Chapter 1

Introduction

Many real world phenomena require the analysis of systems in a probabilistic rather than deterministic setting. Stochastic models are becoming increasingly important for understanding and for assessing performance evaluation of complex systems in broad spectrum of fields such as Operations Research, Computer Science, Telecommunication and Engineering. In this thesis $T$-policy is implemented to the $(s, S)$ inventory system with random lead time and also repair in the reliability of $k$-out-of-$n$ system.

In this thesis we analyze an $(s, S)$ inventory system with random lead time under $T$-policy and also a repairable $k$-out-of-$n$ system with control policy governed by $T$-policy.

Inventory may be defined as a physical stock of goods kept in a system for the smooth and efficient business transactions. Inventory system may be considered as the system of keeping records of the amounts of commodities in stock. In an inventory problem lead time is defined as the time between the placement of order and the actual time at which units reach system. Several policies may be used to control an inventory system, of these, the most widely used is the $(s, S)$ policy. Under this policy, whenever the position inventory reaches a level less than or equal to $s$ for the first time measured from the previous replenishment epoch, a procurement is made to bring its level to $S$. Under a continuous review system, the $(s, S)$ policy will usually imply the procurement of a fixed quantity $M = S - s$ of the commodity, while in periodic review systems the procurement quantity will vary. The $(s, S)$ policy incorporates two decision variables $s$ and $S$. The variable $s$ is called the reorder level, which identifies when to order, while $S - s$ identifies how much to order.
During the lead time it may happen that there is no backlog, finite backlog (which will be met immediately on replenishment) or a large number of lost sales. In the latter two cases there is every chance of loss of customer goodwill and consequent loss to the system. In order to overcome this, $T$-policy is introduced during lead time.

We define the $T$-policy as follows: a replenishment does not occur within $T$ units (a r.v) after the placement of an order, a local purchase is made either (i) bring the inventory level to $S$ cancelling the replenishment order placed or (ii) to bring the inventory level to $s$ or (iii) to bring the inventory level to 0 without cancelling the order (that is to meet all the backlogs, if any, without cancelling the order).

Local purchases by shop keepers are very common. This will ensure goodwill of customer to a great extent. Situations of this sort arise in practice. In shops when certain goods run out of stock and reaches a threshold (a negative level) due to backlogging the owner goes for local purchase. The local purchase involve higher cost to the system. The introduction of $T$-policy ensures the minimum number of loss of demands by taking decision at the right moment.

Inventory system of $(s,S)$ type had been extensively studied in the past. A systematic account of such inventory system was first provided by Arrow, Karlin and Scarf [1958]. Further details of work carried out in this field can be found in Hadley and Whittin [1963], Veinott [1966], Sivazlian [1974]. $(s,S)$ inventory policy with renewal demands and general lead time distribution was first considered by Srinivasan [1979]. Sahin [1979] deals with $(s,S)$ policy where demand quantity is a continuous random variable and lead time is a constant. Sahin [1983] compute the binomial moments of the inventory level in an $(s,S)$ inventory with compound renewal demand and arbitrarily distributed lead time. Manoharan, Krishnamoorthy and Madhusoodan [1987] investigate $(s,S)$ inventory policy with unit demand and non-identically distributed inter-arrival times of demands having arbitrary lead time distribution.

Several models for perishable inventory systems can be found in the review article by Nahmias [1982]. $N$-policy in the queueing setup has been discussed by several authors (see Artalejo [1992], Gakis et.al [1995], Teghem Jr.[1986], Heyman [1967], Balachandran [1973].

$(s,S)$ inventory system with $N$-policy during lead time have been introduced and in-
vestigated through a series of paper by Krishnamoorthy and Raju [1998, 1999] and Raju [1998]. In $N$-policy a local purchase is made when the number of backlogs reaches $N$.

We can make a note on some control policies in queueing system. Consider a steady state $M/G/1$ queueing system. Server remains in the system till all waiting customers are served. When the number of customers in the system reaches $N$, where $N \geq 1$, for the first time after the server is removed, it returns immediately and provides service until there are no customers in the system. This operating policy is called the $N$-policy in queueing context. In the $T$-policy the removed server returns to the system and provides service, on the elapse of $T$ time units from the epoch of server removal, if there is at least one customer present in the waiting line. He continues to serve until there are no customers in the system, at which time the server is removed again to return after $T$ time units. This process continues. Finally, if the workload or backlog, which is equal to the sum of the service time of waiting customers, exceeds $D$ (where $D > 0$) for the first time after removal of the server, it returns to the system and provides service to all customers when the system is empty.

