CHAPTER 2

PROCESS CHARACTERISTICS AND
CONTROL STRATEGIES

In this chapter, the process characteristics and the methods to obtain them using simple process information is presented. Different types of control strategies using adaptive control, fuzzy logic and neural network architectures are also presented. The contents presented in this chapter are most useful in tuning controllers, evaluating control system stability and in designing advanced control strategies.

2.1 TYPES OF PROCESS

A process can be classified into self-regulating and non-self-regulating processes depending on how they respond to an input change. The self-regulating process has a typical response to step change as shown in Figure 2.1. The process undergoes a finite change in output in response to a bounded change in input. The output reaches a new operating point and remains there. That is, the process regulates itself to a new operating condition.
Figure 2.1  Servo response of a self-regulating process in response to a step input

A non-self-regulating process is one, in which, upon a bounded change in input, the output does not regulate itself to a new operating condition. Two different responses of a non-self-regulating process are presented here. In the first case, the output has a constant rate of change (slope) and in the second case the output changes exponentially. Those processes with the type of response shown in Figure 2.2 are generally referred to as integrating process. An example of this process is the liquid level in a tank, as shown in Figure 2.4. As the control signal to the outflow valve is decreased, the level in the tank (process output) starts to increase and reaches a steady rate of change. Processes with the response shown in Figure 2.3 are referred to as open-loop unstable.
Figure 2.2 Servo response of a non-self-regulating integrating process (step response)

Figure 2.3 Step response of a non-self-regulating open-loop unstable process
2.2 PROCESS CHARACTERISTICS

The three important characteristics of a process, that represent the system behaviors are (i) process gain (ii) process time constant and (iii) dead time.

2.2.1 Process Gain (K)

The gain is a steady state characteristic of the process and is defined as the ratio of the change in output to the change in input or the excitation function. The process gain is a measure of the sensitivity of the output variable to a change in the input variable. The process gain can be positive or negative and specifies the direction in which the input and output variables change. The gain does not specify the dynamics of the process, that is, it does not give information as to how fast the process variations occur.

2.2.2 Process Time Constant (τ)

This is a measure of how fast the process responds to a change in input. This term relates the dynamics of the process. The speed of response of a process and the time constant τ are inversely related.
2.2.3 Process Dead Time ($t_d$)

This is defined as the finite amount of time between the change in input variable and when the output variable starts to respond. The parameters $\tau$ and $t_d$ are shown in Figure 2.5. The numerical values of $K$, $\tau$, and $t_d$ depend on the physical parameters of the process, such as size, calibration, etc.

![Figure 2.5 Servo response of a plant used in the measurement of $t_d$ and $\tau$](image)

2.3 PROCESS NONLINEARITIES

A linear process is one in which the numerical values of $K$, $\tau$, and $t_d$ are constant over the entire operating range. If one or more of these parameters vary with operating range, then such processes are classified as nonlinear processes. An example of the nonlinear process is shown in Figure 2.6. The figure shows a conical tank. Since the cross section of the tank at height $h_1$ is less than at $h_2$, the level at $h_1$ will respond faster (to changes in inlet or outlet flow) than at level $h_2$. Thus the dynamics of the process are faster at $h_1$ than at $h_2$. In other words, the gain varies as a nonlinear function of the pressure drop across the valve, which in turn
depends on the liquid head in the tank. The shape of the tank contributes to the nonlinearity in the above process. The identification of the process characteristics for a nonlinear system, is required for designing an parameter adaptive controller, in which the controller tunings will vary with operating conditions.

![Diagram](image)

