CHAPTER – II
LITERATURE REVIEW

Researchers have extensively studied flow in the vicinity of the stenosis because of its interesting features, such as flow separation, pressure and velocity fluctuations, wall shear stress, flow disturbances and clinical implications from a fluid dynamics point of view. In the recent past quite a good number of theoretical and experimental investigations related to blood flow in the arteries in the presence of stenosis have been carried out with various perspectives in the realm of arterial biomechanics depending upon the objectivity of the problems of the life sciences. Mathematical treatment of such problems has been subjected to constant changes and modifications to account for modern conceptions based on new evidences uncovered through improved experimental measurements. The initial research into the blood flow within stenosed arteries concentrated on experimental simulations and reasonably simple geometries or steady-state axisymmetrical simulations using numerical simulations.


Md. A. Ikbal et al. [35] have analyzed the theoretical investigation of atherosclerotic arteries deals with mathematical model that represent non-Newtonian flow of blood through a stenosed artery in the presence of
transverse magnetic field using power law model. The finite difference scheme has been adopted to estimate the effects of Hartmann number, Power law index, generalized Reynolds number, severity of the stenosis on various parameters such as flow velocity, flux, and wall shear stress and the results showing that the above factors affect the flow characteristics significantly, especially the development of several recirculation zones upstream the constricted site and also the magnetic field causes substantial reduction of the flow rate.

D.S. Sankar and K. Hemalatha [31], [36], [38] have analysed the pulsatile flow of blood through stenosed Artery and non-Newtonian behaviour of blood represented by Herschel-Bulkley fluid and assumed that the thickness of the plug core region is non-uniform and changing with axial distance. A perturbation method has been used to find the expression for the variation of the plug core radius with time and axial distance. The variation of pressure gradient with steady flow rate and wall shear stress distribution as well as resistance to flow with axial distance values of time and for different values of yield stress have been given and compared with the Casson fluid model. Also the effects of Catheterization and non–Newtonian nature of blood in small arteries of diameter less than 100μm considered blood as a Herschel-Bulkley fluid with parameters n and θ and catheter by coaxial rigid circular cylinders have been considered. The equations have been solved using Regula-Falsi method and Trapezoidal Rule. The velocity decreases and the width of the plug core region increases as the yield stress increases. The velocity of Herschel-Bulkley fluid is higher than that of Casson fluid and lower than that of power law
fluid. In the same way the variation flow rate, wall shear stress and the frictional resistance are discussed and the results are compared with Casson and Newtonian fluids. They analysed the steady flow of blood through a tapered tube by assuming blood as Casson fluid and Herschel-Bulkley fluid. The expressions for pressure drop, wall shear stress and resistance to flow have been obtained and also found that for all fluids, the pressure drop increases with increasing angle of the taper. For both the fluids the resistance to flow as well as the wall shear stress increase with increasing yield stress and also for the increasing axial distance for a given taper tube Reynolds number, angle of taper and yield stress. The pulsatile flow of Herschel –Bulkley fluid through catheterized artery has been considered.

Perturbation method has been used to solve the resulting quasi-steady nonlinear coupled implicit system of differential equations. The effects of catheterization and non-Newtonian nature of blood on yield plane locations, velocity, flow rate, wall shear stress and longitudinal impedance of the artery have been determined.

