Chapter 1

Introduction

1.1 Quantum Computation and Information

1.1.1 What is quantum computation?

"Because the nature isn't classical, damn it..." - Richard Feynman.

Quantum computation and information [1] is a rapidly progressing subject nowadays. Often in science exciting results come from combining two apparently unrelated ideas into one. Exactly in the same way came the idea of quantum computation, by combining quantum mechanics and computer together.

All our present day computers obey the laws of classical physics. But if someday it so happens that the bits of a computer shrink to atomic scale, then a quantum description of a bit state may become necessary [2]. Here comes the idea of quantum computation. That the quantum system can perform computation was first explicitly pointed out by Paul Benioff [3] and Richard Feynman [4], independently in 1982. In 1985 Feynman optimistically concluded [5]: "it seems that the laws of physics present no barrier to reducing the size of computers until the bits are the size of atoms and quantum behavior holds dominant sway". Since then there have been enormous theoretical as well as experimental progress in the field of quantum computation and information processing. The major milestones in this subject are Shor's factorization, Grover's search algorithm, quantum teleportation, quantum cryptography etc [1]. Though till date the subject of quantum computation has grown up with a significant theoretical progress, but the picture
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for experimental realization of these ideas is not very charming. It is extremely
difficult to precisely control the quantum system. However the development of
some powerful error correcting code is very hopeful.

1.1.2 Difference between classical and quantum computer

A conventional digital computer works with bits - the Boolean states. Classical
information theory considers transmissions of classical states. In a properly
functioning classical computer, all the bits always have a definite state, say,
011100101...at any instant of time. In contrast to this, quantum computation
involves quantum states. The state of a quantum computer is described by a
wave-function or a state in the Hilbert space and quantum information theory
considers transmission of quantum states from source to receiver. For example
the state of the bits in a quantum computer might look like

\[ |\psi\rangle = a|011100101... \rangle + b|111010001... \rangle + ... \]  \hspace{1cm} (1.1)

The coefficients \( a, b, ... \) are complex numbers and determine the probability that
the computer is in a certain state. These coefficients can describe interference
among different states of the computer, which is a very useful process for com-
putation. The quantum wave function declares that the computer exists in a
superposition of all of its states so long as that state is not measured.

1.1.3 What is the advantage of a quantum computer?

Quantum computation and information has many unusual properties, which has
made them distinct and advantageous from classical computation. This became
very clear when Peter Shor in 1994 [6], demonstrated that at least in principle a
quantum computer can factorize a very large number efficiently. It is very strongly
believed that the time required to find the factors of a \( n \)-digit number is super-
polynomial in \( \log(n) \). The best known algorithm (the "number field sieve") requires
[7] time of the order of \( \exp\left[c\left(\ln(n)\right)^{1/3}(\ln(\ln(n)))^{2/3}\right] \), where, \( c = (64/9)^{1/3} \approx 1.9 \). Using
this one can estimate that factorization of a 400 digit number would take about
\( 10^{10} \) years, which is of the order of the age of the universe. So even with vast
improvements in technology, factorizing a 400 digit number will be out of reach for
a while. The exciting new result that Shor found is that, quantum computer can factorize a $n$-digit number in a polynomial time, e.g., in time $O(\ln(n))^{3/4}$. Thus using Shor’s algorithm, a quantum computer can factorize a 400 digit number in less than 3 years. Similar idea also appeared in 1985, in a paper by Deutsch [8] that shows that a single evaluation is sufficient for a quantum computer to determine whether a bivalued function is constant or balanced. Contrary to this, a classical computer would require at least three evaluations of the same function. This enormous speed of quantum computer has its origin in quantum parallelism. Because quantum mechanics allows superposition of quantum states, any quantum operation can be performed on all these states simultaneously. Further, since the work of Turing, it was believed that in classical computation the answer to the question of whether any given problem could be solved in a time that was polynomial or greater than polynomial in the size of its inputs was independent of the physical apparatus used for the computation. But quantum computer can solve in polynomial time problems that have no polynomial time solution on any classical machine.

The role of energy resources is not very important in computer science. However there is an important relationship between energy and information processing. Landauer in 1961 [9] pointed out that erasing of information is necessarily a dissipative process. Erasing one bit of information dissipates $kT \ln 2$ amount of energy, where $k$ is the Boltzmann’s constant and $T$ is the temperature involved. This amount is orders of magnitude smaller than the energy dissipation by a computer and thus is negligible. But as computing hardware continues to shrink in size it may become difficult to ignore this small amount further. Charles Bennett in 1973 [10] pointed out that any computation can be performed using only reversible steps. Using reversible process, information can be erased without any energy cost. Since quantum dynamics is governed by unitary operators, all quantum mechanical processes are essentially reversible. Thus a quantum computer is not expected to dissipate energy. Here lies the importance of a quantum computer.
1.1.4 The quantum bit

The bit is the fundamental resource of classical computation and information. In analogy, quantum computation and information is built upon the concept of quantum bit or qubit. Any two state system can serve as a qubit. The states of the qubit are generally designated as $|0\rangle$ and $|1\rangle$, in correspondence with the 0 and 1 states of the classical bit. The most essential property of qubit which makes it different from a classical bit is the possibility of coherence and superposition. Apart from the states $|0\rangle$ and $|1\rangle$, a qubit can also be in a state of coherent superposition of them. Thus a general qubit state can be written as

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle ,$$  \hspace{1cm} (1.2)

where $\alpha$ and $\beta$ are complex numbers and $|\alpha|^2 + |\beta|^2 = 1$. The above state is called a coherent superposition state because $\alpha$ and $\beta$ have a constant phase relation. There is always a definite basis where the state of the qubit is well defined. In an incoherent mixture of states, it always remains a mixture irrespective of the basis. The general state of a qubit $|\psi\rangle$ can be visualized as a point in a unit three-dimensional sphere, known as Bloch sphere [see in Fig. 1.1] as

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle .$$  \hspace{1cm} (1.3)

The angles $\theta$, $\phi$ define a point on the Bloch sphere, $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. On measurement in $\{|0\rangle, |1\rangle\}$ basis, the states $|0\rangle$ and $|1\rangle$ are obtained with
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probabilities $\cos^2 \frac{\theta}{2}$ and $\sin^2 \frac{\theta}{2}$ respectively. The surface points of the Bloch sphere represent pure states and the other points represent mixed states. We will discuss the pure state and the mixed state in detail in a while.

The state of a qubit is a ray in the Hilbert space. The dimension of the Hilbert space is the number of basis states of the Hilbert space. For example, for a $N$-qubit system the dimension of the Hilbert space is $2^N$. The basis states of a two-qubit system are $\{|0,0\}, |0,1\}, |1,0\}, |1,1\}$. A general two-qubit state can thus be written as $|\psi\rangle = c_1|0,0\rangle + c_2|0,1\rangle + c_3|1,0\rangle + c_4|1,1\rangle$, where $c_i$'s are complex numbers with the normalization condition $\sum_{i=1}^{4} |c_i|^2 = 1$.