Together with these, six different dyadic policies which are different combinations of the $T$-policy, $N$-policy and the $D$-policy are also studied in queueing literature. They are (i) the $T^M/N$-policy (ii) the $T^M/D$-policy (iii) the $\min(N, D)$-policy (iv) $\min(T, N)$-policy (v) the $\min(T, D)$-policy (vi) the $\max(N, D)$-policy. In the $T^M/N$-policy, a $T$-policy is first used once the server becomes idle. If following an idle period no customer appears in the first $MT$ time units, where $M = 1, 2, \ldots$ is a given quantity, then the server switches to an $N$-policy. Thus, an $N$-policy is used if the server remains idle for $MT$ time units, the $N$-policy is initiated at the end of $MT$ time units. In the $T^M/D$ policy a $T$-policy is again used first once the server becomes idle. If no customer appears during the first $MT$ time units, where $M = 1, 2, \ldots$, is a given quantity, the server switches to a $D$-policy. Thus a $D$-policy is used if the server remains idle for $MT$ time units.

Reliability of $k$-out-of-$n$ system under $D$-policy has been studied by A Krishnamoor-thy and P.V. Ushakumari [2000]. In the $\min(N, D)$ policy, following the start of an idle period or on completion of an idle period the server restarts serving and hence initiates a busy period, if either $N$ customers have accumulated in the system ($N \geq 1$) or the total accumulated backlog of customers service time exceed $D$, whichever occurs first. Similar interpretations can be given to other policies also. For further details one may refer
to Yadin and Naor [1963], Heyman [1977], Levy and Yechiali [1975], Balachandran and Tijms [1975], Bell [1971,73,80], Tegham [1986] and Gakis, Rhee and Sivazlian [1995].

We have also introduced the repair of a $k$-out-of-$n$ system under $T$-policy. Several models are analysed under this set up.

Reliability is generally characterized or measured by the probability that an entity can perform one or several required functions under given conditions for a given time interval. The term 'entity' is used here to denote any component, subsystem, system or equipment that can be individually considered and tested separately. According to the entities, the notion of time interval should be replaced by the notion of number of cycles, distance travelled etc.

Reliability is defined as the ability of an entity to perform a required function under given conditions for a given time interval. It is measured by the probability that an entity $E$ can perform a required function under given conditions for the time interval $[0, t]$. Thus $R(t) = P(E \text{ does not fail during } [0, t])$. The reverse of this ability is called unreliability.

A system is a deterministic entity comprising an interconnected or interacting collection of discrete elements.

Suppose that a system has finite number $n$ of independent components labelled $1, 2, \ldots, n$ and that the system is capable of just two modes of performance. Represent the mode of performance of the system by the Bernoulli r.v. $X$. Suppose that, given the structure of a system, the knowledge of its performance can be determined from that of its components. The system structures generally considered are described below.

i) Series system The system functions iff all the $n$ components functions. We have $X = \min(X_1, X_2, \ldots, X_n)$, the reliability of $n$ components is given by $P = P(X = 1) = P(\min(X_1, X_2, \ldots, X_n) = 1) = P(X_1 = 1, X_2 = 1, \ldots, X_n = 1) = \prod_{i=1}^{n} P_i$

(Here $X_i = 1$ indicates that $i$th component is operational and $P_i = P(X_i = 1)$, $i = 1, 2, \ldots, n$)

ii) Parallel system: The system functions iff at least one of the $n$ components functions.
We have $X = \max(X_1, X_2, \ldots, X_n)$. The system reliability is given by

$$P = P(X = 1) = P(\max(X_1, X_2, \ldots, X_n) = 1) = 1 - P(X_1 = 0, X_2 = 0, \ldots, X_n = 0) = 1 - \prod_{i=1}^{n}(1 - P_i)$$

iii) $k$-out-of-$n$ system: The system function iff at least $k(1 \leq k \leq n)$ of the $n$ components functions. As particular cases, we get the series system for $k = n$ and the parallel system for $k = 1$.

$k$-out-of-$n$ system have been studied extensively (see, for example Angus [1988], Godbole, Potter and Sklar [1998], Pham and Upadhyaya [1988]. Madhu Jain and Ghimira [1997] discuss the reliability of $k$-out-of-$n$ system. $k$-out-of-$n$ system in discrete time with multiple repair facilities has been discussed in kapur, Garg, Sehgal and Jha [1997].