D.S.Sankar and Usik Lee [34], [39], [40-42] have studied the pulsatile flow of blood through the mild stenosed artery and non-Newtonian behaviour of blood as Herschel-Bulkley fluid. A perturbation method has been used to analyze the flow characteristics. The effects of asymmetric of the stenosis observe the plug core radius, pressure drop and wall shear stress increase with the increase of yield stress or the stenosis height. The estimates of the increase in the longitudinal impedance increase with increase of the axial distance or with increase of the stenosis height. They have also analyzed the pulsatile flow
of blood through mild stenosed narrow arteries by treating the blood in the core region as a Casson fluid and the plasma in the peripheral layer as a Newtonian fluid. A Perturbation method has been used to solve the coupled implicit system of non-linear differential equations and the percentage of increase in the resistance to flow over the uniform diameter tube is considerably very low for the two fluid model compared with those of the single fluid model. The pulsatile flow of blood through mild stenosed narrow arteries by treating the blood in the core region as a Casson fluid and the plasma in the peripheral layer as a Newtonian fluid have been investigated. A Perturbation method has been used to solve nonlinear partial differential equations. From the results the wall shear stress and longitudinal impedance to flow are considerably lower for the two-fluid Casson model than of the single–fluid Casson model. It has been found that the presence of body acceleration and a peripheral layer influences the mean flow rate and the mean velocity by increasing their magnitude significantly in arteries. The blood flow through a catheterized artery, assuming the flow is steady and blood is treated as a two-fluid model with the suspension of all the erythrocytes in the core region as a Casson fluid and the plasma in the peripheral region as a Newtonian fluid. The equations have been solved using Regula-Falsi method and Simpson’s rule. The expressions for velocity, flow rate wall shear stress and frictional resistance have been obtained and reported that the increase in the frictional resistance is significantly very small for the two-fluid model than the single-fluid Casson model.
D.S. Sankar [43] has discussed extensively the pulsatile flow of a two phase model for blood flow through axisymmetric and asymmetric stenosed narrow arteries, treating the blood as a two phase model with core region as the Herschel-Bulkley material and plasma in the peripheral layer as the Newtonian fluid. A perturbation method has been used to solve partial differential equation and the estimated flow characteristics have showed that the increase in longitudinal impedance to the flow of the two phase Herschel-Bulkley material are significantly lower than those of single phase flow in small diameter arteries with stenosis. The pulsatile flow of blood through a catheterized artery assuming the blood as a two-fluid model with the suspension of all the erythrocytes in the core region as a Casson fluid and the peripheral region of plasma as a Newtonian fluid has been studied. The resulting non-linear implicit system of partial differential equations has been solved by using perturbation method. The expressions for shear stress, velocity, flow rate, wall shear stress and longitudinal impedance have been obtained. The variations of these flow quantities with yield stress, catheter radius ratio, amplitude, pulsatile Reynolds number ratio and peripheral layer thickness have been discussed. It has been observed that the velocity distribution and flow rate decrease, while, the wall shear, width of the plug flow region and longitudinal impedance increase when the yield stress increases and also it has been identified that the velocity increases, but the longitudinal impedance decreases when the thickness of the peripheral layer increases. The wall shear stress decreases non-linearly, while, the longitudinal impedance increases non-linearly when the catheter radius ratio increases. The estimates of the increase in the longitudinal impedance are
considerably lower for the present two-fluid model than those of the single-fluid model.

D.S.Sankar and Ahmad Izani Md Ismail [44] have investigated that the pulsatile flow of blood through stenosed arteries by assuming blood as a two-fluid model with the suspension of all erythrocytes in the core region as a non-Newtonian fluid (i) Herschel-Bulkley fluid and (ii) Casson fluid perturbation method have been used to solve the system of non-linear partial differential equations. They have concluded that two-fluid Casson model would be more useful than the two-fluid Herschel–Bulkley model.

V.P.Srivastava [45 - 49] has studied the flow of a couple stress fluid, representing blood through an artery with mild stenosis consisting of a core region of suspension of erythrocytes assumed to be a couple stress fluid (Non-Newtonian) and also a peripheral layer of plasma (Newtonian fluid). The expressions of blood flow characteristics have been derived and compared with similar results. From the results it has been concluded that the magnitudes of the flow characteristics increase with non-Newtonian behaviour of the core fluid and also in the presence of a peripheral layer causes significant reduction in the magnitude of the flow resistance and the wall shear stress. Also he has tried to find the effects of the stenosis shape and red cell concentration (i.e) hematocrit on blood flow characteristics due to the presence of a stenosis. The analytical results have been found and used to estimate the blood flow characteristics of normal and diseased blood. From the results, the resistance decreases with increasing shape parameter but increases with hematocrit.
Prashantakumar Mandal [50] has considered non-Newtonian and non-linear blood flow through a stenosed artery characterized by the generalized power-law model also the vascular wall deformability is taken to be elastic. An extensive quantitative analysis has been performed through numerical computations and graphical representations using finite difference method. Also he has solved numerically the problem of non-Newtonian and nonlinear blood flow through a stenosed artery characterized by the generalized power-law model. The finite difference scheme has been used to solve the unsteady nonlinear Navier-Stokes equations and a quantitative analysis has been performed through numerical computation. The effects of the taper angle, wall deformation, severity of the stenosis in fixed length, steeper stenosis of the same severity, nonlinearity and non-Newtonian rheology have been found.

Prashantakumar Mandal et.al.[51] has also developed a mathematical model by treating blood as a non-Newtonian fluid characterized by the generalized Power–law model incorporating both the shear-thinning and shear thickening characteristics of the streaming blood. The Unsteady flow mechanism under the influence of externally imposed periodic body acceleration has been discussed.