1.1.5 Density matrix formalism: Pure state vs mixed state

A given physical system is characterized by a state vector $|\psi(t)\rangle$ whose dynamics is governed by the Schrödinger equation as

$$i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H |\psi(t)\rangle.$$ (1.4)

Here $H$ is the total Hamiltonian operator for the system which includes all its interactions with external agencies. The interaction may be explicitly time dependent like the interaction of an atom with classical, monochromatic wave and therefore the Hamiltonian depends explicitly on time in general. The state vector $|\psi(t)\rangle$ can be expanded in terms of an orthonormal basis $\{|i\rangle\}$ as $|\psi(t)\rangle = \sum_i c_i(t) |i\rangle$, where $\sum_i |c_i(t)|^2 = 1$. This state is called a pure state [11]. The expectation value of an operator $A$ at the instant $t$ is given by

$$\langle A \rangle_t = \langle \psi(t)|A|\psi(t)\rangle = \sum_{i,j} c_i^*(t)c_j(t) A_{ij}.$$ (1.5)

where, $A_{ij} = \langle i|A|j\rangle$ are the matrix elements of the operator $A$ in the basis $\{|i\rangle, i = 1,2,..\}$. The coefficients $c_i^*(t)c_j(t)$ in the above sum can be interpreted as the matrix element of an operator $|\psi(t)\rangle \langle \psi(t)|$ between the states $|j\rangle$ and $|i\rangle$, i.e., $c_i^*(t)c_j(t) = \langle j|\psi(t)\rangle \langle \psi(t)|i\rangle$. The operator $\rho(t) = |\psi(t)\rangle \langle \psi(t)|$ is defined as the density operator of the system. The matrix formed by the matrix elements $\rho_{ij}(t)$ of the density operator is called the density matrix. Some of the important properties of the density operator are now in order: (1) $\text{Tr} \rho(t) = 1$, which indicates the conservation
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of probability. (2) The expectation value of any operator \( A \) in terms of \( \rho(t) \) is given by

\[
\langle A \rangle_t = \langle \psi(t)|A|\psi(t) \rangle = \text{Tr}[\rho(t)A] = \text{Tr}[\rho A].
\]  (1.6)

(3) \( \text{Tr}[\rho^2(t)] = 1 \) for a pure state. (4) \( \rho(t) \) is hermitian and positive definite.

In many physical situation of practical interest the state \( \psi(t) \) is not known; rather one may have an ensemble of such identical systems \( \psi_i(t) \). What one is left with is only a set of certain probabilities \( \{p_i\} \) to be in the state \( \psi_i(t) \). For example the state of radiation field inside a cavity which is in equilibrium with a finite temperature thermal reservoir, can be characterized only by a statistical distribution of photon numbers. Clearly the wave-function approach is not suitable here. The density operator of the system in this mixed state ensemble \( \{ \psi_i(t) \} \) can be written as

\[
\rho(t) = \sum_i p_i \psi_i(t) \psi_i^\dagger(t) = \sum_p p_p \rho_p.
\]

The expectation value of an operator \( A \) is still given by Eq. (1.6), but the average \( \langle A \rangle_t \) would now imply not only the quantum mechanical average but also an ensemble average in the mixed state. Further for the mixed state \( \text{Tr}[\rho(t)] = 1 \) with \( \text{Tr}[\rho(t)] = 1 \). It can be shown from Schrödinger equation that the density operator satisfies the following differential equation

\[
\dot{\rho}(t) = -\frac{i}{\hbar} [H, \rho].
\]  (1.8)

which is called the Liouville equation. Note that the Schrödinger equation is very difficult to work with for an ensemble of identical systems. In contrary, Eq. (1.8) is quite general in nature, because it can be applied in treating the system irrespective of its state. Further, it gives both the quantum-mechanical as well as statistical information about the system. In any physical system, there lies a spectrum of incoherent processes, like spontaneous emission of the atoms, decay of the radiation field from the cavity, etc. In that case the above Liouville equation is not quite realistic. One always has to incorporate these incoherent processes in this equation. Thus we obtain our working equation as

\[
\dot{\rho}(t) = -\frac{i}{\hbar} [H, \rho] + \mathcal{L} \rho. 
\]  (1.9)
where $\mathcal{L}$ represents the matrix containing the decay terms. The explicit form of this matrix can be derived using rigorous master equation formalism [12].

1.1.6 Entanglement

"...a correlation that is stronger than any classical correlation". J. Bell

The role of entanglement in quantum information processing is many-folded. It is nowadays viewed as a resource for certain tasks that can be performed faster or in a more secure way than classically. In 1935 E. Schrödinger, in his famous paper proposed the idea of entanglement, to explain the intriguing feature of composite system [13]. An entangled state can be defined as the state of a composite system which cannot be written as a direct product of the states of each subsystem. To understand entanglement in the qubit language, let us consider an unnormalized two-qubit state $|0,0\rangle + |1,0\rangle$. This can be rewritten as the tensor product of single qubit states as $(|0\rangle + |1\rangle) \otimes |0\rangle$. But a two-qubit state of the form

$$
(|0,1\rangle + |1,0\rangle) / \sqrt{2}
$$

(1.10)
cannot be separated into direct product of the individual qubit states, and therefore is called entangled (or nonseparable) state. Mathematically it can be understood in the following way: Any bipartite pure state with the subsystems $A$ and $B$ can be expanded via Schmidt decomposition as [7]

$$
|\psi\rangle_{AB} = \sum_{i,\mu} c_{i,\mu} |i\rangle_A |\mu\rangle_B = \sum_i \sqrt{p_i} |i\rangle_A |\hat{i}\rangle_B.
$$

(1.11)

where $|\hat{i}\rangle_B = \sum_{\mu} \frac{1}{\sqrt{p_i}} a_{i,\mu} |\mu\rangle_B$. The number of terms in the above decomposition is called the Schmidt number. If the Schmidt number is greater than one, then the state is said to be entangled, otherwise separable. Entanglement exploits the correlation between the subsystems. However correlations carried by entangled states are, in some sense, different from classical correlations. Classical correlations are restricted by Bell inequalities [14] whereas the entangled states may violate them. This is why the correlations contained in entangled states led to many new phenomena that never occur using classical correlation. It has the important paradoxical feature: "non-locality" [15]. To explain, the state (1.10) has
the interesting property that neither of the two qubits carries a definite value, but as the state of one qubit is measured, the measurement being completely random, the other one will be immediately found in a corresponding state. This is the conundrum of quantum non-locality as the two qubits could be separated by arbitrary distances at the time of measurement. The behavior of entangled states has been demonstrated in numerous experiments [16].

