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The experimental study of low frequency instabilities in toroidal plasma has been conducted in the 'BETA' device. The device has been designed to study basic plasma phenomena in pure toroidal magnetized plasma. A view of the device along with its sub-system is shown in Figure 2-1.

Figure 2-1 A view of BETA machine along with its sub-system.
Chapter 2 Experimental setup, diagnostics and data analysis.

The device consists of few major sub-systems and may be classified as vacuum system, magnetic field system (toroidal field coils (TF) and vertical field coils (VF)) and supporting structures. The details of the device is already described elsewhere (Bora, 1989). However, for the sake of completeness, the striking features of the machine are described in brief in section 1. Different methods are used to form plasma in this device. Microwave produced plasma and filament produced plasma are mainly studied and are described in detail in section 2. Major diagnostics, which are used in the experiment, are Langmuir probes and probe systems, as they are well known and reliable for the measurements of plasma parameter in low temperature laboratory plasmas. The details of the diagnostics and signal processing are described in section 3. To characterize different microwave components that are used in the experiment, we have used various instruments (like spectrum analyzer and vector network analyzer). The details of the instrument and measurements obtained from it are discussed in section 4. Standard software’s from Ansoft HFSS (High Frequency Structure Simulation) is used to analyze various microwave components used in our experiment. The components are designed and analyzed using the software and the results are compared with the experiment measurements. These results are discussed in detail in section 5. Once the experiment data is acquired it is analyzed using standard data analysis techniques. These techniques are discussed in section 6.

1 BETA Device

The experiments are carried out in BETA device, which is able to produce, in the steady state condition, a toroidal magnetized plasma with no toroidal current. The device consists of few major subsystems and are discussed as follows.

Vacuum system

Vacuum system consists of the toroidal vacuum vessel connected to two pumping lines. Its major radius is 45 cm and minor radius is 15 cm. It is made from four quadrants of stainless steel 304 elbows of circular cross section. The toroidal vacuum vessel has a wall
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thickness of 6mm. The machine has twelve 20 cm diameter radial circular ports and twenty 10 cm diameter vertical circular port. Eight such ports are located on the top of the machine whereas twelve are located on the bottom side of the machine. In addition there are eight smaller ports of diameter 5 cm, located at the topside of the vessel, two in each quadrant. Viton ‘O’ rings are used as sealant to make the four quadrants leak tight and vacuum compatible. Vacuum is maintained in the vessel with the help of two-diffstak pumps, connected to two radial ports situated 180° apart. The pumping capacity of these pumps is 2000 litres/sec. A rotary pump with a pumping speed of 10,000 litres second backs the diffusion pump. With the above mentioned pumping system, a base pressure of \(10^{-6}\) Torr. is easily maintained before the experiment. The typical vacuum vessel parameter of the machine is shown in Table 2-1.

<table>
<thead>
<tr>
<th>Vacuum Vessel Parameter</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius of the vacuum vessel</td>
<td>45 cm</td>
</tr>
<tr>
<td>Minor radius of the vacuum vessel</td>
<td>15 cm</td>
</tr>
<tr>
<td>Number of quadrants</td>
<td>4</td>
</tr>
<tr>
<td>No. of ports</td>
<td>40</td>
</tr>
<tr>
<td>Vessel material</td>
<td>Stainless steel 304</td>
</tr>
<tr>
<td>Base pressure</td>
<td>(10^{-8}) Torr</td>
</tr>
<tr>
<td>Working pressure</td>
<td>(10^{-4}) Torr - (10^{-5}) Torr</td>
</tr>
<tr>
<td>Working gas</td>
<td>Hydrogen and Argon</td>
</tr>
</tbody>
</table>

Table 2-1 Typical vacuum vessel parameter of BETA machine

**Magnetic field system**
The magnetic field system comprises of toroidal field (TF) coils and vertical field (VF) coils system. Both the system are discussed in brief as follows:

**Toroidal field coils system**
Sixteen toroidal field coils are used to produce toroidal magnetic field. The toroidal field coils are designed to produce 5 kG of toroidal magnetic field at minor axis \((R = 45\text{cm})\). The toroidal field coils are of picture frame shape with square aperture of 50 cm by 50
cm and have a major radius of 50 cm. It is to be noted that the major axis of the coils and the vacuum vessel do not coincide and are shifted by 5 cm. This is done to obtain minimum ripple inside the torus. Each coil consists of three turns and they are connected in series. Each layer is insulated from the other by a fiberglass sheet. The three turns of the coil along with interlayer fiberglass insulation are held together by insulating steel bolts. The main parameter of toroidal field coils of BETA machine is shown in Table 2-2. In order to cool the coils, a copper tube is soldered to the inner side of each turn of the coils.

<table>
<thead>
<tr>
<th>Major radius</th>
<th>50 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of coils</td>
<td>16</td>
</tr>
<tr>
<td>Width of each leg</td>
<td>5 cm</td>
</tr>
<tr>
<td>Radial aperture</td>
<td>50 cm</td>
</tr>
<tr>
<td>Vertical aperture</td>
<td>50 cm</td>
</tr>
<tr>
<td>Outer vertical dimension</td>
<td>60 cm</td>
</tr>
<tr>
<td>Outer horizontal dimension</td>
<td>60 cm</td>
</tr>
<tr>
<td>Thickness of each turn</td>
<td>1 cm</td>
</tr>
<tr>
<td>Inner turn insulation thickness</td>
<td>0.1 cm</td>
</tr>
<tr>
<td>Total outer dimension of the coil</td>
<td>60.2 cm x 60.2 cm x 3.4 cm</td>
</tr>
</tbody>
</table>

Table 2-2 Typical parameter of toroidal field coil of BETA machine

<table>
<thead>
<tr>
<th>Coolant</th>
<th>Chilled water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coolant temperature</td>
<td>15 °C</td>
</tr>
<tr>
<td>Coolant temperature at outlet</td>
<td>31 °C</td>
</tr>
<tr>
<td>Total flow rate</td>
<td>15 lpm</td>
</tr>
<tr>
<td>Number of cooling path</td>
<td>4</td>
</tr>
<tr>
<td>Total pressure head</td>
<td>3 atm</td>
</tr>
</tbody>
</table>

Table 2-3 Typical parameter of cooling unit of BETA machine

The temperatures of the coils are maintained at 25 °C during the experiment by circulating chilled demineralised water through these tubes. The temperature rise of coils
when 5 kA of current is passed through the coils for a duration of one second is estimated to be 10 °C. The typical cooling parameter is shown in Table 2-3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance</td>
<td>10 mΩ</td>
</tr>
<tr>
<td>Inductor</td>
<td>0.38 mH</td>
</tr>
<tr>
<td>L/R time constant</td>
<td>38 msec</td>
</tr>
<tr>
<td>Stored energy</td>
<td>150 kJ max</td>
</tr>
<tr>
<td>Peak current</td>
<td>26 kA</td>
</tr>
<tr>
<td>Voltage drop</td>
<td>4 kV</td>
</tr>
<tr>
<td>Magnetic field at axis</td>
<td>1.2 kG</td>
</tr>
</tbody>
</table>

Table 2-4 Typical electrical parameter of toroidal field coil of BETA machine

Toroidal field is produced using a DC power supply. The toroidal field power supply can be operated in continuous mode below 1 kA. If the temperature rise is above 40 °C, then the temperature sensors mounted on each of the coil trips off the toroidal field power supply. In a pulse mode, the toroidal field current rises to a preset value within 50 msec and remains constant upto 1.2 seconds. The ripple in toroidal field current is about 2% for currents above 1 kA. The main electrical parameter for the toroidal magnetic field coils is shown in Table 2-4.