$k$-out-of-$n$ system with the $N$-policy for repair of failed units has been discussed in detail by Krishnamoorthy, Ushakumari and Lakshmi [1998] under the assumption of exponential life times for components. Under this policy a server is called for repair as soon as the number of failed units reach $N(\leq n - k)$. Further Ushakumari and Krishnamoorthy [1998] examine the control problem of obtaining the optimal $N$ value when the service times of units have arbitrary distribution. They analyze the semi-Markov process and the embedded Markov chain arising in this setup.

The optimal number of repairs in the context of analysing systems subject to shocks have been considered by Shen and Griffith [1996]. This can be regarded as the optimal $N$-policy for replacement. Rangan and Sarada [1992 a,b] discuss the optimal strategies of replacement for deteriorating system with changing failure distributions. Lam Yeh [1990] analyses a single repairable replacement model and in [1991] he obtains the optimal number of repairs before replacement. Rangan and Grace [1989] provide the optimal replacement policies for deteriorating systems with imperfect maintenance. Ushakumari [1998] has analyses a $k$-out-of-$n$ system with repair of failed units under $(N, T)$-policy. Here the amount of time for which the server is not available in the system is a random variable which is the minimum of an exponentially distributed time duration $T$ and the sum of $N$
independent exponentially distributed random variables that are not necessarily identically  
distributed (ie. a generalized Erlang variate).

For some of the combination policies it is impossible to get analytical solution (eg.  
probability distribution of the system state). In such cases one can resort to numerical  
studies and also analyse certain performance characteristics.

In this thesis, we have considered a $k$-out-of-$n$ system with repair under $T$-policy.  
Server is activated after the elapse of $T$ time units where $T$ is exponentially distributed  
with parameter $\alpha$ from the epoch at which it was inactivated after completion of repair of  
all failed units in the previous cycle, or the moment $n - k$ failed units accumulate, whichever  
occurr first. Thus server is activated at the moment which is $\min\{T, E_{n-k,\lambda}\}$ after his  
previous departure where $E_{n-k,\lambda}$ is an Erlang distributed r.v. with parameters $n - k$ and $\lambda$.  
He continues to remain active until all the failed units are repaired and then inactivated. The  
process continues in this fashion. The repaired units are assumed to be as good as new.  
Life time of components and service time (repair times) are assumed to be exponentially  
distributed with rates $\lambda$ and $\mu$, respectively. We consider three different situations: (a) cold  
system (b) warm system (c) hot system. A $k$-out-of-$n$ system is called cold, warm or hot  
according as the functional units do not fail, fail at a lower rate or fail at the same rate when  
system is shown as that when it is up.

$k$-out-of-$n$ system with repair and two modes of service under $N$-policy has been in-  
trودuced by A. Krishnamoorthy and P.V. Ushakumari [1999]. In this thesis, we consider  
$k$-out-of-$n$ system with repair and two modes of service under $T$-policy. In this case first  
server is available always and second server is activated on elapse of $T$ time units. Re-  
liability of a $k$-out-of-$n$ system with repair and retrial of failed units has been introduced  
by A. Krishnamoorthy and P.V. Ushakumari [1999]. Retrial queues have been extensively  
studied by many researchers, an excellent account of which can be found in Falin and  
Templeton[1997]
Basic concepts

1.1 Definition: Renewal Process

Consider a specific phenomenon that occurs randomly in time. Let \( w_1, w_2, \ldots \) be the times between its successive occurrences. Write \( S_0 = 0; S_{n+1} = S_n + W_{n+1}, n \in \mathbb{N} \). This sequence defines the times of occurrence of the event assuming that the time origin is taken to be an instant of such an occurrence. The sequence \( S = \{S_n, n \in \mathbb{N}\} \) is called a renewal process provided that \( w_1, w_2, \ldots \) are independent and identically distributed non-negative random variables. Then the \( S_{n}, n \in \mathbb{N} \) is called the \( n \)th renewal epoch.