In 1964 Bell suggested the following two-particle entangled states [14]

\[
|\psi^+\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle + |1,1\rangle), \\
|\psi^-\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle - |1,1\rangle), \\
|\psi^+\rangle = \frac{1}{\sqrt{2}}(|0,1\rangle + |1,0\rangle), \\
|\psi^-\rangle = \frac{1}{\sqrt{2}}(|0,1\rangle - |1,0\rangle),
\]

called the Bell states, which together form a basis for two particle entangled states and play a very important role in quantum information. Note that the state \(|\psi^-\rangle\) is also known as the EPR state after the name of A. Einstein, B. Podolsky and N. Rosen. There are two inequivalent classes of three-particle entangled states: The GHZ state [17]

\[
|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0,0,0\rangle + |1,1,1\rangle),
\]

and the W state [18]

\[
|\text{W}\rangle = \frac{1}{\sqrt{3}}(|0,0,1\rangle + |0,1,0\rangle + |1,0,0\rangle).
\]

The entanglement in the first one is very weak in the sense, if one particle is lost, then all the entanglement disappears whereas the second one retains some entanglement even if one particle is lost [18]. In fact, the entanglement in W-state has the highest degree of endurance against qubit loss. There is another kind of entangled state, known as the two-qutrit entangled state [19]. When quantum systems are defined in three dimensional Hilbert space, they are known as 'qutrit' instead of qubit. Entangled states of two such qutrits are known as two-qutrit entangled states. Recently two-qutrit entangled states have found to be much useful in quantum communication due to their high loss resistance and in quantum cryptography [20, 21].
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So far we have discussed about the entanglement in pure states. To define mixed state entanglement we have to use the density matrix formalism. The definition of a mixed state \( \rho \) defined in terms of entanglement, is equivalent to the following mathematical characterization: \( \rho \) is separable if there exists \( p_k \geq 0 \) and \( \{ \rho_A^k \} \in H_A \) and \( \{ \rho_B^k \} \in H_B \) such that

\[
\rho = \sum_k p_k \rho_A^k \rho_B^k .
\]  

otherwise entangled. A particular useful family of mixed states is the so called Werner states [22], since Werner was the first one who introduced them. The quantification of entanglement, i.e., to determine whether a state is entangled or not, or what is the degree of entanglement is a very crucial and open question in the subject of quantum information. We will have a detail discussion on this issue in Chap. 5.

1.1.7 Quantum gates

A quantum logic gate is a unitary operator acting on the states of a certain sets of qubits. If the number of such qubit is \( N \), the quantum gate is represented by a \( 2^N \times 2^N \) matrix in the unitary group \( U(2^N) \). Qubits can be manipulated in a controlled way through this unitary operation \( U \) such that \( \psi \rightarrow U \psi \). It is thus a reversible gate and we can reverse the action, thereby recovering the initial quantum state from the final one. \( U \) can be decomposed as a product of gates belonging to a small set, the so called universal set of gates [23]. If we are able to perform the set of these gates, we will be able to perform any computation. There are many sets of universal gates, and they are all equivalent [24, 25]. The most convenient set is the one, that contains one two-qubit gate and and a set of single-qubit gates. Diagrammatically a quantum gate is represented by a "black box", wherein the operation takes place, and a number of input (output) lines equal to the number of qubits involved in the computation is used to wire up a set of gates.
Quantum NOT gate

The quantum NOT gate is a single-qubit gate, represented by the unitary operator $U_{\text{NOT}}$. This gate flips the state of a qubit as

$$
|0\rangle \xrightarrow{U_{\text{NOT}}} |1\rangle, \quad |1\rangle \xrightarrow{U_{\text{NOT}}} |0\rangle.
$$

$U_{\text{NOT}}$ can be represented by the following $2 \times 2$ matrix in the basis $\{0, 1\}$,

$$
U_{\text{NOT}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.
$$

Phase shift gate

This is another single-qubit gate defined by,

$$
|0\rangle \xrightarrow{U_{\varphi}} |0\rangle, \quad |1\rangle \xrightarrow{U_{\varphi} = e^{i\varphi}} |1\rangle.
$$

In matrix notation it is represented in the basis $\{0, 1\}$ as

$$
U_{\varphi} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}.
$$

The Hadamard gate

The Hadamard gate is the most useful and common single-qubit quantum gate, which doesn't have any counterpart in classical digital circuit. The corresponding unitary operator is represented as

$$
U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.
$$

It creates superposition while operating on a state of a qubit as

$$
|0\rangle \xrightarrow{U_H} (|0\rangle + |1\rangle)/\sqrt{2}, \quad \text{and} \quad |1\rangle \xrightarrow{U_H} (|0\rangle - |1\rangle)/\sqrt{2}.
$$

We show the block diagrams representing the above single-qubit gates in Fig. 1.2.
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\[ \begin{array}{ccc} 
|0\rangle & \times & |1\rangle \\
H & & U_H |0\rangle \\
\Phi & & U_\phi |0\rangle \\
|1\rangle & \times & |0\rangle \\
H & & U_H |1\rangle \\
\Phi & & U_\phi |1\rangle 
\end{array} \]

Figure 1.2: One qubit gates: (a) NOT gate, (b) Hadamard gate, (c) phase-shift gate

Controlled-NOT gate

The controlled-NOT (CNOT) gate is the most popular two-qubit gate and is equivalent to the XOR gate in classical digital circuit. It flips the second qubit (controlled bit) iff the first qubit (control bit) is in |1⟩ state. The gate is represented by the following unitary matrix

\[
U_{\text{CNOT}} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}.
\]  

written in the \{|00⟩, |01⟩, |10⟩, |11⟩\} computational basis. Under the operation of \(U_{\text{CNOT}}\), a general two-qubit state transforms as

\[
\alpha|0,0⟩ + \beta|0,1⟩ + \gamma|1,0⟩ + \delta|1,1⟩ \xrightarrow{U_{\text{CNOT}}} \alpha|0,0⟩ + \beta|0,1⟩ + \gamma|1,1⟩ + \delta|1,0⟩.
\]  

There is a three-qubit version of the CNOT gate called the CCNOT gate, also known as Toffoli gate. In this gate the CNOT operation occurs in the second and third qubit, iff the first qubit is in state |1⟩.
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\[ |x_1> \ \ (a) \ |x_1> \]
\[ x_2> \ \ |x_2\oplus x_1> \]
\[ x_2> \ \ e^{i\phi} \ |x_2> \]
\[ |x_1> \ \ (b) \ |x_1> \]
\[ x_2> \ \ |x_2\oplus x_1> \]
\[ x_2> \ \ \]
\[ |x_1> \ \ (c) \ |x_1> \]
\[ |x_2> \ \ |x_1> \]

Figure 1.3: Two-qubit gates: (a) CNOT gate, (b) Conditional-phase gate, (c) SWAP gate.

Conditional phase gate

Another common two-qubit gate is the conditional phase gate or a quantum phase gate (QPG) represented by the unitary matrix \( U_{\text{QPG}} \):

\[
U_{\text{QPG}} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & \text{e}^{i\phi}
\end{pmatrix}
\]  \hspace{1cm} (1.24)

written in the computational basis \{\ket{00}, \ket{01}, \ket{10}, \ket{11}\}. It shows that a relative phase \( \phi \) is introduced only if both the qubits are in \ket{1} state. For \( \phi = \pi \) it is an ideal QPG. One can realize the CNOT gate by Hadamard rotation of the second qubit before and after the ideal QPG operation.

SWAP gate

Another kind of two-qubit gate under the operation of which the states of the two qubits get interchanged can be represented by the following unitary matrix

\[
U_{\text{SWAP}} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]  \hspace{1cm} (1.25)
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The cavity-QED implementation of the CNOT, QPG, and SWAP gate operations will be discussed in Chap. 2.

