Chapter 1

Introduction

1.1 Basic plasma phenomena

The electromagnetic force is generally observed to create structure: e.g., stable atoms and molecules, crystalline solids. In fact, the most widely studied consequences of the electromagnetic force form the subject matter of Chemistry and Solid-State Physics, which are both disciplines developed to understand essentially static structures. Structured systems have binding energies larger than the ambient thermal energy. Placed in a sufficiently hot environment, they decompose: e.g., crystals melt, molecules disassociate. At temperatures near or exceeding atomic ionization energies, atoms similarly decompose into negatively charged electrons and positively charged ions. These charged particles are by no means free: in fact, they are strongly affected by each others’ electromagnetic fields. Nevertheless, because the charges are no longer bound, their assemblage becomes capable of collective motions of great vigor and complexity. Such an assemblage is termed a plasma. Of course, bound systems can display extreme complexity of structure: e.g., a protein molecule. Complexity in a plasma is somewhat different, being expressed temporally as much as spatially. It is predominately characterized by the excitation of an enormous variety of collective dynamical modes. Since thermal decomposition breaks interatomic bonds before ionizing, most terrestrial plasmas begin as gases. In fact, a plasma is sometimes defined as a gas that is sufficiently
ionized to exhibit plasma-like behaviour. Note that plasma-like behaviour ensues after a remarkably small fraction of the gas has undergone ionization. Thus, fractionally ionized gases exhibit most of the exotic phenomena characteristic of fully ionized gases. Plasmas resulting from ionization of neutral gases generally contain equal numbers of positive and negative charge carriers. In this situation, the oppositely charged fluids are strongly coupled, and tend to electrically neutralize one another on macroscopic length-scales. Such plasmas are termed quasi-neutral (“quasi” because the small deviations from exact neutrality have important dynamical consequences for certain types of plasma mode). Strongly non-neutral plasmas, which may even contain charges of only one sign, occur primarily in laboratory experiments: their equilibrium depends on the existence of intense magnetic fields, about which the charged fluid rotates.

Important example of plasmas are interstellar gas, stars, stellar atmospheres, and on earth, lightning, arcs, aurorae, fluorescent tubes, the ionosphere, material processing plasmas and fusion plasmas etc.

Application of plasma physics:

Plasma exists almost everywhere in the universe except in the outer layer of earth including the seven seas and few kilometers above. In laboratory conditions also plasma is produced in vacuum discharges, arcs and flames. In early days plasma physics research was not so sophisticated. Few examples were the propagation of radio waves in the ionosphere and the design of fluorescent light tubes and mercury arc rectifier. In 1934, Bonnet and in 1939, Tonk theoretically analyzed the plasma pinch phenomena, the tendency of a plasma column carrying a large current. Though in 1928, Atkinson and Houtermans gave the idea of thermonuclear reaction in hydrogen at high temperature as the source of solar energy, the applications of plasma physics to nuclear fusion was not suggested. But after the second world war, scientists combining these two ideas proposed to use the pinch effect to create in the laboratory a dense high temperature plasma within which thermonuclear reaction could occur. In the 1950’s the main research in this area was done in USA, USSR and UK. The important publications were
by Thonemann (1950), Cousine and Wane (1951). The 2nd international conference on ‘peaceful use of atomic energy’ held in 1958 was perhaps the beginning of modern fusion plasma research. Another important area of application of plasma research is in astrophysics. Except for a few isolated regions of the universe, which consists of solids, liquids and gases, most of the universe is in plasma state. We mention below some of the interesting plasma phenomena occurring in the universe.

(a) The depth of intergalactic space probably consists of low density plasma.

(b) The pulsars are of great interest because an extremely large magnetic field is associated with them.

(c) The stellar galactic area is of interest because they are the large bodies of observable plasmas in this region and exhibit a large number of plasma behaviours.

(d) Nebulae; these are vast region of gas dust and plasmas.

(e) The Sun; this has a hot central regions, extending through most of the solar radius, in which fusion reaction takes place and whose temperature is $10^7$ K. The sun is spherically symmetric and it exhibits a range of localized phenomena. These include sunspots, solar flares, protuberances and spicules, all bearing witness of the temperature activity of the solar surface. All of these effects are associated to solar magnetic field and the magnetic pressure associated with the field. This value is comparable with the plasma pressure at the solar surface and a number of theories has been invoked to explain the observed dynamic feature. The solar wind emerges from the surface of the sun and its dynamic aspects correlate with solar flares and sunspot activity. This flux of plasma, encounters the earth’s dipole magnetic field and sets up a bow shock, known as magnetopause. Plasma shock theory and instability effect must be involved to explain this boundary situation. A considerable flux of high energy ($>10$ KeV) particles get through this boundary and populates the Van Allen Belts, the plasma belts confined at a few earth’s radii between magnetic mirrors formed by the earth’s dipole field. Plasmas are also found in the ionosphere and aurora in the upper atmosphere and lightning and corona discharged in the lower atmosphere and finally there is the ever-interesting variety of man made plasma, ranging from reentry vehicle nose cone shocks to the light of Piccadilly. Another important area of plasma research is
laser plasma research. The He-Ne laser is commonly used for alignment and serving. The CO₂ laser is used for cutting tools. There are other areas where plasma research is going on. Material technology, Electrical engineering, Plasma chemistry, isotope separations are some of the important fields.

1.1.1 Waves in plasmas

A plasma contains a wide variety of waves because of its fluid like behavior and also because of its long range interaction between the particles in it. It is well known that plasma is a dispersive media. Again from the study of plasma oscillation it is obvious that plasma waves can propagate in a dispersive media. So in plasma medium plasma particles and waves can coexist and they can interact with each other and the oscillation can occur. The plasma waves have a direct application to human information exchange by means of radio waves. Waves are also important for large-scale processes in nature. The light waves in the solar radiation heat the earth, but this heating is balanced by cooling due to emission of long wavelength thermal wave radiation from the earth. Plasma waves in space near a planet may “erode” the planetary atmosphere, accelerating ionized particles to speeds above the escape velocity.

The study of plasma waves in space plasmas involves the measurement of the characteristic frequencies of the plasma in order to understand basic properties of the plasma such as its density and the effect of the magnetic field which may be threading the plasma. Since the charged particles in a plasma respond to static and oscillatory electromagnetic fields, strong interactions can occur between these plasma waves and the underlying charged particles in the plasma. These strong interactions are often called instabilities. Electron plasma oscillations at the plasma frequency (sometimes called Langmuir waves) are one example of an instability in a plasma. In many cases, plasma waves and instabilities are important in understanding the state of the plasma, the evolution of energy and the flux of plasma in a magnetized plasma, and a number of other interesting phenomena. One example is the case of strong whistler mode waves in
the magnetosphere of a planet. These waves have phase velocities which nearly match
the motion of electrons around the magnetic field and can, therefore, have a profound
effect on the motion of the electrons. In this case, the result can be a scattering process
which would dump electrons otherwise trapped in the Earth’s Van Allen radiation belts
into the atmosphere causing the aurora or northern lights.