**Vertical field coils system**

The vertical field is produced by two coils with ten turns each placed at a distance of 60 cm from the equatorial plane. The radius of the coils is 115 cm. These coils have been used to produce a magnetic field perpendicular to the toroidal magnetic field. This produces a small shear in the field lines, which can modify the levels of fluctuations. With the help of this vertical magnetic field we could open or close the toroidal magnetic field lines. A small component of vertical magnetic field when added to the toroidal magnetic field, supported encircling of magnetic filed lines with different number of turns within the vacuum vessel. With the help of this vertical magnetic field coil, we would show later in chapter 4, how we exploited this experimental set-up in supporting toroidal plasma with different parallel wave number. To generate a magnetic field of 3 Gauss at equatorial plane, a current of 25 Amp. is required to pass through the coils. The main parameter for the vertical magnetic field coils is shown in Table 2-5.
Supporting structure
As mentioned in the last section, the vessel is a metallic torus made out of SS304 and dynamic forces act on the coils. To support the vessel weight and transfer these dynamic forces, supporting structure has been constructed. The supporting structure for the vacuum vessel and toroidal field coils has been constructed in four quadrants. It is made out of aluminium. One of the quadrants is fixed while the other quadrants can be moved with reference to the fixed quadrant. A buckling cylinder with toroidal discontinuity is placed at the center, concentric to the major axis on a stationary table. Stainless steel channel made out of three 'L' sections is mounted on the top of the aluminium structure to support the toroidal field coils. The toroidal field coils are supported on top by similar structure. One of them is connected to an insulated central disc placed on top of the buckling cylinder and the other end (outer end) is connected to a octagonal frame of stainless steel strips. Cross bars are used to resist the torque acting on the structure due to out of plane forces. The toroidal field coils lean against the buckling cylinder. The buckling cylinder is made out of stainless steel with a toroidal electrical break in it.
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2 Plasma sources

In BETA device, mostly plasma is produced using hot cathode discharge method using a filament. In our experiment, we have also formed the plasma in BETA device using microwave source. Both the methods of plasma production are discussed in the following sub-sections.

Filament produced plasma

Hot cathode discharge is used to produce the plasma in the device. Tungsten filament of 2 mm diameter is placed vertically at the center on the minor axis. The filament is heated to incandescence by passing about 140 Amperes of DC current. The filament is negatively biased to about 110 V with respect to the BETA vessel, which is grounded. A circular limiter of radius 9 cm is placed within the vacuum vessel, such that its center coincides with the minor axis of the device and is grounded. The limiter provides an equilibrium to the toroidal plasma.

![Figure 2-2 Schematic of electrical connections of BETA device.](image)
The plasma is produced by striking a discharge between the anode (vessel wall, which is grounded) and the cathode (filament, which is negatively biased with respect to the grounded vessel). The confinement of the plasma is further improved with the help of toroidal magnetic field. A set of vertical magnetic field coils, in Helmholtz coil configuration, is used to impose a small vertical magnetic field component, to correct the error fields, due to misalignment of the toroidal field coils. The schematic of electrical connection of BETA device is shown in Figure 2-2.

![Figure 2-2 Schematic of electrical connection of BETA device](image)

Figure 2-3 Typical waveform of averaged ion saturation current obtained using Langmuir probe. The ripple in the first 400milliseconds of the shot is due to the ripple in the toroidal magnetic field. Horizontal axis represents time (0.2sec div) and vertical axis represent voltage (0.2V div).

The curved toroidal magnetic field produces charge separation as a result of which an electric field in the vertical direction is setup, which is short circuited by a conducting limiter of 9 cm radius. This defines the major plasma column. The radial location close to filament (± 2 cm) is dominated by non-Maxwellian (hot) components. Hence the experimental measurements are always made away from the filament location. Plasma discharge depends on several external parameters such as discharge voltage, magnetic field, filament current and operating gas pressure. The discharge characteristics obtained earlier (Mattoo, et. al., 1986), in this device suggest that when the filament is placed with angle, 0, with respect to the toroidal magnetic field, the orientation with 75° < 0 < 90° has lesser dependence on external parameters such as gas pressure, discharge voltage and magnetic field. Hence, in our experiments, we have placed the filament vertical (0 – 90°) to minimize the dependence of discharge on various external parameters. A typical ion saturation current obtained by Langmuir probe is shown in Figure 2-3. The ripple during
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the initial stage of the plasma formation is due to ripple in the toroidal magnetic field mentioned earlier.