S.Chakravarty and A.K. Sannigrahi [52] have investigated the blood flow subject to body acceleration in an irregular stenosed arterial segment as an isotropic elastic tube. The effects of the connective tissues surrounding the arterial wall have been computed. The equations of the Laplace transform space and the transformed domain have been solved. Laplace inversion has been carried out by numerical techniques. The effects of body acceleration on
the flow velocity, the effects of body acceleration on the flow velocity, the flux, the resistive impedances and the wall shear stresses have been computed.

J.C.Misra and S.K.Ghosh [53] have developed a mathematical model to study the transport of interstitial or plasma fluid in the wall of a constricted artery by taking into account the micro rotation of erythrocytes of blood. The movement of the interstitial fluid has been described by the Debye-Brinkman equation. The displacements of the solid matrix of the porous interstitial space, the velocity of the interstitial or plasma fluid movement, the pressure distribution in the constricted arterial segment and also the wall shear stress have been evaluated.

R.Ponalagusamy and R.TamilSelvi [54] have developed a two – layered model of blood flow through a stenosed artery with axially variable peripheral layer thickness and variable slip velocity at the wall. The core region is assumed to be Casson fluid and the peripheral of plasma as a Newtonian fluid. The expression for peripheral layer thickness, core viscosity, slips velocity wall shear stress and resistance to flow has been obtained and the computed values have been compared with the experimental values.

Nidhiverma and R.S Parihar [55],[56] have developed a mathematical model of blood flow through a Tapered Artery with mild stenosis. The expressions for flow characteristics have been obtained. They have concluded from the above study that the high wall shear stress cause the innermost membrane of an artery and the height of the stenosis is more important factor influencing blood flow than tapering. Also they have analyzed the fluid flow of blood flow through multi stenosis arteries in the presence of a magnetic field. By
Frobenious method the equations have been solved and it has been concluded that the rise in systolic pressure and fall in diastolic pressure are very harmful for weak hearts.

R.Ponalagusamy [57] has investigated the blood flow through an artery with mild stenosis and modelled a two-layer artery with axially variable peripheral layer thickness and variable slip at the wall. The analytical expressions for the thickness of the peripheral layer slip and core viscosity have been obtained. The core viscosity has been obtained by the formula and by calculating the red cell concentration in the core also the concentration versus relative viscosity curve has been drawn.

V.P Srivastava et.al [58-65] have investigated the effects of an overlapping stenosis on blood flow characteristics in a narrow artery. The non-Newtonian behaviour of the blood has been represented by a Casson fluid. The equations have been solved by computer codes and the expressions of the flow characteristics have been derived. From the results it has been concluded that the impedance increases with the non-Newtonian behaviour of blood as well as the stenosis size.

B.Pincombe et.al [66] have investigated the effects of multiple stenoses and post-stenotic dilatation on fully developed one-dimensional Casson flow through a single vessel to varying radius of low Reynolds number blood flow in small stenosed coronary arteries. The formula for the resistance-to-flow ratio has been derived and concluded that the resistance-to-flow ratio moves closer to unity as yield stress increases and blood viscosity or flux decreases.
Santabrata Chakravarty and Prashantakumar Mandal [67] have developed a mathematical model of non-linear two dimensional blood flows in tapered arteries in the presence of stenosis. An extensive quantitative analysis has been performed through numerical computations of the desired quantities and have been presented graphically.

B. Pincombe and J. Mazamdar [68] have considered Bingham fluid for the flow through both constrictions and dilatations. Considering the effects of both a single diseased portion and pairs of abnormal wall segments in close proximity to each other and comparing with earlier studies, they concluded that the Bingham model is the closest analogue to blood to make predictions when severe arterial blockages are under consideration.

A. Razavi et al. [69] have compared symmetrical 30 - 60% stenosis in a common carotid artery under unsteady flow conditions for Newtonian and six non-Newtonian viscosity models. The numerical simulations have been used to show that Power – law model produces a higher deviation in terms of velocity and wall shear stress in comparison with other models. They reported that the Generalized Power-law and modified Casson are underestimated for non-Newtonian behavior while Korea and Korea – Yasude represent moderate IG (Global Importance factor) values and also the global non-Newtonian importance factors have been introduced.