Consider the number of renewals \( N_t \) in the interval \( [0, t] \); this is \( N_t(w) = \sum_{n=0}^{\infty} I_{[0,t]}(S_n(w)), t \geq 0, w \in \Omega \), where \( I_A(x) = 1 \) or \( 0 \) according as \( x \in A \) or \( x \notin A \). Note that \( N_0(w) \geq 1 \) always, and that \( N_t(w) = \inf\{n \in \mathbb{N} : S_n(w) > t\} \). Thus the event \( \{N_t = k\} \) is equal to the event \( \{S_{k-1} \leq t; S_k > t\} \cap \{S_k \leq t\} \) and \( \{S_k \leq t\} \subseteq \{S_{k-1} \leq t\} \). Since \( S_k > S_{k-1} \). Thus, for any \( k = 1, 2, \ldots \) \( P(N_t = k) = P(S_{k-1} \leq t) - P(S_k \leq t) = F^{(k-1)}(t) - F^{(k)}(t) \) where \( F^{(l)}(\cdot) \) is the \( l \)-fold convolution of \( F \) with itself. One can compute the expected member of renewals in \( [0, t] \) by using this distribution:

\[
R(t) = E[N_t] = \sum_{n=0}^{\infty} E[I_{[0,t]}(S_n)] = \sum_{n=0}^{\infty} P(S_n \leq t) = \sum_{n=0}^{\infty} F^n(t)
\]

The function \( R = 1 + F + F^2 + \ldots \) is called the renewal function corresponding to the distribution \( F \).

1.2 Definition: Regenerative Process

Consider a stochastic process \( Z = \{Z_t, t \geq 0\} \) with state space \( E \). Suppose that every time a specified event occurs, the future of the process \( Z \) after that time becomes a probabilistic replica of the past. Such times (usually random) are called regeneration times of \( Z \), and the process \( Z \) is then said to be regenerative.

Let \( Z \) be a regenerative process with a discrete state space, and consider the probability \( f(t) \) that \( Z_t = i \) for some fixed state \( i \). We condition the event \( \{Z_t = i\} \) on the time \( S_t \).
of first regeneration, and argue as follows. The process $Z$ regenerates itself at $S_t$ and the future process $\tilde{Z}$ defined by $\tilde{Z}_n = Z_{S_t + n}$ has the same probability law as $Z$ itself. Given $S_t$, if $S_1 = S \leq t$, then $Z_t = \tilde{Z}_{t-s}$, and therefore

$$P(Z_t = i/S_1) = P(\tilde{Z}_{t-s} = i) = f(t-s)$$

Hence, if we define $g(t) = P\{Z_t = i, S_1 > t\}$ then we have $f(t) = g(t) + \int_{[0,t]} F(ds)f(t-s)$. This equation is called a renewal equation. Renewal theory is the study of the renewal equation $f = g + F \ast f$ where $F$ is a distribution function on $R^+$ and $f$ and $g$ are function which are bounded over finite intervals. The renewal equation has one and only one solution $f = R \ast g$ where $R = \sum F^n$ is the renewal function corresponding to $F$.

It is well known that with probability 1, $\frac{\Delta Z}{t} \to \frac{1}{\mu}$ as $t \to \infty$ where $\mu = \int_0^\infty xdF(x)$ (see for example Ross [1970])

### 1.3 Markov Renewal Process

Definition: The Stochastic process $(X, T) = \{X_n, T_n, n \in N\}$ is said to be a Markov renewal process with state space $E$ provided that

$$P\{X_{n+1} = j, T_{n+1} - T_n \leq t/X_n = i, T_0, \ldots, X_n, T_0, \ldots, T_n\}$$

$$P\{X_{n+1} = j, T_{n+1} - T_n \leq t/X_n\} = Q(i, j, t) \quad \text{for all } n \in N, j \in E$$

and $t \in R^+$ (Cinlar [1975]).

Markov renewal theory combines renewal theory with the theory of Markov chains to create tools that are more powerful than those which either could provide. Consider a process which moves from one state to another with random sojourn times in between such that the successive states visited form a Markov chain and the sojourn time has distribution which depends on the state being visited as well as the next state to be entered.

The family of probabilities $Q = \{Q(i, j, t) : i, j \in E, t \in R^+\}$ is called a semi-Markov kernel over $E$. For each pair $(i, j)$ the function $t \to Q(i, j, t)$ has all the properties of a distribution function except that $P(i, j) = \lim_{t \to \infty} Q(i, j, t)$ is not necessarily equal to one, we can see that $P(i, j) \geq \sum_{j \in E} P'(i, j) = 1$ that is, the $P'(i, j)$ are the transition probabilities for some Markov chain with state space $E$. 
1.4 Proposition

$X = \{X_n, n \in \mathbb{N}\}$ is a Markov chain with state space $E$ and transition matrix $P$.