**Ion acoustic waves:**

Ion acoustic waves propagating in plasma are nearly similar to ordinary sound waves
in neutral gas, they are longitudinal waves consisting of compressions and rarefaction
progressing in the medium. The role of ions is the same as that of neutral atoms in
ordinary sound waves. A difference is that, unlike sound waves, ion acoustic waves can
also propagate in collisionless medium, because the charged ions interact at long dis-
tances via their electromagnetic field. A second difference is that plasma also contains
electrons which have their effect on the wave dispersion equation. Due to their small
mass the electrons are very mobile and they quickly follow the ion motion trying to
preserve the charge neutrality. The electron motion is caused by a small electric field
internally generated by the plasma as a result of variations in the local ion density. Ion
acoustic wave is the low frequency plasma wave. Here the electron and ion fluids must
be considered together.

1.1.2 dusty plasma

Dusty plasmas have opened up a completely new line of research in the field of plasma
physics. Dusty plasmas are low-temperature multispecies ionized gases comprising elec-
trons, ions, and negatively (or positively) charged dust grains typically micrometer or
submicrometer size and neutral atoms. Dust grains may be metallic, conducting, or
made of ice particulates. The size and shape of dust grains will be different, unless
they are man-made. The dust particles can be charged due to the collection of electron
and ion currents from the background plasma. Therefore the dust charge becomes an-
other dynamical variable that distinguishes a dusty plasma from an ideal electron-ion plasma. Dusty plasmas are ubiquitous in different parts of our solar system, namely planetary rings, circumsolar dust rings, the interplanetary medium, cometary comae and tails, as well as in interstellar molecular clouds etc. Dusty plasmas also occur in noctilucent clouds in the arctic troposphere and mesosphere, cloud-to-ground lightening in thunderstorms containing smoke-contaminated air over the United States, in the flame of a candle, as well as in microelectronic processing devices, in low temperature laboratory discharges, and in tokamaks. Dusty plasma physics has appeared as one of the most rapidly growing field of science. In fact, it is a truly interdisciplinary science because it has many potential applications in astrophysics (viz. in understanding the formation of dust clusters and structures, instabilities of interstellar molecular clouds and star formation, decoupling of magnetic fields from plasmas, etc.) as well as in the planetary magnetospheres of our solar system and in strongly coupled laboratory dusty plasmas.

A fascinating property of dusty plasmas is that the particles can arrange in ordered crystal like structures, so-called plasma crystals. In the plasma, the particles acquire high negative charges of hundreds or thousands of elementary charges due to the inflow of electrons and ions. Then, the Coulomb interaction of neighboring particles by far exceeds their thermal energy; the system is strongly coupled. The spatial and time scales of the particle motion allow easy observation by video microscopy. Weak frictional damping ensures that the dynamics and kinetics of individual particles become observable. Thus, dusty plasmas enable the investigation of crystal structure, solid and liquid plasmas, phase transitions, waves and many more phenomena on the kinetic particle level.

Dusty plasma is a normal electron-ion plasma with an additional charge component of macro particles. A plasma with dust particles or grains can be termed as either 'dust in a plasma' or 'a dusty plasma'. If \( r_d \ll \lambda_D < a \) then the plasma is called 'dust in a plasma' and the situation \( r_d \ll a < \lambda_D \) corresponds to 'a dusty plasma'. Where \( r_d \) is dust grain radius, \( \lambda_D \) is Debye length and 'a' is average inter-grain distance.

(1) A criterion for an ionized gas to be a plasma is that it be dense so enough that the
Debye length $\lambda_D$ is much smaller than L i.e $\lambda_D << L$

(2) If there are only one or two particles in the sheath region. Debye shielding would not be a statistically valid concept. The number of particles within the Debye sphere $N_D >> 1$.

(3) A third condition has to do with collisions. If $\omega$ is the frequency of typical plasma oscillations and $\tau$ is the mean between collisions with neutral atoms, we require $\omega\tau > 1$ for the gas to behave like plasma rather than a neutral gas.

**Dust acoustic waves (DAWs):**

The dust acoustic waves have been theoretically investigated by Rao et al. (1990) in a multi component collisionless dusty plasma whose constituents are the electrons, ions and negatively charged dust grains. The phase velocity of the dust acoustic waves is much smaller than the electron and ion thermal speeds. The low frequency dust acoustic waves have been observed experimentally by Barkan et al. (1995) and Thompson et al. (1999). The dispersion relation for the dust acoustic waves is given by

$$\omega^2 = 3k^2V^2_{Td} + \frac{k^2C^2_D}{1 + k^2\lambda^2_D}$$

where $C_D = \omega_{pd}\lambda_D$ is the dust acoustic speed, $\omega$ and $k$ are the frequency and wave vector respectively, $\omega_{pd}$ is the dust plasma frequency, $\lambda_D$ is the dusty plasma Debye radius and $V_{Td}$ is the dust thermal speed.

**Dust ion acoustic waves (DIAWs):**

The dust ion acoustic waves were predicted by Shukla and Silin (1992). The phase velocity of the dust ion acoustic waves is much smaller (larger) than the electron thermal speed (ion and dust thermal speeds). The dust ion acoustic waves have been observed experimentally by Barkan et al. (1996) and Merlino et al. (1998). The dispersion relation (Shukla and Silin, 1992) of dust ion acoustic waves is given by

$$1 + \frac{k^2_{Di}}{k^2} - \frac{\omega^2_{pi} + \omega^2_{pd}}{\omega^2} = 0$$
Because of the large mass of the dust grains, the ion plasma frequency $\omega_{pi}$ is much larger than the dust plasma frequency $\omega_{pd}$. Hence the above equation yields

$$\omega^2 = \frac{k^2 C_s^2}{1 + k^2 \lambda_{De}^2}$$

where $C_s = \omega_{pi} \lambda_{De}$, $\lambda_{De}$ is the electron Debye length.

### 1.2 Nonlinearity and solitons

#### 1.2.1 Introduction

Nonlinear science is believed by many to be the most deeply important frontier for understanding nature. Nonlinear wave phenomena are of great importance in the physical world. Nonlinear waves are abundantly present in the world around us, manifesting themselves often in a destructive manner. Explosions, tidal waves, sonic blasts, these are typical examples of shock waves arising when a wave-conducting medium is driven beyond its linear response regime. It is not surprising that nonlinear wave phenomena attract a lot of interest in numerous fields of physics, ranging from energy transport in biological molecules to high temperature plasma physics. Nonlinear wave phenomena are also exploited in state-of-the-art technology, like femtosecond pulsed lasers and modern telecommunication.