Zubaila Ismail et al. [70] have considered the mathematical model of non-Newtonian blood flow through a tapered overlapping stenotic artery characterized by the generalized Power–law model. The finite difference scheme has been used to find the blood flow characteristics such as the axial
velocity profiles, flow rate and wall shear stress. They have reported that the axial velocity profiles, flow rate and wall shear stress have lower values, while the resistive impedances have higher values than the Newtonian model.

Jan Vimmr and Alenajonasova [71] have investigated numerically non-Newtonian steady blood flow in a complete idealized 3D bypass model with occluded native artery. The numerical simulations have been performed by pseudo-compressibility approach, cell–centered finite volume method and for the time integration the fourth order Runge-Kutta method has been used. The non-Newtonian and Newtonian flows through the coronary and femoral bypasses have been focused.

Cheng Tu and Michel Deville [72] have investigated the problem of blood flow with a partial occlusion through stenoses, using the incompressible generalised Newtonian model to incorporate the Herschel-Bulkley, Bingham and power-law model. A Galerkin finite element method and a predictor–corrector time marching scheme have been used and concluded that the memory effects taken into account in the model affect deeply the flow compared with the Newtonian reference case and the disturbances are stronger by their vorticity intensity and persist after the geometrical obstacle.

K.W.Lee and X.Y.Xu [73] have investigated pulsatile flow and vessel wall behaviour in a simplified model of a stenosed vessel. The Geometry of a 45% axisymmetrically stenosed (by area) cylindrical tube and a sinusoidal inflow waveform has been simulated. The fluid flow and wall motion have been carried out separately using two commercial codes CFX4.2 and ABAQUS7 respectively. Their combined effects and interactions have been investigated
through an iteratively coupled algorithm. The comparison between the rigid and compliant models has revealed that, the flow separation layer distal to the stenosis has been thicker and longer, and wall shear stress is slightly lower in the compliant model by less than 7.2%. Under the flow, structural conditions have been investigated and the effects of wall compliance were found to be small and have not changed the flow.

S.U Siddiqui, et.al [74] have determined the effects of non-Newtonian nature of blood and pulsatility on flow through a stenosed artery and noted that the thickness of the viscous flow region changes with axial distance. An important result has been concluded that the mean and steady flow rates decrease as the yield stress increases. The mean resistance to flow is greater than its steady flow value, whereas the mean value of the wall shear for pulsatile blood flow is equal to steady wall shear stress. The velocity profiles and associated physiological characteristics involved in the analysis have been determined.

S.Nadeem et.al [75] have analyzed blood flow through a tapered artery with a stenosis by assuming the flow is steady and blood is treated as non-Newtonian power law fluid model. The exact solution has been evaluated for velocity, resistance impedance, wall shear stress and shearing stress at the stenosis throat. The graphical results of different types of tapered arteries (i.e. converging tapering, diverging tapering, and non-tapered artery) have been examined for different parameters of interest.
T. Bodnar et al., [76] have described and discussed the results of numerical comparative study performed in order to demonstrate and quantify some of the most relevant non-Newtonian characteristics of blood flow in medium-sized blood vessels, namely its shear-thinning and viscoelastic behavior. The classical Newtonian and Oldroyd-B models, as well as their generalized (shear-thinning) modifications have been considered. On three-dimensional geometries, an idealized axisymmetric stenosis and a realistic stenosed carotid bifurcation reconstructed from medical images have been performed. A finite-volume method on a structured grid has been used and model sensitivity tests have achieved findings with respect to the characteristic flow rate to evaluate its impact on the observed non-Newtonian effects.

K. Venkateswarulu and J. Anand Rao [77] have studied the unsteady blood flow through an indented tube with atherosclerosis. The finite difference method has been used for numerical calculations. From the discussions it is clear that the pressure gradient increases with the increase of hematocrit value indicating that there is a higher value in systolic and lower value in diastolic pressure. In high systolic and low diastolic pressure, peripheral blood flow will increase, but the arterial blood flow will decrease. The effects of hematocrit height of stenosis, parameter determining the shape of the constriction on velocity field, volumetric flow rate, pressure gradient of the fluid in stenotic region and wall shear stress at the surface of the stenosis have been obtained.

Kelvin Wong et al. [78] have mathematically modelled the blood flow through an artery with multiple mild stenoses and post-stenotic dilations. The equations for the resistance to flow ration of an artery have been obtained and
analytical solutions have concluded that the effects of variations in arterial wall geometry have on the blood flow resistance.