1.1.8 Quantum algorithms

The aim of quantum algorithm is to simulate the classical circuits with minimum resources. Quantum parallelism is the fundamental feature of many quantum algorithms. This allows quantum computers to evaluate a function f(x) for many different values of x simultaneously. There are basically three classes of quantum algorithms which provide an advantage over known classical algorithms [1]. The first one is the quantum versions of Fourier transform, a tool which is widely used in classical algorithms. The Deutsch-Jozsa algorithm [26], Shor’s algorithms [6] for factorization and discrete logarithm are this type of algorithms. The purpose the Deutsch-Jozsa algorithm (DJA) is to determine whether a bivalent function of N binary variables is constant or a balanced one, only in a single function call, whereas classically it needs up to 2^{N-1} - 1 enquiries. Shor’s algorithm for factorization is the most celebrated quantum algorithm till date. Given a number N, it determines a number k (not equal to 1 or N) which divides N exactly. For any given N, Shor’s algorithm takes time which is polynomial in the number of digits of N, but classically the algorithm runs in a time of order exp(\log N + \log \log N + \epsilon). The second class is the quantum search algorithms, e.g., Grover’s search algorithm [27]. It shows that searching a specific state from an unsorted database of N states requires only \(O(\sqrt{N})\) steps whereas in classical methods it requires \(O(N)\) steps. The third class of algorithms is quantum simulation, whereby a quantum computer is used to simulate a quantum system. The DJA and its implementation will be discussed in detail in Chap. 7.

1.1.9 Quantum communication

Quantum communication, i.e., transfer of information is a very essential requirement in quantum information theory. There are various protocols nowadays to perform quantum communication. For example, “teleportation” of a quantum state refers to information transfer from a sender (Alice) to a receiver (Bob), where
Alice is able to communicate the unknown state of a given particle to Bob at another location without sending the particle itself. Bennett et al. [28] first proposed a scheme of quantum teleportation from Alice to Bob, where both of them share common EPR state of two particles. During their protocol, Alice performs a Bell state measurement (BSM) on the particle to be replicated and one of the EPR pair. Then she sends a classical information as an outcome of her measurement to Bob. According to this classical information, Bob performs an unitary operation on his particle to reconstruct the initial state of Alice. This efficient scheme clearly requires a classical channel between Alice and Bob and a BSM. There are other methods to transfer quantum information like "quantum networking" [29]. The basic idea of the quantum network is to transfer a quantum state from one node to another node with the help of a carrier (a quantum channel) such that it arrives intact. In between, one has to perform a process of quantum state transfer (QST) to transfer the state from one node to the carrier and again from the carrier to the destination node. Note that in the present case to transfer information from one place to other one need not perform any BSM. We will discuss about a cavity QED implementation of QST and quantum network in detail in Chap. 3.

1.1.10 Decoherence

The biggest obstacle in the way to realize a quantum computer is the problem of decoherence. Every system, in reality, interacts with the environment. The most striking feature of quantum systems is that, they can be in superposition state and hence show interference. As soon as the quantum systems interact with the environment, the superposition state is destroyed and the system loses information. The more components are involved in a quantum network, the more likely it is that quantum information will spread outside the quantum computer and be lost into the environment, thus spoiling the computation process. This process is called decoherence. To illustrate the origin of decoherence mechanism, let us consider a joint unitary time evolution operator for the qubit and the environment which acts as follows [30]

\[
|0angle|E\rangle \xrightarrow{U(\tau)} |0\rangle|E_0(\tau)\rangle, \quad |1\rangle|E\rangle \xrightarrow{U(\tau)} |1\rangle|E_1(\tau)\rangle.
\]

(1.26)
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Here the first ket is the state of the qubit and the second ket is the state of the environment. Now if the qubit is in a superposition state initially, then the unitary operator leads to an entangled state of the qubit and the environment given by

\[ (a_0|0) + a_1|1]\otimes |\mathcal{E}(t)\rangle = a_0|0\rangle |\mathcal{E}_0(t)\rangle + a_1|1\rangle |\mathcal{E}_1(t)\rangle. \]  

(1.27)

Decoherence occurs due to such entanglement. The non-unitarity of the process becomes obvious when we write down the reduced density matrix of the qubit of the above state as

\[ \rho_q(t) = \text{Tr}_E(\rho_{q+E}) = \begin{bmatrix} a_0^2 & a_0a_1^*\mathcal{E}_0\mathcal{E}_1^* \n a_0a_1\mathcal{E}_0^*\mathcal{E}_1 & a_1^2 \end{bmatrix} \]  

(1.28)

The off-diagonal terms of the density matrix (coherence) decay in time

\[ \langle \mathcal{E}_0(t)|\mathcal{E}_1(t)\rangle = e^{-\Gamma(t)}. \]  

(1.29)

where the specific form of \( \Gamma(t) \) will depend on the details of the coupling between the qubit and the environment. For more number qubits, the collective interaction of the qubits with the environment leads to faster decay rates, though for multiple qubits there also exists a subspace which is decoupled from the environment, called the decoherence free subspace (DFS).

1.1.11 Implementation

Till date it has been found extremely difficult and challenging to practically implement the quantum logics though there is quite a good amount of theoretical progress in the field of quantum computation and information processing. There are five basic requirements a physical system should satisfy for the implementation of quantum computation [31]:

1. A scalable physical system with well characterized qubits.

2. The ability to prepare a well defined input state.

3. Long relevant decoherence times, much longer than the required logic operation.
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5. A qubit-specific measurement capability.

There are a few physical systems that have emerged as promising tools to implement quantum computation. We summarize a few of them below.

**Nuclear Magnetic Resonance (NMR)**: In this case qubits are represented by nuclear spins within the same molecules and the manipulation takes place using the NMR technique (interaction of spins with external magnetic field). Though it is difficult to prepare the initial pure state and to address a single qubit, quantum logic with *maximum* number of qubits has been realized using these systems.

**Solid state systems**: In this case electrons are used as qubits. There are two types of solid state systems, namely, quantum dots and Cooper pairs, to implement quantum computation depending on the interaction between the electrons. Quantum dots are electrons in semiconductors interacting via controlled Coulomb interaction. Controlled Heisenberg coupling is used for coupled qubit operations. However uncontrolled distant charge motion is an important source of decoherence in this system. Secondly 'Cooper pairs' are electrons in superconductors interacting via phonons. The external magnetic field is employed for the coupled qubit operations. In this case decoherence arises from the spontaneous emission of electro-magnetic photons.

**Linear optical systems**: Optical photons are used as qubits which can be guided along long distance. Nonlinear optical media are used to mediate the interactions between them. However it is difficult to realize such a medium with low absorption loss.