µ-wave produced plasma
The plasma is produced in the device, using a magnetron-based microwave source. The source is designed using National make magnetron. It operates at 2.45 GHz and delivers a maximum rf power of 850 W during CW operation. The compact magnetic circuit has very low stray magnetic field and allows it to be placed near the radial port without distorting the ambient toroidal magnetic field of BETA machine. The electrical and mechanical properties of the magnetron are shown in Table 2-6 for ready reference.

<table>
<thead>
<tr>
<th>Electrical</th>
<th>Mechanical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power supply</td>
<td>Mounting position</td>
</tr>
<tr>
<td>Full wave stabilized DC</td>
<td>Any</td>
</tr>
<tr>
<td>power supply</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>RF coupler</td>
</tr>
<tr>
<td>2.45 GHz</td>
<td>WR430 waveguide type</td>
</tr>
<tr>
<td>Load VSWR</td>
<td>Magnetic system</td>
</tr>
<tr>
<td>4.0 Max.</td>
<td>Ferrite magnet</td>
</tr>
<tr>
<td>Anode voltage</td>
<td>Anode temperature</td>
</tr>
<tr>
<td>-4.1 kV</td>
<td>250° max</td>
</tr>
<tr>
<td>Anode current</td>
<td>Cooling air flow</td>
</tr>
<tr>
<td>-300 mA</td>
<td>800 lpm</td>
</tr>
<tr>
<td>Peak anode current</td>
<td>Weight</td>
</tr>
<tr>
<td>-1200 mA max</td>
<td>0.95 kg (2.1 lbs.)</td>
</tr>
<tr>
<td>Output power</td>
<td>Cold filament resistance</td>
</tr>
<tr>
<td>850 W</td>
<td>0.047 Ω</td>
</tr>
<tr>
<td>Tube efficiency</td>
<td>Preheating time</td>
</tr>
<tr>
<td>~71 %</td>
<td>Zero seconds</td>
</tr>
<tr>
<td>Filament voltage</td>
<td></td>
</tr>
<tr>
<td>-3.3 V</td>
<td></td>
</tr>
<tr>
<td>Filament current (operating)</td>
<td></td>
</tr>
<tr>
<td>-10.5 A</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-6 Typical electrical and mechanical properties of magnetron.
Figure 2.4 Typical layout of microwave components for launching the microwave power into BETA device to form the plasma. Power supply circuit diagram for the magnetron is also shown.
The magnetron is mounted on WR340 rectangular waveguide coupler. A three-port circulator is connected to the source to protect it from reflected power. To monitor the forward and reflected power, a dual directional coupler is connected in the rectangular transmission line. Finally, the output port of the transmission line is connected to the device through one of the radial port. Quartz window is mounted on the radial port of the machine with the help of viton O’ ring to make the joints vacuum compatible. A schematic layout of the microwave component and electrical connections for the magnetron is shown in Figure 2-4. Measurements reveal that ~500 W of microwave is coupled to the plasma in our experimental set-up. The power thus coupled, forms and maintains the plasma. In principle, the plasma can be formed for long durations (limited by available toroidal magnetic field pulse length), however the pulse duration of the plasma chosen, in our experiment, is typically about 1.5 seconds. To avoid the effect of ripple of toroidal magnetic field on the formation of plasma, the microwave power is switched on, about 2 seconds after the toroidal magnetic field is switched on. The filament is removed from the vessel completely to form electrodeless plasma. The oscillating electric field ionizes the gas, in the presence of collisions and thus plasma is formed. A limiter of radius 9 cm limits the plasma and provides an equilibrium to the toroidal plasma by short circuiting the vertical electric field formed due to curvature and VB drift. The plasma is formed, where the ECR region is located. For example, at a toroidal magnetic field of 0.0875 T, the ECR region is located near the minor axis and hence plasma peaks near ECR region. The strength of the toroidal magnetic field is changed in order to obtain different radial density profiles.