Devajyothi Biswas and Udayshankarchakraborty [79], [80] have given a brief review on blood flow modeling in Arteries and have discussed some frequently used blood flow models and have concluded that since the nature of blood is very complicated, it is not easy to choose an appropriate fluid model for it. They have also investigated a steady laminar flow of blood through an annulus, enclosed between an arterial stenosis, developed along a tapering wall and a uniform catheter, co-axial to it. A velocity slip condition has been employed at the catheterized wall with different sizes of stenosis and zero –slip at the catheter boundary. Analytical expressions have been obtained for axial velocity flow rate, wall shear stress and apparent viscosity.

Guo-Tao Liu, et.al [80] have computed the numerical simulation of pulsating blood flow through models of stenotic and tapered arteries. The Equations have been solved numerically by finite difference method. The effects of tapering and stenosis have been considered and the authors have concluded that the stenosis disturbs the flow field at the vicinity of the stenosis, especially at the throat and downstream and easily leads to the formation of a flow separation region in the post-stenotic region.

Bijendra Singh, et.al [82] have investigated the blood flow through an artery having radially non-symmetric multiple stenoses located at equispaced points. The graphical analysis has been made. It has been concluded that the increasing values of α show the lower variations for different values of δ/R as well as the length of the stenosis.
Kyoung Chul Ro and Hong Sun Ryou [83] have investigated the effects of periodic body acceleration and bifurcation angle in the stenosed artery bifurcation. Considering the blood to be pulsatile a non-Newtonian fluid based on Carreau viscosity model has been used. The software FLUENT, 3D analyses have been performed for six simulation cases with different body acceleration and bifurcation angles. It has been concluded that the flow variables, flow rate and WSS increase with body acceleration and decrease with bifurcation angle.

G.C. Layek et.al [84], [85] have studied the unsteady viscous flow with variable viscosity in a vascular tube with an overlapping constriction. The finite-difference techniques with staggered grid distribution have been used to solve the equations. The results have been analyzed. From the results it has been noticed that the arterial WSS, pressure distribution and flow rate have been altered (i.e.) the WSS decrease with increasing hematocrit parameter but the flow separation region has increased with increasing hematocrit. Also they have investigated the effects of an overlapping stenosis on flow characteristics considering the pressure variation in both the radial and axial directions of the arterial segment under consideration.

Rashid Ali, et.al [86] have analysed the blood flow through stenosed artery represented by Newtonian fluid. A perturbation technique has been used and for numerical analysis an integral momentum method has been used to solve a problem of industrial importance. The centre line velocity and WSS distribution have been obtained using the model with stenoses simulating a part of the vertebral artery.
A.K.Politis, et.al [87], [88] have pointed out the composite arterial coronary grafts. The computational fluid dynamics techniques have been applied for the simulation of multi-branched CACGs under physiologically realistic inflow waveforms. The numerical solutions have been obtained by a finite-volume method. The stenotic effects have been investigated by comparing computational results for three different degrees of area constriction. The interactions between the grafts and coronary flows in terms of spatial and temporal variations of velocity and WSS distribution have been discussed.

Moloy Kumar Banerjee, et.al [89] have investigated blood flow in a rigid artery to observe the variations in flow pattern and hemodynamic parameters under the influence of multiple stenoses. Grid sensitivity analysis has been carried out for the Newtonian model of blood in the artery. Simulations have been performed with two successive cosine shaped stenoses in a coronary artery with various interspacing distances, degrees of stenoses and Reynolds number.

Helge-I.Anderson and Terje.H.Toften [91] have investigated the power-law fluid boundary layer problem. The transformed momentum equations of first order have been solved by the implicit Keller box difference scheme. Numerical results have been presented for the similarity boundary layer flow along a flat plate and for the non-similar boundary layer along a cylindrical surface. It has been observed that the latter led to substantial savings in computational effort, except for some similarity in boundary layers of pseudo-plastic fluids.
Sunil Appanaboyina, et.al [92] have mentioned the use of stents as flow diverters for the treatment of intracranial aneurysms is the potential occlusion of a perforating artery. The vascular modeling 3D and blood flow modeling have been used with boundary conditions. They have concluded that the small reductions in the blood flow rate inside arterial branches jailed by stents used to treat intracranial aneurysms are expected unless the side branch is completely occluded.

David.N.Ku[93], reviews have focused on selected areas of cardiology and vascular surgery and basic normal flows in arteries and biological response to these flows. The one-dimensional models and measurement techniques have been explained. The complete understanding of the relationship between pressure flow and symptoms for cardiovascular stenoses remains a critical problem. It has been concluded that the fluid dynamics would continue to play an important role in the future diagnosis, understanding and treatment of cardiovascular diseases.