**Quantum optical systems**: Quantum optical systems are very important as single qubit manipulation can be performed very efficiently in the laboratory. There are mainly two types of quantum optical systems, viz. trapped ions and neutral atoms. Though experiments using both these systems basically implement Jaynes-Cummings type Hamiltonians and thus the same dynamics (see next section for details), each technique has its own assets and drawbacks. Specially in ion-trap technique, electronic levels and vibrational modes of ions (phonon) are used as relevant qubit states. The manipulation of these states is performed using
laser pulse whereas, qubits interact with each other via phonon states. However these systems experience difficulties due to short lifetimes of the phonons and in preparing ions in motional ground states. On the other hand the interactions of neutral atoms with single or two-mode cavities are used to implement quantum logic. The interaction is governed by cavity-QED dynamics. Cavity-QED system, however, can decohere due to leakage of photons from the cavity and spontaneous decay of atomic states. In the present thesis we will mainly concentrate in quantum information processing with cavity-QED and neutral atoms and will discuss in detail about these systems in the next section. Note that both the above techniques (trapped ion and neutral atom) however lack scalability.

All the above techniques have their own characteristics in terms of qubit interactions and decoherence. We outline a comparison among these techniques in Table 1.1.

1.2 Quantum computation using cavity-QED

In this section we will discuss the basic tools of quantum computation using cavity-QED. We will also review some works on quantum computation and information processing using cavity-QED and neutral atoms.

1.2.1 Atom-field interaction in a cavity

The interaction of atom with fully quantized electro-magnetic field has been proved to be superior than the semiclassical model where the field is treated classically. The semiclassical model of atom-filed interaction fails to explain many experimentally verified effects such as atomic decay, Lamb shift, Casimir effects, etc. [11]. Spontaneous emission of atomic system, which is described phenomenologically in a semiclassical model, comes as a direct consequence of quantum theory. For a rigorous treatment of spontaneous atomic decay in free space, one needs to consider the interaction of atom with the vacuum modes of the universe [12]. The situation becomes simpler when the atom interacts with a cavity. In that case any one or two field modes are available for the interaction with the atom. In the simplest case where a two-level atom interacts with a single mode of the field is a
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Table 1.1: Comparison of different tools for implementation of quantum computation

<table>
<thead>
<tr>
<th>Qubits</th>
<th>Nuclear Magnetic Resonance (NMR)</th>
<th>Solid State Systems</th>
<th>Linear Optical Systems</th>
<th>Quantum Optical Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nuclear Spins</td>
<td>Quantum Dots</td>
<td>Cooper Pair</td>
<td>Photon</td>
</tr>
<tr>
<td>Interactions</td>
<td>Zeeman Interaction, J-coupling</td>
<td>Hystems</td>
<td>Josephson Interactions</td>
<td>JC Interaction</td>
</tr>
<tr>
<td>Qubit Control</td>
<td>Magnetic Field</td>
<td>Tunnel Junction</td>
<td>Magnetic Field,</td>
<td>Beam</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Electrostatic Gates</td>
<td>Beam</td>
</tr>
<tr>
<td>Interaction</td>
<td>1 μs-1 ms</td>
<td>10^{-13} s</td>
<td>100 ps</td>
<td>Passage time of photon</td>
</tr>
<tr>
<td>Source of Decoherence</td>
<td>Spin Relaxation</td>
<td>Distant Charge Motion</td>
<td>spontaneous emission of photons</td>
<td>Photon Decay</td>
</tr>
<tr>
<td>Time of Decoherence</td>
<td>10^{-2} s-10^8 s</td>
<td>100 fs-100 ps</td>
<td>1 μs</td>
<td>10^{-1} s</td>
</tr>
</tbody>
</table>

The model is known as Jaynes-Cummings (JC) model [33]. Even in this case the dynamics of the atom is quite different from that in semiclassical theory. In absence of decay the semiclassical theory predicts Rabi oscillations for the atomic inversion whereas the quantum theory (JC model) predicts certain collapse and revival phenomena arising due to quantum aspects of the field. In the next subsection we will discuss some of the properties of JC model and review some related experiments.
1.2.2 Jaynes-Cummings model

A two-level system interacting with a radiation field certainly provides a simplified quantum model for the interaction of radiation with matter. When a single atom interacts with a single mode cavity, then only a pair of atomic levels take part into the interaction with which the field mode frequency is closely resonant. The rest of the atomic level pairs are far off-resonant with the field mode frequency. The atomic transitions to those levels are virtually negligible and the whole atom can be practically regarded as a two-level system. A model where a two-level atom interacting with a single mode cavity under rotating wave approximation (RWA) was first studied by Jaynes and Cummings and hence is known as Jaynes-Cummings model [34]. Let us consider a two-level atom with $\sigma$ and $\bar{\sigma}$ as the excited and ground energy states respectively. The interaction of the radiation field $\vec{E}$ with a two-level atom can be described by the Hamiltonian in dipole approximation as

$$H = H_A + H_F \cdot \vec{d} \cdot \vec{E},$$

where $H_A$ and $H_F$ are the free Hamiltonians of the atom and the radiation field. $\vec{d}$ is the dipole-moment vector of the atom. For a single mode $H_F$ is given by

$$H_F = \hbar \omega \hat{a} \hat{a}^\dagger,$$

where $a$ and $a^\dagger$ are the creation and annihilation operators of the radiation field and we have omitted the zero-point energy term. The quantized electric-field for the atom at the origin can be expressed as

$$\tilde{E} = i \left( \frac{\hbar \omega}{2 \varepsilon_0 L^3} \right)^{1/2} \hat{\sigma} (\hat{a} - \hat{a}^\dagger).$$

Now using the closure relation $|e\rangle \langle e| + |g\rangle \langle g| = \hat{1}$, $H_A$ and $\tilde{d}$ can be written as

$$H_A = \langle |e\rangle \langle e| + |g\rangle \langle g| \rangle H_A \langle |e\rangle \langle e| + |g\rangle \langle g| \rangle$$

and

$$\tilde{d} = \left[ d_{eg} |e\rangle \langle g| + d_{ge} |g\rangle \langle e| \right],$$

where $E_i$ ($i = e, g$) are the energies of the excited and ground state respectively and $d_{ij}$ = $(i|d|j)$ ($i, j = e, g$) are the electric dipole matrix elements. The diagonal
elements $\tilde{d}_{ii}$ of the matrix elements vanish due to parity. We note that for a two-level atom $\tilde{d}_{eg} = \tilde{d}_{ge}$. Thus the total Hamiltonian can be written in a simplified form as

$$H = \hbar \omega \hat{\alpha} + \frac{\hbar \omega_{eg}}{2} \langle \langle e \rangle|\langle e| - \langle g \rangle|\langle g| \rangle - i \hbar g \langle \langle g \rangle|\langle g| \rangle \left( \hat{a} \hat{a}^\dagger - \hat{a}^\dagger \hat{a} \right). \quad [1.34]$$

where $\omega_{eg}$ is the transition frequency of the atom given by $\omega_{eg} = (E_m - E_g) \hbar$ and is very close to the frequency $\omega$ of the cavity. $g$ is the atom-cavity coupling strength and is given by

$$g = \left( \frac{\hbar \omega}{2 \epsilon_b L^3} \right)^{\frac{1}{2}} \tilde{d}_{eg, \hat{\sigma}}. \quad [1.35]$$