When the amplitudes of the waves are sufficiently large, nonlinearities cannot be ignored. The nonlinearities come from the harmonic generation involving fluid advection, the nonlinear Lorentz force, trapping of particles in the wave potential, ponderomotive force, etc. The nonlinearities in plasmas contribute to the localization of waves, leading to different types of interesting coherent structures (namely solitary structures, shock waves, vortices, double layers, etc.) which are important from both theoretical and experimental points of view. The nonlinear structures, which represent the plasma states far from thermodynamic equilibrium, are either spontaneously created in laboratory and space plasmas on account of free energy sources or externally launched in
1.2.2 Solitary waves and solitons

Many types of nonlinear waves are seen in the space plasmas. A solitary wave is a hump or dip shaped nonlinear wave of permanent profile. It arises because of the interplay between the effects of the nonlinearity and the dispersion (when the effect of dissipation is negligible compared to those of the nonlinearity and dispersion). However, when the dissipative effect is comparable to or more dominant than the dispersive effect, one encounters shock waves.

Solitons are a specific type of solitary waves with the remarkable feature that, when two (or more) of them collide, they do not scatter but emerge with the same shape and velocity. The word ‘soliton’ was coined by Zabusky and Kruskal (1965) after ‘photon’, ‘proton’, etc. to emphasize that a soliton is a localized entity which may keep its identity after an interaction. In the absence of nonlinearity, dispersion can destroy a solitary wave as the various components of the wave propagate at different velocities. Introducing nonlinearity without dispersion again rules out the possibility of solitary waves because the pulse energy is continuously injected into high frequency modes. But with both dispersion and nonlinearity, solitary waves can again form.

The history of solitons is an interesting one (Allen, 1998), with solitons first being seen as water waves in canals in England (Russell, 1845). By studying the nature of waves, Russell claimed that the propagation of isolated wave, was a consequence of the property of the medium rather than the circumstances of the wave generation. Since then it took rather a long time to establish that some special nonlinear wave equations admit solutions consisting of isolated wave that can propagate and undergo collisions without loosing their respective identities. The first theoretical work describing solitons was done by Rayleigh (1879), and in 1895, Korteweg and de Vries found the first equation describing a solitary wave (the KdV equation). It was found that the solitary wave appeared as a special solution of the KdV equation. That is, the solitary wave
became an object of mathematical analysis. After the historical discovery of the inverse scattering transform (Gardner et al., 1967) for exact solution of a class of nonlinear partial differential equations, including many of physical interest the concept of soliton has had a significant influence and consequences in various branches of mathematics, physics, and of engineering as well. In the last stage of the 20th century, soliton theory made its impact in industry also.

When the term soliton is mentioned in the history of science, numerical simulations played an important role. Along with the inverse scattering transform, numerical simulations have been powerful tools to unveil the mysterious characteristics of solitons. Zabusky and Kruskal (1965) investigated a numerical study of KdV equation. They observed that a single solitary wave behaves like a particle in its interaction with another one. They also observed that under certain conditions any initial pulse can break up into a number of solitons which can move in a plasma with different phase velocities. Also the solitons interact with each other and after the interaction they emerge out without any change in their shape and velocity. Some important nonlinear equations which give rise to solitary waves in plasma are KdV equation, modified KdV equation, Gardner equation, nonlinear Schrödinger equation etc.

1.2.3 Shock waves in plasmas

A shock wave (also called shock front or simply ”shock”) is a type of propagating disturbance. Like an ordinary wave, it carries energy and can propagate through a medium (solid, liquid, gas or plasma) or in some cases in the absence of a material medium, through a field such as the electromagnetic field. Shock waves are characterized by an abrupt, nearly discontinuous change in the characteristics of the medium (Anderson 2001). Across a shock there is always an extremely rapid rise in pressure, temperature and density of the flow. In supersonic flows, expansion is achieved through an expansion fan. A shock wave travels through most media at a higher speed than an ordinary wave. Unlike solitons (another kind of nonlinear wave), the energy of a shock
wave dissipates relatively quickly with distance. Also, the accompanying expansion wave approaches and eventually merges with the shock wave, partially cancelling it out. Thus the sonic boom associated with the passage of a supersonic aircraft is the sound wave resulting from the degradation and merging of the shock wave and the expansion wave produced by the aircraft. When a shock wave passes through matter, the total energy is preserved but the energy which can be extracted as work decreases and entropy increases. This, for example, creates additional drag force on aircraft with shocks.

Solitary structures which arise only when the dissipative effects are negligible in comparison with the dispersive effect have already been discussed. However, in practice there are some plasmas in which the dissipative effects may be comparable or even dominant over the dispersive effect. In such a circumstance, the nonlinear plasma waves may appear in the form of shock structures instead of solitary structures. Shock waves can be found in a planet’s magnetotails and are formed in the solar corona and solar wind. Wherever plasmas and field energy flow there will be shock waves. Shocks are places where the plasma and field go through intense changes such as density, temperature, field strength and flow speed. The study of plasma shocks surfaced during the 1950s, with interest in fusion plasmas and shocks caused by explosions in the upper atmosphere. A shock wave travels faster than the speed of sound and changes the state of the medium through which it travels. Shocks occur in the solar atmosphere (corona) during solar ares and other active events. Flares and coronal mass ejections inject energy and material into the solar wind, causing interplanetary shocks, which are travelling shocks propagating out through the solar system. Unlike ordinary sound waves, the speed of a shock wave varies with its amplitude. The speed of a shock wave is always greater than the speed of sound in the fluid and decreases as the amplitude of the wave decreases. When the shock wave speed equals the normal speed, the shock wave dies and is reduced to an ordinary sound wave.

More precisely, Shock wave is a wave of composition (saturation) discontinuity that results from a self sharpening wave. Waves originating from the same point (e.g., constant initial and boundary conditions) must have nondecreasing velocities in the
direction of flow. This is another way of saying that when several waves originate at the same time, the slower waves can not be ahead of the faster waves. If slower waves from compositions close to the initial conditions originate ahead of faster waves, a shock will form as the faster waves overtake the slower waves. This is equivalent to the statement that a sharpening wave can not originate from a point; it will immediately form a shock.

Astrophysical environments feature many different types of shock waves. Some common examples are supernovae shock waves or blast waves traveling through the interstellar medium, the bow shock caused by the Earth’s magnetic field colliding with the solar wind and shock waves caused by galaxies colliding with each other. Another interesting type of shock in astrophysics is the quasi-steady reverse shock or termination shock that terminates the ultra relativistic wind from young pulsars.