Santabrata Chakravarty and AnantadebDatta [94] have analysed and modelled the effects of multiple stenoses on the flow-behaviour of blood in a stenosed arterial segment. The arterial wall has been treated to be composed of two different layers, the media and adventitia. The equations governing the motion of the system have been sought in the Laplace transform space and their solutions have been obtained in the transformed domain through the use of an appropriate finite difference technique. The Laplace inversion has been carried out by employing numerical techniques.
Adarsh Kumar and Usha Awasthi [95] have developed a mathematical model to examine the pressure gradient and wall shear stress with hematocrit of red blood cell. The blood indicates the usefulness of its rheological character in the functioning of the diseased arterial circulation.

Srivastava V.P and Rati Rastogi [96] have investigated the problem of blood flow through a narrowed catheterized artery with an axially non-symmetrical stenosis. A two phase, macroscopic model has been introduced. The coupled differential equations for both fluid and particle phases have been solved and the expressions of flow characteristics have been derived.

Jayaraman and Dash [97] have addressed a numerical study of blood flow in catheterized curved artery with constriction. MacDonald [98], [99] has considered the pulsatile blood flow in a catheterized artery and obtained theoretical estimates for pressure gradient corrections for catheters. Dash et al. [30], [100] have considered the steady and pulsatile flow of blood in a narrow catheterized artery estimated the increase in frictional resistance in the artery due to catheterization, using a Casson fluid model. Sarkar and Jayaraman [102] have obtained the correction to flow rate-pressure drop relationship in coronary angioplasty with steady steaming effect. Back et al. [103 - 106] using analytical flow modeling have coupled with in vitro experimental evidence and angiographic data, studied the important hemodynamical characteristics like the wall shear stress, pressure drop, and frictional resistance in a catheterized coronary artery under normal as well as the pathological situation of a stenosis present and have estimated the mean flow resistance increase due to catheterization for concentric and eccentric catheter configurations.
L.Bjorno and H. Petterson [107-110], have studied extensively the hydro-and hemodynamic effects of catheterization of vessels with and without stenosis with the help of various experimental models. Daripa et.al., [111] for solving Poisson equation inside a circular disk to an annular domain, and to implement and apply the extended fast algorithm to solve an applied physical problem.

Chakravarty and Mandal [112],[113] have studied the effects of an overlapping stenosis on the arterial flow problem of blood. An effort has been made in the present work to study the effects of an overlapping stenosis on the flow characteristics of blood taking into account that flowing blood is represented by a two-layered model.

Kanai et al. [114], supported by experimental results on controlled dog, have found analytically how to reduce the error due to the wave reflection at the tip of the catheter for each experiment, and have expressed that catheter of appropriate diameter must be used. Rao and Padmavathi [29], have shown the changed flow patterns for a moving catheter positioned concentrically as well as eccentrically.

Fung [4] has reported that the blood that accounts for 7% of the human body weight has an average density of approximately 1060 kg/m$^3$, that is very close to the pure water’s density of 1000 kg/m$^3$ and an average adult bears a blood volume of roughly 5 litres. It is already reported that blood is a suspension of various tiny particles, in continuous saline solution plasma.
Schlichting[115] has pointed out that plasma behaves as a Newtonian fluid whereas the whole blood, being a suspension of cells and highly viscous nature, exhibits the property of a non-Newtonian fluid.

Merrill and Taylor [116] have pointed out that though blood shows a non-Newtonian character at low shear rate but at high shear rate, usually available in large arteries, blood behaves like a Newtonian fluid. As blood possesses a finite non-zero yield stress as well as it indicates a power-law behaviour, it can be accounted by considering it, in behaving like a Casson fluid as well as a Power-law fluid.

The following authors have considered the Newtonian Fluid Model. Fung and Taylor [4], [116] have reported that at high shear rates and in larger vessels, blood behaves like a Newtonian fluid. Young [117, 118] has analysed the effects of stenosis on flow characteristics of blood treating blood as a Newtonian. He has reported that an increase in stenosis size increases both the impedance to flow and wall shear stress. As an extension to Young’s work, Forrester and Young [119] have included the effects of flow separation in an artery with a mild stenosis. Young and Tsai [17] have presented the results of experimental work on models of an arterial stenosis. Ahmed and Giddens [120] have investigated the velocity field in the neighbourhood of an axisymmetric constriction in rigid tubes, using Laser Doppler Anemometry (LDA).