In the interaction picture the Hamiltonian (1.34) takes the form [36]

$$H = -i \hbar g \langle \langle g \rangle|\langle g| \rangle \hat{a} \hat{a}^\dagger \hat{\alpha} - \hat{a}^\dagger \hat{a} \hat{\alpha} e^{i \Delta t} \hat{\alpha}^\dagger \hat{\alpha} \hat{\alpha}^{\dagger} \hat{\alpha}. \quad [1.36]$$

where we have used the rotating wave approximation and $\Delta \ll \omega$. The state vector $|\psi(t)\rangle$ of the atom-cavity system can be written as a linear combination of the states $|e,n\rangle$ and $|g,n\rangle$ where $n$ is the number of photons. The state vector is therefore given by

$$|\psi(t)\rangle = \sum_n (c_{e,n}(t)|e,n\rangle + c_{g,n}(t)|g,n\rangle). \quad [1.37]$$

Since the Hamiltonian (1.36) couples only the state $|e,n\rangle$ and $|g,n+1\rangle$, we will consider the evolution of the coefficients $c_{e,n}$ and $c_{g,n+1}$ only. From Schrödinger equation, one can find the following equations for the amplitudes $c_{e,n}$ and $c_{g,n+1}$ as

$$\dot{c}_{e,n} = -g^* \sqrt{n} + \frac{1}{\Delta t} c_{g,n+1}, \quad [1.38a]$$

$$\dot{c}_{g,n+1} = g \sqrt{n} + \frac{1}{\Delta t} c_{e,n}. \quad [1.38b]$$

If the atom initially is in the state $|e,n\rangle$ then the probability $P_{e,n}(t)$ that the atom remains in the initial excited state at time $t$ after the interaction is found to be

$$P_{e,n}(t) = \cos^2 \left( \frac{\Omega_n t}{2} \right) + \frac{\Omega^2}{\Omega_n^2} \sin^2 \left( \frac{\Omega_n t}{2} \right), \quad [1.39]$$

where $\Omega_n = \sqrt{\Delta^2 + 4g^2(n+1)}$. Since the probability is conserved, the probability $P_{g,n+1}(t)$ that the atom goes to the ground state with the emission of one photon in the cavity is given by $1 - P_{e,n}(t)$. The solution (1.39) shows that the excited
state probability oscillates with an angular frequency $2g\sqrt{n+1}$ for the resonant case $\Delta = 0$. This is called the Rabi oscillation due to historical reason and the frequency is called the Rabi frequency. As $\Delta$ increases, the frequency of oscillation increases but the amplitude decreases. It is very interesting to note that even for an initial vacuum state $(n = 0)$ of the cavity and $\Delta = 0$, the Rabi oscillation takes place with a frequency $2g$. This is the simplest example of spontaneous emission of an atom inside the cavity in which the spontaneously emitted photon contributes to the single mode of the cavity. The spectrum of spontaneous emission was shown to exhibit a doublet rather than a single Lorentzian line as of a free space atom. The splitting of this single line is called the vacuum field Rabi splitting [38]. The emission spectrum is also very sensitive to the number of atoms interacting with the cavity and many new lines were predicted even for two atoms [39]. Eberly et al. [40] showed that for an initial coherent state of the field in the cavity, the spontaneous and stimulated processes in the atomic emission lead to periodic collapse and revival of the atomic inversion. Later this was verified experimentally in Rydberg atomic transitions by Rempe et al. [41].

1.2.3 Implementation of quantum computation using neutral atoms and cavity-QED

There are two different kinds of systems in the present context:

1. Microwave cavity and atom in the Rydberg levels

2. Optical cavity and atom in its ground levels

Microwave cavity and Rydberg atoms

The pioneering works with Rydberg atoms and microwave cavity have been performed by Haroche and coworkers. They use an experimental setup where a two-level atom in Rydberg states interacts with a single-mode microwave cavity [42]. Circular Rydberg states correspond to large principal number and maximum orbital and magnetic quantum number. The experiments of Haroche and

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1I. I. Rabi (1937) discovered that the probability of a spin-1/2 particle oscillates between its two spin states with the radio-frequency of the applied magnetic field [37].
coworkers involve three circular Rydberg states with principal quantum numbers 51, 50, and 49, respectively, with atomic transition frequency \( \approx 50 \text{GHz} \). The radiative life-time of these levels are of the order of 30 ms. Their cavity is an open Febry-Perot resonator which has a large \( Q \) factor \( (Q = 3 \times 10^4) \), corresponding to a photon storage time \( T_r = 1 \text{ms} \) \( (Q = 2\pi T_r) \). The resonant interaction of the atom with the cavity is governed by the JC Hamiltonian (1.36). The interaction time is controlled by controlling the atomic velocity through the cavity. If the initial state of the atom-cavity system is \(|e, 0\rangle \) [i.e., the atom is initially in state \( e \) and there is no photon in the cavity], then the state of the system at time \( t \) can be written as

\[
|\psi(t)\rangle = \cos(gt)|e, 0\rangle + \sin(gt)|g, 1\rangle,
\]

where \( g = d\alpha_0/h \) is the atom-field coupling constant [see Eq. (1.35)], \( d \) and \( \alpha_0 \) being the dipole moment between the levels \( e \) and \( g \) and cavity field amplitude, respectively. Similarly if the atom-cavity system is initially prepared in \( g, 0 \) state, then at time \( t \) the state of the system will be

\[
|\psi_g(t)\rangle = \cos(gt)|g, 1\rangle + \sin(gt)|e, 0\rangle,
\]

Now if one chooses the interaction time \( T \) in such a way that \( 2\sqrt{T} = 2 \) then, the atom-cavity system initially in state \(|g, 1\rangle\) will be prepared in a state

\[
|\psi_T\rangle = \frac{1}{\sqrt{2}}(|g, 1\rangle - |e, 0\rangle).
\]

This is an EPR state in the atom-cavity basis. Similarly if the atom experiences a \( \pi \)-pulse \( (2gT = \pi) \) while passing through the cavity, then the atom-cavity system evolves as \(|e, 0\rangle \rightarrow |g, 1\rangle \) and \(|g, 1\rangle \rightarrow -|e, 0\rangle \). This refers to swapping of atom and cavity excitations. A global phase shift of \( \pi \) to the atom-cavity state can be incorporated if one chooses \( 2gT = 2\pi \) so that \(|e, 0\rangle \rightarrow -|e, 0\rangle, \ |g, 1\rangle \rightarrow -|g, 1\rangle \). We should further mention that any two-qubit (atom and cavity) operation can be implemented using the JC Hamiltonian, whereas the single qubit operation for the Rydberg atoms can be performed using classical Ramsey pulse [42].