1.2.4 Double layers

A double layer is defined as a monotonic transition of the electric potential connecting smoothly two differently biased plasmas. According to Poisson’s equation, a positively charged layer gives rise to a region of negative curvature of the potential and vice versa, and hence two oppositely charged layers are needed to build up the double layer structure (Schamel, 1986). Double layers consist of two layers of separated charges that have an electric field between them. Since these double layers are formed by separated charges, they must violate quasi-neutrality. Quasi-neutrality is the assumption that the charges in the plasma balance each other out leaving a plasma that is essentially neutral over macroscopic scales. The scale over which charges in a plasma are screened out by charges of the opposite sign is the Debye length \( \lambda_D = \sqrt{\frac{kT}{4\pi ne^2}} \), though this assumes that the potential caused by the charge is small \( \phi \ll kT \). Strong double layers are structures where the electrostatic potential drop associated with charge separation is much larger than the temperatures or beam energies of the ions or electrons. In a strong double layer net charge builds up on scales \( \geq \lambda_D \) due to the flow of ions.
and electrons into the structure (Borovsky, 1992). Weak double layers, on the other hand, have electrostatic potential drops which are less than the order of the electron temperature. A weak double layer is a solitary wave where the electric field signal of the pulse is asymmetric, so that there is a net potential drop across the structure. Electrostatic shocks are believed to be strong double layers which are aligned at an angle to the background magnetic field, instead of parallel to the field like double layers (Ergun et al., 1998; McFadden et al., 1999). Since these structures are at oblique angle to the magnetic field, they accelerate particles both parallel and perpendicular to the background magnetic field.

1.3 Importance of studying waves in nonplanar geometry

The extensive application of soliton solutions to one dimensional problems in many areas of physics is well known (Scott et al. 1973). In the past few decades, the propagation of nonlinear ion-acoustic waves (or dust-acoustic waves), especially ion-acoustic solitary waves (IASWs) (or dust acoustic solitary waves (DASWs)) in space and dusty plasmas with an unbounded planar geometry has been extensively studied theoretically (Ikezi et al. 1970; Nakamura and Sarma 2001; Cairns et al. 1996; Mamun and Cairns 1996; Rao et al. 1990; Goertz 1989; Mamun et al. 1996a). But most of these studies are limited to unbounded planar geometry, which may not be a realistic situation in laboratory devices and space and the geometry distortion on waves always exist. The solitary waves in unmagnetized plasma without the dissipation and the geometry distortion effects can be described by Korteweg-de Vries (KdV) equation or Kadomtsev-Petviashvili (KP) equation. The waves observed in laboratory and space are certainly not bounded in one dimension. Franz et al. (1998) have shown that a purely one dimensional model cannot account for the observed features in the auroral region, especially at the higher polar altitudes. Recent theoretical studies indicates
that the properties of solitary waves (IASWs/DASWs) in bounded nonplanar (cylindrical/spherical) geometry are very different from those in unbounded planar geometry. Sometimes ago Maxon and Viecelli (1974a, 1974b) discussed the problem. It may be mentioned that cylindrical and spherical symmetric solitons have been observed in plasmas (Tsukabayashi et al. 1981; Nakamura and Ogino 1982).

1.3.1 The governing equations of motion in nonplanar (cylindrical/spherical) system

Continuity equation (principle of conservation of mass) in cylindrical and spherical coordinates:

We know from fluid motion theory, the equation of continuity in cylindrical coordinate system

\[
\frac{\partial n}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(r n u_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(n u_\theta) + \frac{\partial}{\partial z}(n u_z) = 0,
\]

(1.1)

where the component of the velocity vector given by \( \vec{u} = (u_r, u_\theta, u_z) \). In this three-dimensional cylindrical coordinate system, a point P is represented by the triple \( (r, \theta, z) \) where \( r \) and \( \theta \) are the polar coordinates of the projection of P onto the xy-plane and \( z \) has the same meaning as in Cartesian coordinates.

In spherical coordinate system, with the components of the velocity vector given by \( \vec{u} = (u_r, u_\theta, u_\phi) \), the continuity equation is

\[
\frac{\partial n}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 n u_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(n u_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}(n u_\phi) = 0
\]

(1.2)

where, \( r \) is the radius, \( \theta \) is the polar angle, has the same meaning as in polar and cylindrical coordinates and \( \phi \) is the azimuthal angle(or colatitude angle), is the angle between the positive z-axis and the line from the origin to P in three-dimensional
spherical coordinate system.

Momentum balance equation (Navier-Stokes equation) in cylindrical and spherical coordinates:

The Navier-Stokes equation for incompressible fluid motion gives,

\[ n \left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla P + \mu \nabla^2 \vec{u} + \vec{F} \]  

(1.3)

where left side represents inertia per volume (specifically, first term is for unsteady acceleration and second term is for convective acceleration), first two terms of right side represents divergence of stress (specifically, first term is for pressure gradient and second term is for viscosity) and third term is for other body forces.

In cylindrical coordinates, the Navier-Stokes equation along r direction is given by

\[ n \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta u_r}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_r^2}{r} \right) = - \frac{\partial P}{\partial r} + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (ru_r) \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - 2 \frac{\partial u_\theta}{r^2 \partial \theta} \right] + F_r, \]  

(1.4)

along \( \theta \) direction is given by

\[ n \left( \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta u_r}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} \right) = - \frac{1}{r} \frac{\partial P}{\partial \theta} + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (ru_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + 2 \frac{\partial u_r}{r^2 \partial \theta} \right] + F_\theta, \]  

(1.5)

and along z direction is given by

\[ n \left( \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta u_z}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) = - \frac{\partial P}{\partial z} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] + F_z. \]  

(1.6)

In spherical coordinates, the Navier-Stokes equation along r direction is given by

\[ n \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta u_r}{r} \frac{\partial u_r}{\partial \theta} + \frac{u_\phi u_r}{r \sin \theta} \frac{\partial u_r}{\partial \phi} - \frac{u_r^2}{r} - \frac{u_\theta^2}{r} \right) = - \frac{\partial P}{\partial r} + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (ru_r) \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial \phi^2} \right] + F_r. \]
\[ + \mu \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u_r}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial u_r}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_r}{\partial \phi^2} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \phi^2} \right] + \]

\[ \mu \left[ - \frac{2 u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} - \frac{2 u_\theta}{r^2} \cot \theta - \frac{2}{r^2 \sin \theta} \frac{\partial u_\phi}{\partial \phi} \right] + F_r, \quad (1.7) \]

along \( \theta \) direction is given by

\[ n \left( \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\theta}{\partial \phi} + \frac{u_r u_\theta}{r} - \frac{u_\theta^2 \cot \theta}{r} \right) = - \frac{1}{r} \frac{\partial P}{\partial \theta} + \]

\[ \mu \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial (u_\theta \sin \theta)}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\theta}{\partial \phi^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{2}{r^2} \cot \theta \frac{\partial u_\theta}{\partial \phi} \right] + F_\theta, \quad (1.8) \]

and along \( \phi \) direction is given by

\[ n \left( \frac{\partial u_\phi}{\partial t} + u_r \frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\phi}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r u_\phi}{r} + \frac{u_\theta u_\phi \cot \theta}{r} \right) = - \frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi} + \]

\[ \mu \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u_\phi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial (u_\phi \sin \theta)}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\phi}{\partial \phi^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{2}{r^2} \cot \theta \frac{\partial u_\theta}{\partial \phi} \right] + F_\phi \quad (1.9) \]