_Figure 2.6 Example of a nonlinear process_

### 2.4 FIRST-ORDER-PLUS-DEAD-TIME (FOPDT) TRANSFER FUNCTION MODEL OF A SELF-REGULATING PROCESS

The transfer function model of a self-regulating process (but not integrating process) is generally described in terms of the three process characteristics \( K, \tau \) and \( t_d \). Higher order processes with multiple time constants in cascade (Sathe Vivek and Chidambaram 2005), are best approximated by a FOPDT transfer function. Thus higher order processes are commonly approximated by low-order-plus-dead-time model. The steps required to obtain the necessary process data are (i) The controller is set in the manual mode, that is, effectively the controller is removed. (ii) A step signal is given to the plant. (iii) The process variations are recorded. From the process curve the process characteristics are obtained using the two-point method. This method is usually called the process reaction curve method (PRC) or the two-point method. This is shown in Figure 2.7. A typical PRC of
a higher order self-regulating process was shown in Figure 2.5. From the PRC, the process characteristics $\tau$, $t_d$ and $K$ are obtained as

\[ \tau = 1.5(t_{0.632,i} - t_{0.283,i}) \]

\[ t_d = (t_{0.632,i} - \tau) \]

\[ K = \frac{\delta_i}{\delta_{co}} \] where $\delta_i$ is the change in the process output and $\delta_{co}$ is the change in the controller output or process input.

Once these three process characteristics are identified, the process transfer function is defined as

\[ \frac{C(s)}{M(s)} = \frac{Ke^{\frac{-y_s}{\tau s}}}{\tau s + 1} \] where $C(s)$ is the process output and $M(s)$ is the controller output (process input).

### 2.5 PI CONTROLLER

The commonly used controllers in industrial processes are the Ziegler-Nichols tuned PI controller with fixed parameters. The transfer function of the PI controller is given by

\[ G_c(s) = K_c \left[ 1 + \frac{1}{sT_i} \right] \] (2.1)
The drawback with these controllers is that, they exhibit poor performance when applied to systems containing nonlinearities (Ibrahim Kaya 2003) such as dead zone, hysteresis and saturation. The usual way to optimize the control action in such situations is to retune the PI settings, but however this cannot cope up with time varying system parameters or system nonlinearities (Gyongy and Clarke 2006, Hagglund and Astrom 2002).

2.6 ADAPTIVE CONTROL THEORY

Adaptive control is an important area of modern control used in the control of processes with changing parameters (Astrom and Whittenmark 1995). The growth of interest in this field during the past three decades has considerably increased. Adaptive control techniques have great potential, as these methods can cope with increasingly complex systems in the presence of extreme changes in the system parameters and input signals. These controllers have a structure similar to PI controller (Hsiano Ping Huang et al 2003), but their parameters are adapted on-line based on parameter estimation requiring certain knowledge of the process.

2.6.1 Motivation for Using Adaptive Control

A real-world plant can be usually characterized by time-varying dynamical properties, most of them as a result of plant non-stationary, non-linearity, and random disturbances. It is clear that the control algorithm used in these circumstances should either be adaptive or should exhibit some robustness properties with respect to poor plant models and changes in the plant dynamics (Clarke 2003). Robustness properties can usually be ensured by the feedback structure of the control system. The feedback compensates for the deviation of the plant output signal value from its set point, irrespective of the factors that has caused deviation. The common factors that cause deviations are exogenous disturbances affecting the plant, improper
plant model structure or a perturbation in the plant model parameters. However, the two latter factors usually cannot be dealt with well enough by the control system feedback structure alone. Large differences in plant model structure and large variations in the plant dynamics may cause the natural robustness properties of the control system to be exhausted thus causing unacceptable degradation of the system performance. These structural perturbations and environmental variations that result in the degradation of system performance can be cited in a number of examples including the process control where there are changes in the rates of inflow and/or outflow. In summary the use of adaptive control is required to take care of the

(i) Modifications in the plant transfer function, either in its order or in the value of some parameters due to variations in the environment, the size and properties of raw materials, the plant throughput, the characteristics involving alterations in the coefficients and wear and tear of some important components.

(ii) Stochastic disturbances and variations in the nature of inputs.

(iii) Non-linear behavior as in the case of complex chemical or biochemical reactions.

(iv) Systems with appreciable dead time.

(v) Unknown parameters that exist when a control system for a new process is commissioned.

2.6.2 Essential Aspects of Adaptive Control

The basic functions common to most adaptive control systems are the following:
(i) To identify the unknown parameters of a plant or to measure an index of performance (IP).

(ii) To take appropriate decision on the control strategy.

(iii) To perform on-line modification of the parameters of the controller or the input signal.