In a typical microwave cavity experiment the velocity of the Rydberg atoms are chosen to be \( v = 500 \text{m/s} \), such that the interaction time of the atom with the cavity of length \( l = 6\text{mm} \) becomes \( T \sim 10\mu \text{s} \). Thus the atom-field interaction, as described above, can be performed well within the cavity decoherence.
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time. Further the life-time of the Rydberg levels $T_{sp} \sim 30$ ms. Thus on the course of interaction the probability of decay due to spontaneous emission from these levels is much less. Thus the time scale for the cavity QED experiments using microwave cavity and Rydberg atoms for quantum computation reads as follows [see Table 1.1]:

$$T \ll T_r < T_{sp}.$$  \hspace{1cm} (1.43)

Haroche and his coworkers have prepared different kinds of entangled states in the atomic basis, namely EPR [43] states and GHZ [44] states, by sequentially addressing the atoms by the cavity. For example, let us assume that the first atom in state $|e\rangle$ state passes through the cavity in vacuum and experiences a $\pi$-pulse, such that the interaction (1.42) occurs. Next, after this atom comes out of the cavity, a second atom in state $|g\rangle$ enters into the cavity and the atomic cavity system is prepared in a state $(|e_1, g_2, 0\rangle - |g_1, e_2, 1\rangle)/\sqrt{2}$. Now if the second atom experiences a $\pi$-pulse, then, the final state of the cavity and two atoms becomes $$(|e_1, g_2\rangle - |g_1, e_2\rangle)/\sqrt{2} \otimes |0\rangle.$$ Thus two atoms are prepared in an EPR state via sequential interaction with the cavity. In a similar fashion, three atoms can be prepared in a GHZ state in properly chosen basis [44] by their sequential interaction with the cavity and using Ramsey pulses.

It has been further shown that quantum phase gate between atom and cavity state can be realized using $2\pi$-pulse technique [45]. Microwave cavity QED with flying atoms can be used also for single-photon quantum non demolition (QND) detection [46], as well as for preparation of Schrödinger cat like state in cavity basis [47]. Moreover entangled state of two modes of a microwave cavity has been prepared using light-shift technique [48].

Optical cavity and atom in its ground states

Next we will discuss the interaction a neutral atom with a high-Q optical cavity. Note that the advantage of working with the optical cavity is that one need not work with the Rydberg states of atom. Rather one can work with the ground states of atom, i.e., one can choose the atomic ground states as the relevant qubit states. Because the atomic ground states are radiatively stable, so in principle the
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Figure 1.4: A typical level configuration showing a three-level atom in A-configuration with \(|g\rangle\) and \(|f\rangle\) as the ground sublevels dipole coupled with the excited state \(|e\rangle\). The atom is interacting with a bimodal cavity with modes \(a\) and \(b\) and \(g_{1,2}\) are the atom-cavity coupling constants for the two modes respectively. \(\Delta_{1,2}\) are the detunings of these modes from the corresponding atomic transition.

The problem of decoherence due to spontaneous emission is entirely ruled out. The present thesis will focus on three-level atoms in A-configuration, with two ground states \(|g\rangle\) and \(|f\rangle\) [see Fig. 1.4] each being dipole-coupled with the common excited state \(|e\rangle\). These two ground states will be considered as qubit states of the atomic qubit. The atoms in this configuration interact with two modes of a bimodal cavity. This interaction can be manipulated in such a way, that the effective Hilbert space will consist of only the atomic ground states and the cavity modes, and hence one can overcome the problem of spontaneous emission from the excited state of the atom.

In this thesis I will discuss two different ways of eliminating excited state from the working subspace in a \(\Lambda\) type atom. In the first case we will make use of the dispersive interaction of the atoms with the cavity, i.e., we will work in the regime of large detuning of the cavity mode frequency from the atomic transition. In this limit the excited state is hardly populated during the evolution, and thus, one can adiabatically eliminate this level from the working subspace. Due to large detuning the ground states experience Stark shifts which play a crucial role in the evolution of the atom-cavity system. With the aid of this Stark shift we will show how to implement different kinds of two-qubit logic gates, namely QPG, CNOT gate, SWAP gate etc. Also we propose a simple and efficient model for quantum state transfer, quantum networking, and quantum memory.

In the second case we will consider the resonant interaction of the atoms with
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the cavity. We will use the idea of a very unique method, named as stimulated Raman adiabatic passage (STIRAP) to eliminate the excited level $|e\rangle$ from the working subspace. In the next subsection we will discuss this method in detail.

1.2.4 STIRAP

Excitation of atoms and transfer of population from one level to another preselected level has been studied since long ago. Let us consider a three-level atom in $A$ configuration with two ground states $|g\rangle$ and $|f\rangle$ and the excited state $|e\rangle$. One wishes to transfer the population from the level $|g\rangle$ to $|f\rangle$ which are dipole forbidden. One obvious way to doing this is to coherently pump the population from $|g\rangle$ to $|e\rangle$ by applying a pump field $\Omega_P$ [see Fig. 1.5(a)]. The level $|e\rangle$ is populated via spontaneous emission form the level $|g\rangle$. This method is called optical pumping. However this method lacks selectivity, as this procedure populates a statistical mixture of possible final states, not a single state. The selectivity can be increased by using coherent field (Stokes field) in the $|e\rangle \rightarrow |f\rangle$ transition [see Fig. 1.5(b)] following the pump field in $|g\rangle \rightarrow |e\rangle$ transition. In this method called

![Diagram showing the optical pumping process to transfer population from the level $|g\rangle$ to $|f\rangle$. (b) A better method to increase the selectivity of the final state where a Stokes pulse $\Omega_S$ is applied in the $|e\rangle \rightarrow |f\rangle$ transition.](image)

stimulated emission pumping (SEP) [49], the transfer efficiency can be at most 25% in strong field regime. If both the fields are applied simultaneously, then this can be enhanced upto 33%. Efficiency of population transfer in SEP method can be increased upto 100% by using pulsed laser for excitation. The pulsed laser with effective pulse area equal to odd multiple of $\pi$ (a "$\pi$"-pulse) first excites the
atom from the level $|g\rangle$ to $|e\rangle$ with 100% efficiency (using Rabi oscillation technique). Then a similar Stokes $\pi$-pulse follows the pump pulse and populates the level $|f\rangle$ with 100% efficiency. However in this method the Stokes pulse has to be applied before the excited level spontaneously decays to other levels. Also the success rate depends upon the effective pulse area. This constraint can be overcome in a method called STIRAP [50, 51], which is at first glance counter-intuitive [52] in the sense that the Stokes pulse precedes the pump pulse. It is the most efficient population transfer process for a typical stimulated Raman excitation scheme (as shown in Fig. 1.5(b)). Why it is called an adiabatic passage technique will be clear from the following discussion.

The basic idea of counter-intuitive sequence of pulses was first proposed by Oreg et al. [53] where they have shown how to transfer population between the dipole forbidden levels in a multilevel system in large detuning regime. Their idea of counterintuitively ordered pulses has been implemented in the method of STIRAP [50].