The Navier-Stokes equation with no body force (i.e. \( \vec{F}=0 \)), \( F_r = 0 = F_\theta = F_z = F_\phi \).

**Poisson’s equation in cylindrical and spherical coordinates:**

In cylindrical coordinates, the Poisson’s equation is given by

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{\partial^2 \psi}{\partial z^2} = -4\pi n, \quad (1.10) \]

And in spherical coordinates, the Poisson’s equation is given by

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2} = -4\pi n \quad (1.11) \]
For axially symmetric cylindrical/spherical geometries, we considered motion takes place along the direction of r only in chapter 2, 3, 5, 6 and 7. And in chapter 4, motion takes place along r and $\theta$ directions.

1.3.2 Review on cylindrical and spherical solitary waves and shock waves in plasmas

Nonplanar solitary waves:
Cylindrical solitons was first studied by Maxon and Viecelli in 1974. In their theoretical investigation they compared the physical properties of cylindrical solitons with spherical solitons. They found that the cylindrical soliton travels slower than the spherical but faster than the one dimensional soliton of the same amplitude. In their future work, they investigate the soliton solutions in a plasma together with a magnetic field including damping.

In 1979, Ko and Kuehl presents the cylindrical and spherical Korteweg-de Vries solitary wave solution through first order in the expansion parameters. They also give numerical results which indicate that these cylindrical and spherical solitary waves are normal mode, i.e. an arbitrary localized initial disturbance approaches a superposition of one or more solitary waves at large time.

After twenty years, Mamun and Shukla (2001) investigated the nonlinear propagation of nonplanar (cylindrical and spherical) dust acoustic waves in an unmagnetized dusty plasma with Boltzmann electrons and ions. By employing the reductive perturbation method (RPM), nonplanar K-dV equation is derived from the dust continuity, momentum and Poisson’s equation. They are found the numerical solution of nonplanar K-dV equation and shows that the propagation characteristics of the cylindrical and spherical solitary waves significantly differ from those of 1-dimensional dust acoustic solitary waves (DASWs). In 2002, they studied the dust-ion acoustic solitary waves (DIASWs) in an unmagnetized dusty plasma in the framework of nonplanar geometry.
They found that properties of the DIASWs in a nonplanar geometry differs from those in a planar 1-dimensional geometry.

Xue (2004), also investigated the combined effects of bounded nonplanar geometry and ion fluid viscosity on ion acoustic solitary waves (IASWS) in an unmagnetized plasma for the first time. By using standard RPM, a nonplanar KdV equation is obtained. The change of IASWs amplitude profile due to geometry factor and dissipation effect is deduced analytically by him. He also verified the analytical results with the numerical calculation of the fully dissipative nonplanar KdV equation.

Sahu and Roychoudhury (2003) investigated the exact solutions of cylindrical and spherical dust-ion acoustic waves. For this they derived the nonplanar KdV equation. They shown that a suitable coordinate transformation reduces the cylindrical KdV equation into the ordinary KdV equation which can be solved analytically. A completely different analytical solution is obtained by them by using the group analysis method. However, for nonplanar KdV equation group analysis method yields trivial analytical solutions. Numerically, solutions to these nonplanar KdV equation are obtained by assuming initial profiles similar to those in one-dimensional planar geometry soliton solutions. After two years (2005), they theoretically investigated the cylindrical and spherical ion acoustic waves in a plasma with nonthermal electrons and warm ions. The effect of nonthermally distributed electron on nonplanar ion acoustic waves are studied by them. They found that the electrons nonthermality has a very significant effect on the properties of ion acoustic waves.

Mirza et al. (2007) studied nonplanar dust acoustic solitary waves in an adiabatically hot dusty plasmas. In their theoretical work, thay found that the amplitude of the dust acoustic waves increases with with the increase of dust temperature in both the geometries. These results may be useful to explain the salient features of multidimensional dust acoustic waves for space and laboratory plasmas. Also Jehan et al. (2007) studied cylindrical and spherical ion-acoustic solitons in adiabatically hot electron-positron-ion plasmas. In their theoretical work they found that the amplitude and width of the ion-acoustic solitons decreases with an increase in the positron concentration. They also found that adiabatically hot ions plays a destructive role in the formation of solitons
in both the geometries.

After two years, Eslami et al. (2011b) studied nonplanar ion-acoustic solitary waves with superthermal electrons in warm plasma. They considered an unmagnetized plasma system consisting of warm adiabatic ions, superthermal electrons and thermal positrons. They shown that, an increase in positron concentration decreases the amplitude of the solitary waves and they also found that the effects of the superthermal parameter kappa ($\kappa$) on the nonplanar ion-acoustic waves. In the same year, they (Eslami et al. 2011d) investigated the nonplanar ion-acoustic solitary waves in electron-positron-ion plasmas with electron following a q-nonextensive distribution. For this they derived the nonplanar KdV equation by RPM. The effects of the nonplanar geometry and q-nonextensive electrons on the properties of the amplitude and width of the solitary structures were examined by them.

From the above review on nonplanar geometry (i.e. in bounded plasma system) we found that, in the last 10-12 years theoretical plasma research on bounded nonplanar geometrical plasma system is of considerable attention by the many researcher. Many authors studied the nonlinear propagation of waves (either solitary waves or shock waves) by considering the electron and positron follows Maxwell’s distribution. But, our aim is to study the nonplanar solitary waves and shock profiles in bounded nonplanar geometry (which is a realistic situation in space and laboratory plasma devices) by considering the Non-Maxwellian distribution (Nonthermal or Superthermal or nonextensive distribution). Here, we also study the nonplanar Gardner solitary waves and double layers in an unmagnetized plasma system with non-Maxwellian distributed electrons or positrons beyond the KdV limit.