Baaijens et al., [121] have studied the numerical analysis of a steady generalised blood flow, by considering blood acting as a Newtonian fluid. Kanai et al., Bjorno and Peterson [107-110] have reported that when a catheter is
inserted in a stenosed artery, it further increases the impedance to flow and changes the pressure distribution.

Jayaraman and Tewari [122] have studied blood flow in a catheterised curved artery, by modeling the artery as a curved pipe and the catheter to be co-axial. In their model, blood is assumed as an incompressible Newtonian fluid and no-slip boundary condition is used to analyse the flow. Sarkar and Jayaraman [102] have proposed a model for blood flow through a catheterized and stenosed artery, in considering blood as a Newtonian fluid and the flow as axi-symmetric and oscillatory in nature.

Bugliarello and Hayden [123] have reported theoretically and experimentally the velocity slip at the flow boundary. Biswas and Chakraborty [124] have presented a Newtonian model of blood flow, through a stenosed artery with a velocity slip condition at the boundary. In this model it has been reported that employment of velocity slip at the stenotic wall increases the axial velocity but reduces the wall shear stress and apparent viscosity.

Many researchers have considered the non-Newtonian Fluid Models but they have limited their work in the single layered model. Misra et al., [125] have experimentally analysed the vicinity of a stenosis, the shear rate of blood is low and therefore the non-Newtonian behaviour of blood in that region is quite prominent. Keeping in view this finding, many researchers have considered blood in behaving as a non-Newtonian fluid, while investigating its flow behaviour inside a constricted artery but they have limited their research for the Power-Law Fluid Models. Kapur et al., have dealt with the Power-law fluids belonging to the class of visco- inelastic fluids that form an important division in
non-Newtonian fluids. Bird et al.,[11] have included that the fluids of this kind are characterised by the rheological equation. Metzner [126] has given the magnitudes of Power-law fluid parameters. Raju and Devanathan [127] have developed an analysis of the flow of a Power-law fluid through a circular tube, under a constant axial pressure gradient. In their work, effects of non-Newtonian viscosity of blood on the flow in a coronary artery casting of human have been studied using a finite element method. Cho and Kensey [18] have examined various constitutive models including a Power-law fluid model, to model the non-Newtonian viscosity of blood and their model constants have been submitted. Edwards et al., [9] have proposed the unsteady laminar flow of a Power-law fluid in pipes.

Chaturani and Palanisamy [128] have proposed a mathematical model to study the pulsatile flow of a Power-law fluid through a rigid circular tube under the influence of periodic body acceleration. Numerical solutions have been obtained in their work, with the help of Crank-Nicholson method. Casson Fluid Models have been considered by several researchers but the two layered model for the catheterised tapered artery with stenosis has not been considered so far. Fung [4], [134] have analysed the effects due to the presence of several substances, like protein, fibrinogen and globulin in aqueous base plasma, and have found that human red blood cells can form a chain-like structure, known as aggregates or rouleaux. It has also been found that the rouleaux behave like a plastic solid and there would be an yield stress that can be identified with the constant yield stress in Casson’s fluid.
Charm and Kurland [14] have mentioned the utility of Casson's equation in the rheology of blood and also have shown that Casson's equation can be used to analyse the blood flow over a wide range of haematocrit and shear rates. Bugliarello and Sevilla [129] have considered a two-layered model with a core and a peripheral layer as Casson fluids, with different yield coefficients and viscosities. Oka [37] has considered the flow of Casson fluids in tapered tubes.

Batra and Koshy [130] have investigated the flow of Casson fluids in non-circular ducts, by using a variational method. Walender et al.,[131] have proposed an approximate Casson fluid model for tube flow of blood. Chaturani and Samy [128] have analysed the problem of blood flow in a constricted artery and in their steady two-layered model, and have found that peripheral layer consists of a Newtonian fluid whereas the core is taken as a Casson fluid. Misra et al.,[125], Chaturani and Palanisamy [128], Dhar et al.,[26], Biswas and Mazumder [132], Dash et al.,[30], [100] Nagarani and Sarojamma [133] have considered Casson fluid models of blood flow in various situations. But they do not consider the tapered catheterized artery in their study.

