (p-R)(t_2-T) \]

which in turn give

\[ (4.2.3) \quad t_1 = \frac{1}{\theta} \log \left[ 1 + \frac{R}{p} \{\exp(\theta T') - 1\} \right] \]

and

\[ (4.2.4) \quad t_2 = T - \frac{R}{p} (T-T') \]

respectively.

Now the total demands during the cycle time is \( RT \) and the total number of units produced during a cycle is

\[ (4.2.5) \quad q = pt_1 + p(T-t_2) = \frac{p}{\theta} \log \left[ 1 + \frac{R}{p} \{\exp(\theta T') - 1\} \right] + R(T-T') \]

so that the number of units that deteriorate during a cycle is given by

\[ (4.2.6) \quad D(T', T) = q - RT = \frac{p}{\theta} \log \frac{Z}{p} - RT' \]

where

\[ (4.2.2) \quad Z = p + R \{\exp(\theta T') - 1\} \]
For obtaining the average number of units carried in inventory per unit of time we again assume that $Q(t)$ is approximately linear in $t$ over $(0, T')$. With this assumption, the average number of units carried in inventory per unit of time during the cycle is

\begin{equation}
I_1(T', T) = \frac{T'}{T} \cdot \frac{1}{2} Q(t_1) = \frac{(p-R)T'(Z-p)}{2T0Z}
\end{equation}

Since $Q(t)$ is exactly linear over $(T', T)$ average shortages per unit of time during the cycle is

\begin{equation}
I_2(T', T) = \frac{1}{T} \int_{T'}^{T} -Q(t) \, dt = \frac{R(T-T')^2}{2T} \left(1 - \frac{R}{p}\right)
\end{equation}

The number of replenishments per unit of time is $1/T$. Hence, using (4.2.6), (4.2.8) and (4.2.9), the total average cost per time unit of the system is

\begin{equation}
K(T', T) = \frac{C}{T} D(T', T) + C_1 I_1(T', T) + C_2 I_2(T', T) + \frac{C_3}{T}
\end{equation}

\[= \frac{Cp}{0T} \log \frac{Z}{p} - \frac{CRT'}{T} + \frac{C_1(p-R)T'}{26T} \left(1 - \frac{R}{p}\right)
\]

\[+ \frac{C_2R(T-T')^2}{2T} \left(1 - \frac{R}{p}\right) + \frac{C_3}{T}
\]

where $Z$ is given by (4.2.7)

For optimum values of $T'$ and $T$, differentiating (4.2.10) w.r.t. $T'$ and $T$ and equating the resulting expressions to zero,
we have respectively,

\[(4.2.11) \quad \frac{C(Z-p)}{Z} + \frac{C(Z-p)}{2Z^2} + \frac{C_1pT'}{(Z-p+R)} + \frac{C_2RT'}{p} = \frac{C_2RT}{p}\]

and

\[(4.2.12) \quad \frac{C_p}{\theta} \log \frac{Z}{p} - CRT' + \frac{C_1}{2Z\theta} (p-R)(Z-p)T' + \frac{C_2R}{2p} (p-R)T'^2 + C_3 = \frac{C_2R}{2p} (p-R)T'^2\]

Eliminating \(T\) from (4.2.11) and (4.2.12) we get

\[(4.2.13) \quad p \left[ (Z-p) \left\{ 2CZ\theta + C_1(Z+p\theta T') \right\} + C_1pR\theta T' \right]^2 + 4C_2Z^2\theta^2T' (Z-p)(2CZ+C_1pT') + 4C_1C_2Z^2\theta^2p^2\theta^2T'^2 - \frac{8C_2Z^4\theta^2}{(p-R)} \left[ \frac{C_p}{\theta} \log \frac{Z}{p} - CRT' + C_3 \right] = 0\]

Solving (4.2.13) for \(T'\) we get optimum value of \(T' = T'_o\) (say) and substituting it in (4.2.11) we obtain optimum cycle time \(T = T_o\) (say). Finally, substituting \((T', T) = (T'_o, T_o)\) in (4.2.5) and (4.2.10) we can obtain the optimum value of \(q = q_o\), the number of units to be produced per cycle and the minimum total average cost \(K_o = K(T'_o, T_o)\) respectively.

If shortages are not allowed, i.e. if we take \(C_2 = \infty\) (in (4.2.11) or (4.2.12) we get \(T' = T\) and hence \(Z = Y\), where
Y is given by (4.1.9). Dividing eqn. (4.2.13) throughout by $C_2$ and taking its limit as $C_2 \to \infty$, this eqn. reduces to eqn. (4.1.14) of the model without shortages.