**Nonplanar shock waves:**

It is a fairly well established fact that shock waves can be excited in a dissipative nonlinear medium. There can be several dissipative processes in a plasma. The important ones are Landau damping, kinematic viscosity among the plasma constituents, as well as the collisions between charged particles and neutrals present in the system. However,
when a medium has both dispersion and dissipation, the propagation characteristics of small amplitude perturbations can then be adequately described by Korteweg-deVries-Burgers (KdVB) in one dimension and Kadomtsev-Petviashvili-Burgers (KPB) equation in a two-dimensional geometry. The dissipative Burgers term in the nonlinear KdVB or KPB equation arises by taking into account the kinematic viscosity among the plasma constituents (Xue 2003c, Sahu and Roychoudhury 2004, 2007). When the wave breaking due to nonlinearity is balanced by the combined effect of dispersion and dissipation, a monotonic or oscillatory dispersive shock wave is generated in a plasma (Vladimirov and Yu 1993, 1994; Xue 2003c; Shukla and Mamun 2003). In 2003c, Xue first time investigated the effect of the bounded nonplanar geometry on dust-ion acoustic shock waves in an unmagnetized plasma. For this, He derived the nonplanar Korteweg-de Vries-Burger’s (KdV-B) equation. Change in the dust-ion acoustic shock wave structure due to the effect of the geometry, dust density and ion temperature was studied by deriving the numerical solution of the nonplanar KdV-B equation. The propagation of cylindrical and spherical ion acoustic shock waves in multielectron temperature collisional plasma is studied by Sahu and Roychoudhury (2004). It is found that in the limits of small $\tau$ ($\tau$ is time variable, occurring in the term $\nu/\tau$, of the spherical and cylindrical Korteweg-de Vries-Burger (KdVB) equation; small $\tau$ means the regime in which it is near to zero, i.e., $\tau = \pm 1, \pm 2$), the considered plasma is conducive for the propagation of shock waves as well as solitons, and as the unperturbed value of the ion temperature is decreased, the soliton amplitude decreases. Roy et al. (2008) studied ion acoustic shock waves in quantum epi plasma through the KdVB equation. Their main findings are that the system can sustain both oscillatory and monotonic shock waves depending on quantum parameters (H), that there exists a limiting value of H at which shock wave breaks up to solitary wave, and that the amplitude of shock wave increases with an increase of kinematic viscosity.

Masood and Rizvi (2009) studied two dimensional planar and nonplanar ion acoustic shock waves in electron-positron-ion plasmas. For this they derived nonplanar Kadomtsev-Petviashvili -Burgers (KPB) equation using the small amplitude perturbation expansion method. The analytical solution of the planar KPB equation is obtained
using the tangent hyperbolic method that is used as the initial profile to numerically solve the nonplanar KPQ equation. They found that the strength of the IA shock waves is maximum for spherical, intermediate for cylindrical and minimum for planar geometry. They observed that the positron concentration and the plasma kinematic viscosity significantly modify the shock structure. Their results may be applicable in the study of small amplitude localized electrostatic shock structures in the electron-positron-ion plasmas.

The nonplanar dust ion acoustic solitary and shock waves in a dusty plasma with electrons following a vortex like distribution are studied by Mamun and Shukla (2010). It is found that the dust charge fluctuation, nonplanar cylindrical spherical geometries, and vortexlike electron distribution significantly modify the shock and solitary structure.

### 1.4 Reductive Perturbation Method (RPM)

In recent times reductive perturbation method is used in almost all the branches of plasma physics. This method enables us to reduce a set of general nonlinear partial differential equations to a system of single solvable nonlinear partial differential equation. This method when applied to more general system including dissipation or dispersion shows that for long waves the system of the above mentioned partial differential equations can be reduced to a burger’s equation or KdV equation respectively. Also it is found that for the propagation of slow modulation of a plane wave of infinitesimal amplitude is governed by the nonlinear Schrodinger equation. Let $\xi$ and $\eta$ be defined as

\[
\xi = \epsilon^m (x - v_0 t) \tag{1.12}
\]

\[
\eta = \epsilon^n t \tag{1.13}
\]

where $v_0$ denotes the group velocity and $\epsilon$ is a parameter measuring the strength of nonlinearity. The powers $m$ and $n$ may be integers or fractions. Also the dependent
variables can be expressed as a power series like

\[ u = u_0 + \epsilon u_1 + \epsilon^2 u_2 + ... \]  

(1.14)

when \( u_0 \) is the unperturbed quantity.

In this theory, for different values of \( m \) and \( n \), a set of coupled nonlinear differential equations are reduced to a single nonlinear evolution equation such as Burger’s, KdV, the nonlinear Schrödinger equation and the modified forms of the above equations. During the past decade or so extensive investigations of the nonlinear wave propagations have been developed systemically on the basis of reductive perturbation theory. The reductive perturbation technique has been applied in plasma physics by Asano (1974), Nishikawa and Kaw (1975), Watanabe (1977) among many others.

1.5 Nonlinear evolution equations

1.5.1 Korteweg-de Vries (KdV) equation

Two of the most important properties of a plasma are nonlinearity and dispersion. We begin in this section by discussing a classic nonlinear partial differential equation, known as the Korteweg-de Vries (KdV) equation, which arises in a variety of physical situations, including problems relevant to plasma physics. The Korteweg-de Vries equation is given by

\[ \frac{\partial \phi}{\partial \tau} + A \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} = 0 \]  

(1.15)

where \( \xi \) and \( \tau \) are independent variables and \( A \) and \( B \) are real, nonzero constants. Equation (1.15) is nonlinear through the convective term \( \phi \frac{\partial \phi}{\partial \xi} \), and dispersive through the term \( \frac{\partial^3 \phi}{\partial \xi^3} \). Historically, Eq. (1.15) was first derived by Korteweg and de Vries (1895) in relation to the problem of long surface waves in water channel of constant depth. Much later, Gardner and Morikawa (1960) derived Eq (1.15) from a cold-plasma hydromagnetic model describing the long time behavior of disturbances propagating
perpendicular to a magnetic field with velocity near the Alfven velocity. Kruskal and Zabusky (1963) derived this Eq. (1.15) for one-dimensional acoustic waves in anharmonic crystals. Moreover, as a further example from plasma physics, Washimi and Taniuti (1966) have shown that Eq. (1.15) gives a weakly nonlinear description of one-dimensional acoustic wave disturbances traveling near the ion sound speed. In view of these many different applications of the KdV equation, it is obvious that some generalizations are in order. In this regard, Su and Gardner (1969) have shown that this equation arises in a broad class of weakly nonlinear dispersive systems, just as Burgers equation (1940) arises in a broad class of weakly nonlinear dissipative system. The propagation of solitary waves in cylindrical or spherical geometry can be described by the **cylindrical or spherical KdV equation (or MKdV equation)**, which is given by

\[
\frac{\partial \phi}{\partial \tau} + \frac{\nu^2 \tau}{2} + A \phi \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} = 0 \tag{1.16}
\]

where \( \nu = 1, 2 \) for cylindrical and spherical geometries respectively. It is obvious that in equation (1.16), the second term, namely \( (\nu/2\tau)\phi \), is due to the effect of the cylindrical and spherical geometry (Maxon and Viecelli, 1974). The solution of the cylindrical or spherical modified KdV equation are elaborately discussed in the results and discussions section of chapter 2 and 3.