Depending on how these functions are brought about, there exist different types of adaptive controllers. To outline the essential aspects of adaptive control, the following definition is considered. “An adaptive system measures a certain index of performance using the inputs, the states and the outputs of the adjustable system. From the comparison of the measured IP values and a set of given ones, the adaptation mechanism modifies the parameters of the adjustable system or generates an auxiliary input in order to maintain the IP values close to the set of given ones”. The usual techniques of changing the controller parameters are through gain scheduling and plant model identification.

2.6.3 Classification of Adaptive Control

Adaptive control systems may be classified according to:

(i) The adaptation mechanism: parameter-adaptive or signal-synthesis adaptive systems.

(ii) The operating conditions: deterministic, stochastic or learning systems.

(iii) The nature of the basic mathematical equations used: linear, distributed, parameter, continuous-time, discrete-time systems or hybrid systems.
(iv) The index of performance specified: static, dynamic, parametric and function of state variables and inputs.

(v) The choice of test signals used to measure the index of performance correctly and quickly.

(vi) The nature of comparison-decision block.

(vii) The nature of uncertainty: parametric or structural uncertainty and time-invariant or time-varying uncertainties.

(viii) The nature of constraints imposed either on the desired system or on the nature of the adaptation process.

There are two principal approaches in designing adaptive controllers, namely, Model Reference Adaptive Control (MRAC) and Self-Tuning Controllers (STC). MRAC can further be classified as parameter adaptation type and signal synthesis type.

In the MRAC parameter adaptation type (Figure 2.8), the error due to the deviation between the output of reference model and adjustable system (plant) is used by the adaptation mechanism to modify the controller parameters, with an objective of minimizing the “error”. Alternately if the adaptation mechanism is adjusted to generate an auxiliary input signal with an objective to minimize the difference between the output of the reference model and that of the adjustable system then the system represents a signal-synthesis MRAC (Figure 2.9). The main problem is to determine the adaptation mechanism, such that not only the error is brought to zero but also the resulting system is stable. If stability is not guaranteed, the adaptive system is not of practical utility.
Figure 2.8 Model reference adaptive controller (parameter adaptation type)

Figure 2.9 Model reference adaptive controller (signal synthesis type)
Further, MRAC can be used with continuous-time as well as discrete-time systems, whereas STC involves only discrete-time systems. MRAC systems are mostly applied in deterministic systems whereas there has been a considerable progress in the development of stochastic STC’s.

2.6.4 Self-Tuning Controllers (STC)

The self-tuning controller is another important form of an adaptive control system. In STC, a design procedure for known plant parameters is first chosen. This is applied to the unknown plant using recursively-estimated values of the parameter. Such a system is shown in Figure 2.10. The block regulator design represents the on-line solution for a system with known parameters. This is called the “underlying design problem”. To evaluate adaptive control schemes, it is helpful to find this underlying design problem, as this will give the characteristics of the system under ideal conditions when the parameters are known exactly. The perturbation signals are not shown in Figure 2.10. However, these are necessary to obtain good estimates. STC’s have become very popular in recent years because of their versatility and the ease with which they can be implemented with microprocessors. The STC shown in figure is called an explicit STC. It is based on estimation of an explicit process model. Sometimes, the process may be represented in terms of the regulator parameters. This results in simpler algorithms as the design calculations are avoided. Such an STC is called an implicit STC, since it is based on estimation of an implicit process model.
2.7 FUZZY LOGIC CONTROL

Fuzzy logic has emerged as one of the most active and fruitful areas for research in the real time of industrial processes (Koppen – Seiger and Frank 1999), which do not lend themselves to control by conventional methods because of a lack of quantitative data relating the input and output. Fuzzy control is based on fuzzy logic, a logic which is much closer in spirit to human thinking and natural language than traditional logical systems.

2.7.1 Fuzzy Knowledge Based Controllers (FKBC)

The Fuzzy Knowledge Based Controller is shown in Figure 2.11. It includes four major components namely

(i) Knowledge Base and Rule Base
(ii) Fuzzification
(iii) Inference Mechanism and
(iv) Defuzzification
2.7.2 Fuzzification

Fuzzification converts a crisp input signal into fuzzified signals that can be identified by grade of membership in fuzzy sets. It involves

a) Measuring the values of plant variables.

b) Performing a scale mapping that transfers the range of values of the measured input variables into corresponding universe of discourse.

c) Performing fuzzification, that assigns suitable linguistic values for the input data, which may be viewed as labels of fuzzy sets.