Using series form of the logarithms and exponentials and assuming $\theta << T'$ we ignore the terms with second and higher order powers of $\theta$. Thus, to the first order approximation in $\theta$, we have

\[
\frac{1}{\theta} \log \frac{Z}{p} = \frac{R \theta T'}{2p} (1 - \frac{R}{p}) + \frac{RT'}{p}
\]

\[
\frac{Z - \theta}{Z} = \frac{R \theta T'}{p} \left( \frac{1}{2} - \frac{R}{p} \right) + \frac{RT'}{p}
\]

\[
\frac{1}{Z^K} = \frac{1}{p^K} \left( 1 - \frac{KR \theta T'}{p} \right), \quad K = 1, 2
\]

\[
\frac{Z - \theta}{Z^K} = \frac{R \theta T'}{p^K}, \quad K = 1, 2
\]

Eqns. (4.2.10), (4.2.11) and (4.2.12) now become

\[
(4.2.14) \quad K(T', T) = \frac{C R \theta T'}{2T} (1 - \frac{R}{p}) + \frac{C_1 R T'}{2T} \left[ 1 + \theta T' \left( \frac{1}{2} - \frac{R}{p} \right) \right] \\
\quad \cdot (1 - \frac{R}{p}) + \frac{C_2 R}{2T} (T - T')^2 (1 - \frac{R}{p}) + \frac{C_3}{T}
\]

\[
(4.2.15) \quad \frac{3}{2} C_1 \theta T' \left( \frac{1}{2} - \frac{R}{p} \right) + (C_0 + C_1 + C_2) T' = C_2 T
\]

and
respectively. Eliminating $T$ between (4.2.15) and (4.2.16), the equation to be solved for $T'$ is

\[
(4.2.17) \quad C(2C_1+C_2)\theta T'^2 + C_1(2C_1+C_2)T'^2 + C_1(3C_1+2C_2)\theta T'^3\left( \frac{1}{2} - \frac{R}{p} \right) - \frac{2C_2C_3}{R(1-\frac{R}{p})} = 0
\]

Solving (4.2.17) for $T'$ we can get its optimum value, say $T' = T'_0$, to first order approximation. Substituting it in (4.2.15) we can obtain optimum cycle time $T = T'_0$ and then (4.2.14) will give the optimum cost $K_0 = K(T'_0, T_0)$ of the system. Finally, the optimum lot-size for a cycle is

\[
(4.2.18) \quad q = q'_0 = RT'_0 + \frac{1}{2} R\theta T'_0\left( \frac{1}{2} - \frac{R}{p} \right)
\]

Dividing (4.2.17) throughout by $C_2$ and taking its limit as $C_2 \to \infty$, this equation reduces to eqn. (4.1.16) of the foregoing Section.

When $p = \infty$, the replenishment rate is infinite and eqn. (4.2.14) to (4.2.17) reduce to the corresponding equations (3.2.13) to (3.2.16) of the order-level lot-size model for deteriorating items.
Also when \( p = \infty \), it can be seen that eqns. (4.2.10) to (4.2.13) reduce to eqns. (3.2.6) to (3.2.9) of the order-level lot-size inventory model for exponentially deteriorating items.

When \( \theta = 0 \) there is no deterioration and eqns. (4.2.17) gives

\[
T'(0) = \left[ \frac{2C_3}{C_1R(1 - R/p)} \cdot \frac{C_2}{C_1 + C_2} \right]^{1/2}
\]

and then (4.2.15) gives

\[
T(0) = \frac{C_1 + C_2}{C_2} T'(0) = \left[ \frac{2C_3}{C_1R(1 - R/p)} \cdot \frac{C_1 + C_2}{C_2} \right]^{1/2}
\]

**Method to obtain numerical solution**: 

For obtaining the numerical solution of (4.2.13) by the Newton-Raphson iterative procedure described in Section 3.2, we take

\[
h(T') = p \left[ (Z - p) A(T') + C_1pR T' \right]^2
\]

\[
+ 4C_2Z^2R^2 \Theta T'(Z - p) B(T') + 4C_1C_2pR^2 \theta^2 Z^2 T'^2
\]

\[- \frac{8C_2R^2 \theta^2 Z^4}{(p - R)} \left( \frac{C_p}{e} \log \frac{Z}{p} - CRT' + C_3 \right),
\]
\[ h'(T') = 2p \left[ (Z-p)A(T') + C_1 p \Delta T' \right] \left[ \Re e \exp(\delta T') A(T') \right] \\
+ \left[ (Z-p)A'(T') + C_1 \Re e \right] + 4C_2 \Re e^2 Z B(T') \\
+ \left[ Z(Z-p) + 2\Re e T' \exp(\delta T')(Z-p) + \Re e T' Z \exp(\delta T') \right] \\
+ 4C_2 \Re e^2 Z T' (Z-p) B'(T') + 8C_1 C_2 p \Re e^2 Z T' \\
\\{ \Re e T' \exp(\delta T') + Z \} - \frac{32C_2^2 \Re e^2 Z^3 \exp(\delta T')}{(p-R)} \\
\left. \left( \frac{\Re e}{\delta} \log \frac{Z}{p} \right. \right. - C \left. \left. + C_3 \right) - \frac{8C_1 C_2^2 \Re e^2 Z^3}{(p-R)} \left( p \exp(\delta T') - Z \right) \right. \\
\]

and for the initial value to the iterative process take

\[ T' = T'(0) \] given by (4.2.19) for the corresponding model for non-deteriorating items, where

\[ A(T') = 2C_2 \Re e + C_1 Z + C_1 p \Delta T' \]
\[ A'(T') = 2C_1 \Re e^2 \exp(\delta T') + C_1 \Re e \exp(\delta T') + C_1 p \delta \]
\[ B(T') = 2C_2 + C_1 p T' \]

and

\[ B'(T') = 2C_1 \Re e \exp(\delta T') + C_1 p \]