### 1.5.2 Standard Gardner (SG) and modified Gardner (MG) equation

Reductive perturbation method (RPM) is an well known technique that is widely used for the analysis of physical system in response to small but finite perturbations. In the field of nonlinear plasma physics, the Korteweg-de Vries (KdV) eqn., which is expressed as

\[
\frac{\partial \phi}{\partial \tau} + A_1 \phi \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} = 0 \tag{1.17}
\]

is one of the most extensively studied equations derived from RPM, since Washimi and Taniuti (1966) first derived it to predict the ion-acoustic solitons in electron-
ion plasmas. The KdV equation can be obtained by applying a stretched coordinate transformation $\xi = \epsilon^{1/2}(x-\nu t)$, $\tau = \epsilon^{3/2}t$ (where $\epsilon$ is a small parameter which measures the weakness of the dispersion and $\nu$ is the normalized phase velocity of the wave) to basic plasma fluid equations of the system originally expressed in $(x,t)$ coordinates. As manifested by the quadratic nature of the nonlinear term $A_1 \phi \frac{\partial \phi}{\partial \xi}$, KdV is the result of the second order calculation in the smallness parameter $\epsilon$ (after expanding each variables in power series of $\epsilon$). For plasmas with more than two species, however, there can arise cases where $A_1$ vanishes at a particular value of a certain parameter $r$, and Eqn. (1.17) fails to describe nonlinear evolution of perturbation. So, at this stage, higher order calculation is needed at this critical value $r = r_c$. Now from the third order calculation, which utilizes another stretched coordinate transformation $\xi = \epsilon(x-\nu t)$, $\tau = \epsilon^3t$, which is different from the one used for KdV eqn., one can obtain the modified KdV (m KdV) equation as

$$\frac{\partial \phi}{\partial \tau} + A_2 \phi^2 \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} = 0 \quad (1.18)$$

The mKdV eqn., like KdV, is known to have N-soliton solutions. The mKdV eqn. (Eqn. (1.18)) is valid only for the critical value $r = r_c$ at which $A_1 = 0$. For $r$ in the vicinity of $r_c$, however $A_1$ is small and can be of order $\epsilon$. Then the term $A_1 \phi \frac{\partial \phi}{\partial \xi}$ as a whole becomes of the same order as the terms in Eqn. (1.18). Including this terms in the m K-dV equation, we obtain

$$\frac{\partial \phi}{\partial \tau} + \tilde{A}_1 \phi \frac{\partial \phi}{\partial \xi} + A_2 \phi^2 \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} = 0 \quad (1.19)$$

which is often called mixed modified KdV equation (MMKdV equation) or combined KdV and MKdV equation or standard Gardner (SG) equation (Wazwaz 2007, Lee 2009). The Gardner equation was first derived rigorously within the asymptotic theory for long internal waves in a two-layer fluid with a density jump at the interface. The competition among dispersion, quadratic and cubic nonlinearities constitutes the main interest. Eqn. (1.19), like the KdV equation, is completely integrable with a Lax pair and inverse scattering transform. Gardner equation is widely used in various branches of physics, such as plasma physics, fluid physics, quantum...
field theory. The equation plays a prominent role in ocean waves. The Gardner equation describes internal solitary waves in shallow seas. The Gardner equation has been investigated in the literature because it is used to model a variety of nonlinear phenomena. The tanh method, the cosh ansatz, and the Hirota’s method will be used to solve this problem.

When we consider nonplanar effects then we get nonlinear dynamical equation of the form:

$$\frac{\partial \phi}{\partial \tau} + \nu \frac{\partial^2 \phi}{\partial \zeta^2} + \tilde{A} \frac{\partial \phi}{\partial \zeta} + A_2 \phi^2 \frac{\partial \phi}{\partial \zeta} + B \frac{\partial^3 \phi}{\partial \zeta^3} = 0 \tag{1.20}$$

Equation (1.20) is the Modified Gardner (MG) equation. The modification is due to the extra term, \((\nu/2\tau)\phi\), which arises due to the effects of the nonplanar geometry. We have already mentioned that \(\nu = 0\) corresponds to a 1D planar geometry which reduces eqn. (1.20) to a SG equation (eqn. (1.19)). It can be shown that Gardner equation can have not only the soliton solution, like KdV or mKdV equation, but also the double layer solution. The solution of the Gardner equation have been extensively studied by Wazwaz (2007). Among the many possible solutions of the Gardner equations, we only consider kink (double layer) and solitary wave-type solutions.

In chapter 5, we studied double layer (kink-soliton) solution of MG equation and in chapter 6, we studied solitary wave type solutions of MG equation. Both kink (double layer) and solitary wave-type solutions of MG equation considered in chapter 7.

1.5.3 Kadomstev-Petviashvili-Burger’s (KPB) equation

The dissipative term in Burger’s equation (in one dimension) arises by considering the kinematic viscosity among the plasma constituents. The Burger’s equation can be obtained by applying a stretched coordinate transformation \(\xi = \epsilon \frac{1}{2} (r - v_0 t), \quad \tau = \epsilon \frac{1}{2} t\) (where \(\epsilon\) is a small parameter which measures the weakness of the dispersion and \(v_0\) is the normalized phase velocity of the wave). When dispersive and transverse effects are neglected, the one dimensional Burger’s equation is given by,
\[ \frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} - C \frac{\partial^2 \phi}{\partial \xi^2} = 0 \] (1.21)

where, A and C are the coefficients of nonlinearity and dissipation. The solution of Burger’s equation gives the shock wave solution.

Now, when dispersive effect taken into account, the KdV-Burger’s (KdVB) equation can be obtained by applying above transformation in polar coordinates is given by,

\[ \frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} - C \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\nu}{2\tau} \phi = 0 \] (1.22)

where, B is the coefficient of dispersion.