2.7.3 Knowledge Base and Rule Base

The database is composed of a knowledge base and rule base. The knowledge base, consisting of input and output membership functions, along with the rule base provides information for the appropriate fuzzification operations, the inference mechanism and defuzzification. The rule base is
made up of set of linguistic control rules relating the fuzzy input variables to the desired fuzzy control actions. Fuzzy control rules are generally derived using the four modes given below:

1. Based on operator experience and control engineering knowledge.
2. Based on operations, control actions.
3. Based on fuzzy model of a process.
4. Based on learning.

These four modes are not mutually exclusive and it seems likely that combination of them would be necessary to construct an effective method for the derivation of fuzzy rules. The basic function of a rule base is to represent in a structured way the control policy of an experienced process operator and/or control engineer in the form of a set of prediction rules such as IF (process state) THEN (control output)

The design parameters involved in the construction of the rule base includes the following:

1. Choice of process state and control output variables.
2. Choice of the contents of the rule antecedent and the rule consequent.
3. Choice of term sets (range of linguistic values) for the process state and control output variable.
4. Derivation of the set of rules.

2.7.4 Inference Mechanism

Inference mechanism is the kernel of fuzzy logic controller. The control rules are evaluated by an inference mechanism. During the rules
evolution step, the combination of the selected rules is evaluated. There are
two basic approaches employed in the design of the inference engine of
FKBC. (i) Composition based inference (firing) and (ii) Individual rule based
inference (firing). Among these two, the second one is predominantly used.

The basic function of the inference engine of the second one is to
compute the overall values of the control output variable based on the
individual contribution of each rule in the output variable as computed by a
single rule. There are several methods, such as min-max algorithm, the
correlation product algorithm, the Mamdani algorithm etc., for the
implementation of the inference mechanism.

2.7.5 Defuzzification

The defuzzification performs a scale mapping which converts the
range of values of output variables into corresponding universe of discourse
and it yields a non-fuzzy (crisp) control action.

The following four strategies are commonly used in defuzzification:

1. Max criterion method.
2. Mean of maximum method.
3. Center of area method (center of gravity) and
4. Weighted average method.

Of the above four methods, the center of area method (center of
gravity) is the most widely used technique since the defuzzified value tends to
move smoothly around the output fuzzy region, i.e., changes in fuzzy set
topology from one model to the next usually result in smooth changes in the
expected value.
2.7.6 **Advantages of Fuzzy Logic Controller**

Fuzzy logic is best applied to nonlinear, time-variant and ill-defined systems. In system control, the fuzzy approach has a distinct edge over conventional methods. “Preprocessing” large values into a small number of membership grades reduces the number of values that a controller has to contend with to make a decision. Fuzzy logic deals with observed variables rather than the measured variables of system. This means that user can indirectly evaluate more variables than with a conventional PID controller. Implementing a control system with fuzzy logic can reduce design complexity to a point where previously insolvable problems can now be solved. Fuzzy systems typically result in a 10:1 rule reduction, requiring less software to implement the same decision-making capability.

2.8 **Neural Networks and their Control Structures**

In this section the possibilities of using neural networks for nonlinear control is reviewed. The neural networks are generally viewed as process modeling formalism or even a knowledge representation framework. The knowledge about the plant dynamics and mapping characteristics is implicitly stored within the network. The nonlinear functional mapping properties of neural networks are central to their use in control applications. Training a neural network using input-output data from a nonlinear plant is considered as a nonlinear functional approximation problem.

2.8.1 **Basic Neural Learning Model**

One type of neural structure used for learning and control is shown in Figure 2.12. To cope with uncertainties regarding plant dynamics and its environment, the controller has to estimate the unknown information during its operation. If this estimated information gradually approaches the true information as time proceeds, then the controller approaches that of an
optimal controller. Such a controller can be viewed as an adaptive controller, due to the gradual improvement of the estimated information.