Another convenient picture in describing a Markov renewal process is provided by the process $Y = \{Y_t : t > 0\}$ defined by putting for each $t \geq 0$ and $w \in \Omega$

$$Y_t(w) = \begin{cases} X_n(w) & \text{if } T_n(w) \leq t < T_{n+1}(w) \\ \Delta & \text{if } t \geq \sup_n T_n(w) \text{ where } \Delta \text{ is a point not in } E \end{cases}$$

The stochastic process $Y = \{Y_t : t \geq 0\}$ defined as above is called the minimal semi-Markov process associated with $(X, T)$

1.5 Markov Renewal Function

Let $(X, T) = \{(X_n, T_n) : n \in \mathbb{N}\}$ be a Markov renewal process with semi-Markov kernel $Q$ over a countable state space $E$. Define

$$Q^n(i, j, t) = P\{X_n = j, T_n \leq t / X_0 = i\}, \; i, j \in E, t \in \mathbb{R}^+$$

for all $n \in \mathbb{N}$, with

$$Q^0(i, j, t) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Then for $n \geq 0$ we have the successive relation $Q^{n+1}(i, k, t) = \sum_{j \in E} \int_0^t Q(i, j, ds) Q^n(j, k, t - s)$. Where the integration is on $[0, t]$.

Consider the function $R(i, j, t) = \sum_{n=0}^\infty P_i(X_n = j, T_n \leq t) = \sum_{n=0}^\infty Q^n(i, j, t)$. The functions $t \to R(i, j, t)$ are called Markov renewal functions and the collection $R = \{R(i, j, .) : i, j \in E\}$ of these function is called a Markov renewal kernel.

Let $j \in E$ be fixed, and define $S'_0, S'_1, \ldots$ as the successive $T_n$ for which $X_n = j$. Then $S'_j = \{S'_n : n \in \mathbb{N}\}$ is a (possibly delayed) renewal process.

Let $F(i, j, t)$ be the distribution of the first passage time from state $i$ to state $j$, that is, let $F(i, j, t) = P_i(S'_0 \leq t), \; i \neq j$ and let $F(j, j, t)$ be the distribution of time between successive occurrence of $j$, that is, let $F(j, j, t) = P_j\{S'_1 \leq t\} (P_j(S'_0 = 0) = 1)$

$$R(j, j, t) = \sum_{n=0}^\infty F^n(j, j, t)$$
and \( R(i, j, t) = \int_0^t F(i, j, ds)R(j, j, t - s), \) \( i \neq j \) where \( F^n(j, j, \cdot) \) is the \( n \)-fold convolution of the distribution \( F(j, j, \cdot) \) with itself.

### 1.6 Scope of the Work

The thesis comprises five chapters. In Chapter 1 a brief summary of the topics relevant to the thesis, including the contributions of the author is given.

In Chapter 2, we introduce \( T \)-policy during lead time in \((s, S)\) inventory system. In \( T \)-policy whenever a replenishment doesn’t occur after the placement of an order within \( T \) units of time (a \( r \) \( v \)) a local purchase is made either to bring the inventory level to \( S \) cancelling the replenishment order placed or to bring the inventory level to \( s \) or to 0 without cancelling the order (the last policy serves to meet all the backlogs if any, without cancelling the order). The demand process is assumed to be Poisson with rate \( \lambda \). As and when the inventory level drops to \( s \), on order is placed for \( M = S - s \) units. The lead time is exponentially distributed with parameter \( \mu \) and \( T \) is exponentially distributed with parameter \( \alpha \). We denote by \( I(t) \) the inventory level at time \( t, t \geq 0 \). \( \{I(t), t \geq 0\} \) is a finite state space Markov chain with state space \( \Lambda = \{-k, -k + 1, \ldots, s, \ldots, S\} \) when \( k \) is the maximum number of backlogs, allowed. We choose \( k \) such that \( M - k > s \) to avoid perpetual order placement. The probability of transition to \( i \) at time \( t \) starting from \( S \) at time 0 is denoted by \( P_i(t), i \in \Lambda \). \( P_{Si}(t) = P(I(t) = i/I(0) = S) \). The time dependent and steady system probabilities are computed. Also the optimal value of \( k \) is found out in the three cases by fixing \( s \) and \( S \). The situation where \( T \) follows a general distribution is also considered. As above, demands are assumed to be Poisson with rate \( \lambda \) and lead time exponential with rate \( \mu \). The replenishment epochs \( T_1, T_2, \ldots \) follow a regenerative process. Here, we consider only the first case. Time dependent probabilities are found out. Cost function is found out by examining the embedded Markov renewal process.