Nonlinear Kadomstev-Petviashvili-Burger’s (KPB) equation is considered as a nonlinear partial differential equation incorporating both convection and diffusion in fluid dynamics. The equation is introduced to capture some of the features of turbulent fluid in a channel caused by the interaction of the opposite effects of convection and diffusion. It is also used to describe the structure of shock waves, traffic flow, and acoustic transmission. The KPB equation (two dimension) can be obtained by applying a stretched coordinate transformation \( \xi = \epsilon^{-\frac{1}{2}}(r - v_0 t) \), \( \chi = \epsilon^{-\frac{1}{2}}\theta \), \( \tau = \epsilon^{\frac{3}{2}} t \)

to basic plasma fluid equations of the system originally expressed in polar coordinates is given by,

\[ \frac{\partial}{\partial \xi} \left( \frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} - C \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\nu}{2\tau} \phi \right) + \frac{D}{\tau^{2\Xi}} \left( \frac{\partial^2 \phi}{\partial \chi^2} + \frac{\Xi(\nu - 1) \partial \phi}{\phi} \right) = 0 \] (1.23)

where \( \Xi = 0, 1 \) for planar and nonplanar geometries, respectively. A and C are the coefficients of nonlinearity and dissipation, respectively, whereas B and D are the coefficients of dispersion. \( \nu = 0 \) and \( \Xi = 0 \) correspond to the KPB equation in planar geometry, whereas \( \nu = 1, 2 \) and \( \Xi = 1 \) correspond to the KPB equations in the cylindrical and spherical geometries, respectively. The solution of KPB equation also gives the shock wave solution. The solution of the KPB equation is elaborately discussed in the results and discussions section of chapter 4.

Here the term \( \frac{1}{2\tau} \frac{\partial \phi}{\partial \xi} \) and the factor \( \tau^2 \) comes due to the nonplanar effects.

If the dissipative effect is neglected (i.e. C=0) but the transverse effect is taken into
account, the KPB equation (Eqn. (1.23)) degenerates into KP equation by using the same technique:

\[
\frac{\partial}{\partial \xi} \left( \frac{\partial \phi}{\partial \tau} + A \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} + \nu \frac{\tau}{2} \phi \right) + \frac{D}{\tau \Xi} \left( \frac{\partial^2 \phi}{\partial \chi^2} + \frac{\Xi (\nu - 1) \partial \phi}{\chi} \right) = 0 \quad (1.24)
\]

If both the transverse effect and the dissipative effect are neglected (i.e. \( C=0 \) and \( D=0 \)), the KPB equation (Eqn. (1.23)) degenerates into nonplanar KdV equation (Eqn. (1.16)): 

\[
\frac{\partial \phi}{\partial \tau} + A \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3} + \nu \frac{\tau}{2} \phi = 0 \quad (1.25)
\]
1.6 Layout

My intention was to write as much a self-contained text as possible, which should be understandable for a reader with a general knowledge of plasma physics, and simultaneously contain most of the information the expert might be interested in. This hopefully explains the size of this thesis, which has been organized as follows.

In the second chapter, we investigate the effects of nonplanar geometry and kappa distribution on the amplitude and width of the ion acoustic solitary waves (IASWs) are described using numerical simulations. In this paper we have consider a normalized ion fluid model with superthermal electrons, superthermal positrons and warm ions. We derive the modified cylindrical or spherical KdV (MKdV) equation using the reductive perturbation method. Our numerical results indicate that the kappa distribution has only a quantitative, not a qualitative effect on the properties of solitary waves.

In the third chapter, we analytically investigate the nonlinear propagation of cylindrical and spherical IASWs in an unmagnetized, collisionless three-component plasma system comprising of warm ion fluid and q-nonextensive velocity distributed electrons and positrons. To studying this, a MKdV equation is derived for cylindrical and spherical IASWs in electron-positron-ion (e-p-i) plasma by using standard reductive perturbation method. It has been found that non-planar geometry and q-nonextensive velocity distributed electrons and positrons have significant effect on IASWs. IASWs has higher potential for spherical geometry compare to cylindrical geometry. Both electrons and positrons nonextensive parameter($q_e$ and $q_p$) has significant effect on the structure of the non-planar IASWs. Our theoretical investigation gives the possibility of developing more information on the non-planar IASWs that may occur in laboratory as well as astrophysical plasmas.

In the fourth chapter, We have investigated the planar and nonplanar ion acoustic (IA) shock waves in an unmagnetized plasma consisting of nonthermal electrons, non-
thermal positrons, and singly charged adiabatically hot positive ions. The dynamics of such wave is described by the two dimensional nonplanar KPB equations. The numerical results reveal that a shock wave can exist in a bounded nonplanar geometry under the transverse perturbation, but the IA shock waves propagating in cylindrical/spherical geometry with transverse perturbation will be deformed as time goes on. It is found that the shock strength and propagation speed of shocks is maximum for the spherical, intermediate for cylindrical, and minimum for the planar geometry.

In the fifth chapter, we theoretically investigate the dust-ion acoustic (DIA) double layers (DLs) in a dusty plasma consisting cold inertial ions, positive as well as negative dust charge grains and non-extensive distributed electrons. By using RPM we have derived MG equation and solved it numerically. It can be observed that effect of nonplaner term gives rise interesting characteristics. It has been also observed that the nonextnsive q- distribution of electrons has a significant effect on the MG equation and also plays an important role beyond the KdV limit.

In the sixth chapter, we analytically investigate the nonplanar (cylindrical and spherical) ion acoustic Gardner solitons (IA GSs) in electron-positron-ion plasma by deriving the MG equation. For this, we consider inertial ions and superthermally distributed electrons and positrons. The basic features of nonplanar IA GSs are discussed. It is found that the properties of nonplanar IA GSs (both positive and negative) are significantly differs as the value of spectral index kappa ($\kappa$) changes. We have seen that as value of kappa increases, the amplitude of the IA (positive and negative) GSs increases and width of the GSs decreases in both cylindrical and spherical geometry.

In the seventh chapter, we theoretically investigate the DIA GSs and DLs in dusty plasma consisting cold inertial ions, positive as well as negative dust charge grains and superthermal (kappa) distributed electrons. By using RPM we have derived MG equation and solved it numerically. It is seen that the properties of nonplanar DIA GSs and DLs are significantly differs as the value of spectral index kappa ($\kappa$) changes. Also, it
can be observed that effect of nonplanar geometry gives rise interesting characteristics. An outline of possible future works are given in chapter 8.

Most of the authors have studied the solitary waves and shock waves in the planar or nonplanar geometry by taking into account the Maxwell’s distributed electrons or positrons. But in our theoretical investigation we considered the non-Maxwellian distribution (i.e. superthermal distribution or q-nonextensive distribution or nonthermal distribution) for electrons and/or positrons. We have studied the effect of spectral index $\kappa$ (for superthermal distribution) or nonextensive parameter $q$ (for q-nonextensive distribution) in unmagnetized plasmas in the framework of nonplanar (cylindrical or spherical) geometries.