The controller learns the unknown information during operation, and this information, in turn, is used as an experience for future decision and controls. A control system is called a learning control system if the information pertaining to the unknown features of the plant or its environment is acquired during operation, and the obtained information is used for future estimation, recognition, classification, control or decision such that the overall system performance is improved. Once learning is complete the control system can compensate for large number of changes in the plant and its environmental conditions. The difference between adaptive and learning system lies in the fact that the former treats every distinct operating situation as novel, whereas the latter correlates the past experience with the present situations and accordingly adapts its behavior.

Figure 2.12 Typical neural learning scheme
A layered feed forward neural network consists of Adalines connected together as shown in Figure 2.13. A layer of Adalines is created by connecting a number of Adalines to the same input vector. Many layers can then be cascaded, with output of one layer connected to the inputs of the next layer, to form a network. It has been proven that a network consisting of only two layers of Adaline can implement any nonlinear function $X$, $d(X)$ given enough Adalines in the first layer (Fredric and Ivica Kostanic 2001). The idea is that each Adaline in the first layer can take a small piece of the function relating $X$ to $d(X)$ and make a linear approximation to that piece. The second layer then adds the pieces together to form a complete approximation to the desired function.

Figure 2.13 Two layer feed forward neural network
2.8.3 Independent Component Analysis

Independent Component Analysis (ICA) is an unsupervised learning technique that in many cases characterizes the data in a natural way. The main application area of ICA is the blind signal separation (BSS). In signal separation, multiple streams of information are extracted from linear mixtures of these signal streams. This process is blind if examples of the source signals, along with their corresponding mixtures, are unavailable for training. In BSS, signals are estimated from their unknown linear mixtures with the assumption that the sources are mutually independent.

2.8.3.1 ICA data model

The ICA operates on M zero mean source signals $s_k(1),...,s_k(m)$, $k=1,2,...$, that are scalar-valued and mutually statistically independent for each sample value $k$. The original sources are unobservable and the input to the ICA are different linear mixtures $x_k(1),...,x_k(L)$ of the sources. The ICA model in vector form is given as $x_k = As_k = \sum_{i=1}^{M} s_k(i)a(i)$, where $s_k = [s_k(1),...,s_k(M)]^T$ represents the source vector of the M source signals $s_k(i)$ ($i=1,...,M$) at the index value $k$ and $A=[a(1),...,a(M)]$ is a constant $LxM$ mixing matrix whose elements are the unknown coefficients of the mixtures. The columns $a(i)$ are the basis vectors of ICA. Usually $M=L$ i.e. the number of source M is assumed to be equal to the number of available different mixtures L, to simplify the derivation of BSS algorithms. However, in practice, it is not necessary for M to be equal to L.

2.8.3.2 Blind source separation

The structure of BSS for instantaneously mixed sources is shown in Figure 2.14. In BSS, the objective is to separate mutually statistically
independent but otherwise known source signals from their linear mixtures without knowing the mixing coefficients. The task is to find individual source signals \( \{s_k\} \), with only the data vectors \( x_k \) and the number of sources \( M \) known.

![ICA network diagram](image)

**Figure 2.14 ICA network**

2.8.3.3 **Blind signal separation algorithm**

Generally, there are two types of BSS problems: those that involve instantaneous mixtures and those involving convolute mixtures. In the present work, the BSS separation algorithm is considered for problems involving instantaneous mixtures. The ICA architecture used to perform the separation of source signals and to estimate basis vectors consists of (i) whitening (ii) separation and (iii) estimation layers. The whitening process is applied to the input vectors so that (i) the data vectors have a zero mean (ii) the variances of the observed signals are normalized to unity and (iii) the separation algorithms have better stability properties and converge faster. The components of the whitened vectors are mutually uncorrelated since it is a necessary prerequisite for the stronger independence condition.
The separation process can be modeled as a single layer neural network with equal number of input and output nodes, where the coefficients $w_{ij}$ of the separation matrix $W$ are simply the weights from input to output nodes. The activation function at the output nodes is used for the training mode only. The threshold activation function plays a central role in blind signal separation and is discussed in detail in an later chapter.