Apart from the two layered model the two-Phase blood Flow models have been analysed by some researchers. The theoretical analyses of Haynes [16] and experimental observations of Cokelet et al., [13] indicate that blood cannot be treated as a single-phase homogeneous viscous fluid in narrow arteries (of diameter less than 1000 mm). Srivastava and Srivastava [60] have reported that the individuality of red cells (of diameter 7.6 mm) is important even in such large vessels with diameter up to 100 cells' diameter and have proposed a two-phase theoretical model to describe pulsatile blood flow in the
entrance region of an artery. Srivastava et al. [135] have applied the theory to study the effects of external body acceleration on blood flow through tubes with small diameter. Srivastava [45-48] has dealt with the problem of blood flow through stenotic vessels representing blood by an erythrocytes-plasma suspension. Srivastava and Rastogi [59] have studied the effect of Hematocrit on wall shear stress and resistance to flow in a stenosed annulus by considering blood as an erythrocytes-plasma suspension.

Some researchers have considered the two-layered Blood Flow Models but the catheterised artery for tapered tube with stenosis has not been analysed by the previous researchers. Bugliarello and Hayden [123] and Bugliarello and Sevilla [129] have reported in the literature that when blood flows through narrow tubes, there exists a cell free plasma layer near the wall. Cokelet [13], Thurston [137] have shown experimentally that for blood flowing through small vessels, there remains cell-free plasma (Newtonian fluid) layer near the vessel wall and a core region, consisting of suspension of almost all the erythrocytes. Sankar and Ismail [44] have described a realistic description of blood flow, that it is appropriate to treat blood flow as a two-fluid model with the suspension of mostly the erythrocytes, in the core region and plasma in the peripheral region. Haynes has presented a two-fluid theoretical model for blood flow, consisting of a core region of suspension of all the erythrocytes as a homogeneous Newtonian viscous fluid and a cell-free plasma layer as a Newtonian fluid of constant viscosity (equal to the viscosity of water). Shukla et al.,[138] have proposed a two-fluid model to discuss the flow of blood through an artery with a mild stenosis. Chaturani et al.,[128] have addressed the flow of blood in small
diameter tubes using the two-layered model of micropolar and couple stress fluids. Srivastava[48] has investigated two-fluid models to observe the effects of a non-symmetrical stenosis on blood flow characteristics. Srivastava et.al.,[64] has proposed a two-layered model consisting of a core region of suspension of all the erythrocytes (particles) in plasma (fluid), assumed to be a particle-fluid mixture and a peripheral layer of cell-free plasma (Newtonian fluid). Apart from above mentioned models, many other models viz. polar fluid model, Bingham plastic model, couple-stress fluid model etc. have been dealt with by many researchers over the years. Since the nature of blood is very complicated, it is not easy to choose an appropriate fluid model for it.

2.1 LIMITATIONS OF THE PREVIOUS STUDIES AS OUT-LINED ABOVE

The previous researchers have investigated a two phase model of blood flow through a narrow catheterized artery with an overlapping stenosis. In their study they have considered a pulsatile flow but severe cases of stenosis like tapering have not been considered. The present investigation proposes to take the severe case of stenosis that is tapering of the stenosis and the angle of tapering, playing a vital role in this study.

A two-layered model for different shapes of stenoses and slip velocity at the wall have been analysed by the researchers but these studies have been limited to the catheterization and effects of non-Newtonian nature of blood. The simultaneous analytical solution using calculus method and numerical method of output has not been given by many researchers.
Many authors developed a two-fluid model for a catheterized artery by assuming the blood to be a two-fluid with the suspension of all the erythrocytes in the core region as a Casson fluid and peripheral region of plasma as a Newtonian fluid. But the two fluid model for the catheterized tapered artery has not been considered.

Some authors have analysed the blood flow through a catheterized artery by assuming the flow is steady and blood is treated as a two-fluid model in the presence of the catheter. Their studies have been limited to the non-stenosed artery.

A single and two fluid mathematical models for blood flow through catheterized tapered arteries with stenosis have not been studied so far. The comparison between single fluid and two fluid Casson and Herschel-Bulkley models for catheterized tapered artery with stenosis have not been analyzed by any researcher. Some researches have propounded that of blood flowing through small vessels, there is an erythrocyte-free plasma (Newtonian) layer adjacent to the vessel wall and a core layer of a suspension of all erythrocytes (non-Newtonian).

Now in this proposed research work it is aimed to contemplate on the various issues which have not been met with and cleared in the previous studies. Hence, as indicated in the introductory chapter, the proposed plan and implementation of the research study would be beneficial to medical practitioners, including information on the flow velocities, pressure gradient, plug flow velocity, catheter radius ratio, wall shear stress and frictional resistance to the blood flow.