To describe the basic mechanism of the process, let us write down the Hamiltonian under rotating wave approximation which describes the coupling of three states by two coherent radiation fields (pump field and Stokes field) [see Fig. 1.5(b)] as

$$ H(t) = \hbar \begin{pmatrix} 0 & \Omega_p(t) & 0 \\ \Omega_p(t) & \Delta_1 & \Omega_S(t) \\ 0 & \Omega_S(t) & (\Delta_1 - \Delta_2) \end{pmatrix} $$

where $\Omega_p(t)$ and $\Omega_S(t)$ are the time-dependent Rabi frequencies of the pump and the Stokes pulses, $\Delta_1$ and $\Delta_2$ are their respective detunings. The population transfer mechanism in STIRAP is easily understood in the Hilbert space consisting of the adiabatic states, which are the instantaneous eigenstates of the Hamiltonian $H(t)$. In Raman resonance ($\Delta_1 = \Delta_2 = \Delta$), the three eigenvalues of this Hamiltonian can be written as

$$ \omega_0 = 0; \quad \omega_{\pm} = \Delta^2 \pm \sqrt{\Delta^2 + \Omega_p^2 + \Omega_S^2}.$$ (1.45)

Note that one of the eigenvalues of the Hamiltonian is zero. The corresponding
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eigenstates which are called adiabatic states are as follows:

\[ |\psi_0\rangle = \cos \Theta |g\rangle + \sin \Theta |f\rangle, \]
\[ |\psi_+\rangle = \sin \Theta \sin \Phi |g\rangle - \cos \Theta |e\rangle - \cos \Theta \sin \Phi |f\rangle, \]
\[ |\psi_-\rangle = \sin \Theta \cos \Phi |g\rangle - \sin \Phi |e\rangle - \cos \Phi |f\rangle, \]

where \( \Theta \) is the time varying mixing angle and is defined by

\[ \tan \Theta = \frac{\Omega_f(t)}{\Omega_g(t)}. \]

and the angle \( \Phi \) is a known function of \( \Omega_p \), \( \Omega_s \), and \( \lambda \). The adiabatic state associated with the null eigenvalue is of particular importance. Since the time dependent Hamiltonian of the system remains in the state \( |\psi_0\rangle \) unlike the other two adiabatic states \( |\psi_\pm\rangle \), the system remains in the state \( |\psi_0\rangle \) there is no change in the population and it acts like a population trapping state [54]. So this state is an appropriate vehicle for transferring population from one level to another level in either of the states \( |\psi_{\pm}\rangle \). This can be utilized in minimizing the adiabatic condition during the evolution.

For any atomic system and a corresponding time dependent Hamiltonian which is smoothly varying, the adiabatic evolution implies that states \( |\psi_{\pm}\rangle \) remain in one eigenstate of \( H(t) \) for all the times [55] if the pulses \( \Omega_p(t) \) and \( \Omega_s(t) \) are applied in a counter intuitive sequence. The system is automatically prepared in the state \( |\psi_0\rangle \). To avoid any transition between the states \( |\psi_0\rangle \) and \( |\psi_\pm\rangle \) the matrix element \( \langle \psi_\pm | \psi_0 \rangle \) should be much smaller than the field induced splitting \( \omega_\pm - \omega_0 \) of the energies of these states, i.e.,

\[ |\psi_\pm - \psi_0| \ll \omega_\pm - \omega_0. \]

This adiabaticity condition may be maintained if \( \Omega_p(t) \) and \( \Omega_s(t) \) are sufficiently smooth.

Under all the above conditions the state vector \( \Psi(t) \) of the system is tied to the zero-eigenvalue adiabatic state \( |\psi_0\rangle \). For the counter-intuitive pulse ordering, the relations \( \Omega_p(t)/\Omega_s(t) \to 0 \) and \( \Omega_p(t)/\Omega_s(t) \to \infty \) apply. Hence as time...
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Figure 1.6: (a) The counter-intuitive sequence of the pump and the Stokes pulse. (b) Time evolution of the mixing angle $\Theta$. (c) Transfer of population from the state $|g\rangle$ to $|f\rangle$ as an effect of the counter-intuitive pulse sequence.

Consequently the state $|\psi_0(t)\rangle$ evolves from the initial state $\langle g | |g\rangle$ to a superposition of states $|g\rangle$ and $|f\rangle$ at intermediate times and finally to the target state $|f\rangle$ at the end of the interaction. The system vector $\Psi(t)$ adiabatically evolves from the initial state $|\psi_0(t)\rangle$ of the adiabatic state $|\psi_0(t)\rangle$. The temporal profile of the pulses, time evolution of the mixing angle and the population in the levels $|g\rangle$ and $|f\rangle$ have been shown in Fig. 1.6.

Pellizzari et al. [56] first used the idea of STIRAP in a model where three-level atoms in $\Lambda$ configuration are interacting with a common single-mode high-$Q$ optical cavity and two externally applied laser pulses. In fact this was the first example of a cavity QED model for a quantum computer. The interaction between the atoms is mediated via the cavity. The atoms get coupled due to exchange of photons between them using the adiabatic passage technique. This allows entanglement between pairs of atoms. The details about the scheme of Ref. [56] will be provided in Chap. 6 in a more proper context. Rempe and coworkers have been doing nice experiments using neutral atoms and high-$Q$ optical cavity. They have demonstrated the STIRAP technique with a three-level atom in $\Lambda$ configuration interacting with the vacuum field of a high-$Q$ optical cavity [57]. They have also observed the cavity mediated long-range force between the strongly coupled atoms [58]. Recently they have prepared a 'single-photon source' on demand using the adiabatic passage technique [59]. This single-photon source has immense importance in the area of quantum information. Some typical parameters used in
their experiments are: the cavity waist is 35\(\mu\)m, atom-cavity coupling coefficient \(g = 2\pi \times 4.5\text{MHz}\), cavity decay constant \(\kappa = 2\pi \times 1.25\text{MHz}\) corresponding to a photon storage time \(T_r = 0.1\mu\text{s}\) (\(\kappa = 1/T_r\)).

In this thesis we will consider the interaction of three-level atoms with high-Q bimodal optical cavity. Using the idea of STIRAP, we show how different kinds of two-qubit, three-qubit, and even two-qutrit entangled states can be prepared. Using the adiabatic passage technique, we study an important entropic inequality for any three-particle system and propose a measure of entanglement in a specific four-particle entangled state. We further propose a method for collective population transfer of many atoms simultaneously which could be called as collective-STIRAP.

1.3 Outline of the thesis

In the present chapter we provide a brief introduction to the subject of quantum computation and information. We have discussed some basic tools which will be used in the later chapters of the thesis. In chapter 2 and chapter 3, we discuss a model where a three-level atom interacts with a bimodal optical cavity dispersively. Specifically, in chapter 2, we show the performance of different kinds of two-qubit logic gates, whereas, in chapter 3, we show an efficient scheme for quantum state transfer, quantum network and quantum memory. In chapter 4, 5, and 6, we discuss the resonant interaction of a number of atoms in \(\Lambda\) configuration with a single bimodal optical cavity. We show how to prepare different kinds of two-qubit, three-qubit, as well as two-qutrit entangled states in chapter 4. In chapter 5 we study the strong subadditivity inequality for any trio of quantum system and propose a measurement of a four particle entangled state under consideration. In chapter 6 we propose a collective population transfer method between the ground states of \(\Lambda\) type atoms. In chapter 7, we consider a completely different optical model where an atomic ensemble consisting of four-level atoms interacts dispersively with a freely propagating photon. Using this model we propose a simple implementation of the Deutsch-Jozsa algorithm.