### 2.8.4 Back Propagation Algorithm

The governing equations for a back propagation net, (Rumelhart et al 1986, Haralambos Sarimveis and George Bafas 2003), such as in Figure 2.15 is briefly reviewed here. The neurons in the input layer simply store the input values. The hidden layer and the output layer neurons each perform two calculations. First, they multiply all inputs with weights and add the bias to form the sum $S_j$

$$S_j = \sum_{i=1}^{N} w_{ij} x_i + b_j \quad (2.2)$$

![Figure 2.15 BPN model of network](image)
Second, the output of the neuron, $O_j$, is calculated using the sigma function of $S_j$ as

$$ O_j = \sigma(S_j) $$ (2.3)

where

$$ \sigma(S_j) = \left[1 + e^{-S_j}\right]^{-1} $$ (2.4)

It is not necessary for all of the nets to use the sigmoidal function given in equation (2.4), but they are the ones commonly used. A back-propagation net learns by making changes in its weights in a direction to minimize the sum of squared errors between its predictions and a training data set. The minimization is done using algorithms such as the steepest descent or the conjugate gradient or Newton’s method. If there are $R$ input-output pairs, $x^{(r)}$, $y^{(r)}$ available for training the net, then after presentation of a pair $r$, the weights are changed as follows:

$$ w_{uv}^{(r)} = w_{uv}^{(r-1)} + \Delta w_{uv}^{(r)} $$ (2.5)

with $\Delta w_{uv}^{(r)}$ given by:

Hidden to output weights:

$$ \Delta w_{jk}^{(r)} = \sigma(S_j) \left[ y_k^{(r)} - O_k^{(r)} \right] O_j $$ (2.6)

Input to hidden weights:

$$ \Delta w_{ik}^{(r)} = \sigma(S_i) \sum_k \left[ \sigma'(S_k) \left[ y_k^{(r)} - O_k^{(r)} \right] w_{jk}^{(r-1)} \right] x_i^{(r)} $$ (2.7)

and

$$ \sigma'(S_k) = \sigma(S_k) \left[1 - \sigma(S_k)\right] $$ (2.8)

The weights are changed with each presentation of a pair.
One might assume that setting all the weights to zero may be an acceptable starting point. If all the weights start with equal values and the solution requires unequal weights to be developed, then the system can never learn. The reason is because the error is propagated back through the neurons in proportion to the value of the weights as shown by equation 2.7. All the error signals to the hidden nodes remain identical, and the system starts out at local minima and remains there. This problem is counteracted by starting the system with a set of randomized weights distributed uniformly between –0.5 and +0.5. The chosen activation function has a special feature. The function cannot reach its final values 0 and 1 without infinitely large inputs. The useful region of the activation is approximately between 0.1 and 0.9, and the variable’s input to the net is scaled within this range.

2.9 DESCRIPTION OF THE REAL TIME PLANT

The prototype model constructed for experimental study consists of the cylindrical tank with a conical bottom open to the atmosphere at the top end. The experimental model is to be used, to study the performance of the proposed intelligent control algorithms by obtaining the servo and regulatory response, in the presence of disturbances, feedback sensor failure and sensor noise. Suitable signals are given to a pneumatic operated control valve to regulate the manipulated variable inflow. Disturbances in the form of random variations in outflow (measurable) and/or changes in outflow coefficient are considered to enter the process. The schematic diagram of the plant is shown in Figure 2.16. The process and instrumentation diagram (P&I) of the non-linear (hopper type tank) liquid level process is shown in Figure 2.17. The process variable level is sensed by means of an RF capacitance probe and using suitable electronics circuitry, a voltage output is obtained. The analog voltage is converted into digital form using an 8-bit A/D converter. The inflow and outflow rates are measured using suitable flow transmitters. The details of the different plant components are given in Table 2.1. The laboratory set up is shown in Figure 2.18.
Figure 2.16 Geometrical cross-section of the tank used in the mathematical modeling

Figure 2.17 P&I diagram of the nonlinear hopper type tank chosen for experimental study

MV – Manually operated valve and FT – Flow Transmitter
### Table 2.1 Specification of the process