In Chapter 3 the reliability of a \( k \)-out-of-\( n \) system with repair under \( T \) policy is studied. \( T \)-policy in the queueing set up has been extensively studied (see Artalejo [1992]) However, this has not been brought to the investigation of the reliability of \( k \)-out-of-\( n \) system with repair inorder to minimize the system reliability. The repair is according to \( T \)-policy, server
is called to the system after the elapse of $T$ time units, where $T$ is exponentially distributed 
with parameter $\alpha$, since his departure after completion of repair of failed unit or the moment 
n - $k$ failed units accumulate whichever occur first. He continue to remain in the system 
until all the failed units are repaired, once he arrives. We consider three different situations 
(a) cold system (b) warm system (c) hot system. The $k$-out-of-$n$ system is called a cold 
system, warm or hot if once the system is down, the functioning components do not fail, 
fail at a lower rate or fail at the same rate.

Life times of units are assumed to have independent exponential distribution with pa­
rameter $\lambda_i$ when $i$ units are functioning. Repair time is also assumed to be exponentially 
distributed with rate $\mu$. We have obtained the profit function and numerically, we have 
found optimal $\alpha$ which maximize the profit.

Next, the distribution of sevice time is taken as general. In this case we examine the sys­
tem state at repair completion epochs. These epochs form a regenerative process provided 
failure time of components are exponentially distributed with rate $\lambda$ and random variable 
$T$ is assumed to follow exponential distribution with parameter value $\alpha$. Here also, we 
consider three different states of components (i) cold (ii) warm and (iii) hot. In all these 
cases it is established that the cost function is convex and hence global minimum exists.

Chapter 4 deals with $T$-policy for $k$-out-of-$n$ system with two modes of service. $k$-out­
of-$n$ system with repair time distribution of the I server exponential with rate $\mu_1$ and that 
of the II server with rate $\mu_2$. Here, we consider only cold system. Here, the II server is 
activated after the elapse of $T$ time units since becoming idle from the time of completion 
of most repair of all failed units. Since, we are considering cold system, functional compo­
nents do not fail after the system is down. We have obtained system rate probabilities and 
some performance measures. Some numerical illustrations are provided.

In Chapter 5 we discuss some special models in reliability of a $k$-out-of-$n$ system with 
repair under $T$-policy. In the first model, the repair is provided by an unreliable server. 
Here, $T$ is assumed to be exponentially distributed with parameter value $\alpha$. Repair time is 
exponentially distributed with rate $\mu$. Server is subject to breakdown. The failure rate is as­
tumed to be exponential with rate $\beta$ and repair of server is also exponential with parameter
value γ. \( X(t) \) denotes the number of failed units

\[
Y(t) = \begin{cases} 
0 & \text{if Server is inactive} \\
1 & \text{if Server is activated} \\
2 & \text{if Server is activated but down}
\end{cases}
\]

system state probabilities and some characteristics are obtained.

In the second model, though, the server is switched on after the elapse of \( T \) time units, he gets activated only after a random length of time. Let \( U \) be the activation time and is assumed to be exponentially distributed with rate \( \theta \). \( T \) is exponentially distributed with rate \( \alpha \) and repair time exponentially distributed with rate \( \mu \). Hence the time elapsed until activation starting from all units operational, has generalized Erlang distribution. Here, \( X(t) \) represent the number of failed units. \( Y(t) \) equals 2, if server is active at time \( t \). 1, if server is only switched on but not activated and 0 otherwise. Steady state probabilities and some performance measures are found out.

In the third model, we consider the time for the server to get inactivated. The system does not go directly to state \((0, 0)\) from \((1, 1)\), it goes to a state \((0, 2)\) and then to \((0, 0)\). Here \( X(t) \) represents the number of failed units at \( t \).

\[
Y(t) = \begin{cases} 
0 & \text{of server is inactive} \\
1 & \text{if server is active} \\
2 & \text{if server is switched off, but not inactivated}
\end{cases}
\]

Here the inactivation time is assumed to be exponentially distributed with rate \( \eta \). All other assumption are as in the above models. Here also system state probabilities and some characteristics are obtained. In all the above three models, some numerical illustrations are provided.