**Tank dimensions**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>Tank cylindrical portion height</td>
<td>50.5cms</td>
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<tr>
<td>Tank conical portion height</td>
<td>32cms</td>
</tr>
<tr>
<td>Conical portion angle</td>
<td>1.88 degrees</td>
</tr>
<tr>
<td>Outer diameter of tank</td>
<td>32cms</td>
</tr>
<tr>
<td>Inner diameter of tank</td>
<td>31.5cms</td>
</tr>
<tr>
<td>Outer circumference</td>
<td>100cms</td>
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**Valve details**

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<th>Value</th>
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</thead>
<tbody>
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<tr>
<td>Drainage valve</td>
<td>¼ “ ball valve</td>
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</tbody>
</table>

**I to P converter**

<table>
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<th>Value</th>
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</thead>
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<td>Input</td>
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<tr>
<td>Output</td>
<td>3-15psi</td>
</tr>
<tr>
<td>Pressure span</td>
<td>760mm of Hg to 1kg/cm²</td>
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<tr>
<td>Power supply</td>
<td>24vd.c.</td>
</tr>
</tbody>
</table>

**Level Sensor**

<table>
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<th>Value</th>
</tr>
</thead>
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<td>Type</td>
<td>RF capacitance probe</td>
</tr>
<tr>
<td>Maximum distance of probe</td>
<td>150cms</td>
</tr>
<tr>
<td>Resolution</td>
<td>1pF</td>
</tr>
<tr>
<td>Repeatability</td>
<td>better than +/-1%</td>
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<tr>
<td>Zero and span range</td>
<td>0 to 2000pF</td>
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<tr>
<td>Supply voltage</td>
<td>14 to 40vd.c.</td>
</tr>
</tbody>
</table>
2.10 TRANSFER FUNCTION OF THE NONLINEAR PROCESS

The mathematical model of the nonlinear hopper type tank is developed, by considering the process as a combination of (i) a cylindrical geometry and (ii) a conical geometry. The plant transfer function is obtained
in terms of the process characteristics, process gain and the process time constant. The dead time $t_d$ is taken as $\approx 0$ and is hence neglected.

### 2.10.1 Modeling for the Cylindrical Portion of the Tank

The cylindrical portion of the hopper type tank (Figure 2.16) is considered with outflow rate proportional to the square root of level. The mass balance equation governing the system is given by

$$\frac{dv}{dt} = F_{in} - F_{out}$$  \hspace{1cm} (2.9)

$$A\frac{dh}{dt} = F_{in} - bh^{0.5}$$  \hspace{1cm} (2.10)

where $A = \pi R^2$

The transfer function relating the height $h$ and the inflow rate $F_{in}$ with parameters $(k, \tau)$ is derived as:

$$G(s) = \frac{H(s)}{F_{in}(s)} = \frac{k}{(1 + \tau s)}$$  \hspace{1cm} (2.11)

where $k = \frac{2h}{U}$; $\tau = \frac{2hA}{U}$; $U = bh^{0.5}$

The nominal transfer function $G_0(s) = \frac{k^0}{(1 + s\tau_0)}$  \hspace{1cm} (2.12)

where $k^0$ and $\tau_0$ are evaluated at a nominal height $h$. 
The actual transfer function (when the operating point is in the cylindrical region) depends on the level $h$.

### 2.10.2 Modeling for the Conical Portion of the Tank

The conical portion of the hopper type tank (Figure 2.16) is considered with outflow rate proportional to the square root of level. The mass balance equation governing the system is given by

\[
\frac{dv}{dt} = F_{in} - F_{out} \tag{2.13}
\]

\[
A \frac{dh}{dt} = F_{in} - bh^{0.5} \tag{2.14}
\]

where $A(h) = \pi r^2 = \frac{\pi R^2 h^2}{H^2}$

When the operating point is in the conical region, we obtain a similar transfer function:

\[
G_1(s) = \frac{k}{(1 + s\tau_1)} \tag{2.15}
\]

where $k = \frac{2h}{U}; \tau_1 = \frac{2hA(h)}{U}; U = bh^{0.5}$

with the major difference that the area $A(h)$ is now a function of the height $h$. 