3 Diagnostics and signal processing electronics

The local plasma parameters, which are of our interest in the experiments, are the plasma potential (\(\phi\)), density (\(n\)) and temperature (\(T\)). To measure the parameter we insert electrical probes into the plasma. The presence of probes in the plasma perturbs the local plasma behaviour. Thus, the difficulty in measuring the plasma parameter is to understand how the probes disturb the plasma locally and how the local plasma parameter is related to the unperturbed plasma far from the probe.
**Basic understanding of probe theory**

One of the earliest and still most useful tool for diagnosing the plasma is the well known Langmuir probe (Langmuir and Mott-Smith, 1924). Its application has been successfully verified and adapted to magnetized, partially ionized, low temperature, ECR formed plasmas (Koo, et. al., 1999) and in numerous microwave discharges electrical probes have been successfully applied (Roussean, et. al., 2002, Ganguli, et. al., 1998, Ganguli, et. al., 1999). Several robust theories have been proposed since the early work of Langmuir in the 1920s (Langmuir and Mott-Smith, 1923). The basis of modern theories was set by the earlier work (Allen, et. al., 1957, Bohm, et. al., 1949) and Laframboise, 1966, extended it further.

For a non-magnetised plasma there exist a well-defined theory in agreement with experimental observations (Bohm, et. al., 1949). The analysis of this probe is well known and described in several works and papers (Langmuir and Mott-Smith, 1924, Chen, 1995, Hutchinson, 1995, Swift and Schwar, 1971). When a magnetic field is introduced less developed theories exist. The theory of the probes in magnetized plasmas has not changed much since last review (Chen, 1965). Excellent reviews (Chung, et. al., 1974, Smy, 1976) and books (Chung, et. al., 1975) are available on this subject. A kinetic theory for small probes in strongly magnetized plasmas is developed (Demidov, et. al., 1999) and there exist fluid theory for larger probes. The probe theory presented in these works is complicated and is beyond the scope of this thesis. Here we present the theory in a non-magnetized plasma and then discuss briefly the problems which arise when the external magnetic field is taken into account.

In a quasi-neutral plasma consisting of equal number of positive ions and electrons, the electrons are far more mobile than the ions. Fast moving electrons would be rapidly lost to the walls, while the less mobile ions are maintained in the plasma, hence the plasma will charge positively with respect to the wall. A positive ion sheath forms near the wall in which \( n_i \gg n_e \). The net positive charge within the sheath leads to a potential profile that is positive in the plasma and falls sharply to zero near the wall, as shown in figure Figure 2-5.
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Figure 2-5 Typical sketch of plasma sheath in contact with conducting walls. Once the sheath is formed, a typical profile for the electron and ion densities, potential and electric field of the sheath is outlined.

The electric field within the sheath points from the plasma to the wall, hence electrons are decelerated and the ions are accelerated in this direction, so that the ions and electrons are lost to the walls at the same time. Hence, the plasma can remain quasi-neutral. A sheath forms close to all material surfaces in the plasma, also near the probe surfaces. It will be shown that the sheath thickness is limited under certain conditions.

When a charge is inserted into the plasma it generates an electric field. The plasma particles tend to redistribute themselves so as to shield the plasma from the perturbing field. The effect of the charge on the plasma may be deduced from Poisson’s equation. Doing this we have to assume that the ion density near the charge is equal to the ion density far from the charge and that the electrons are in thermal equilibrium. Hence, the Boltzmann relation determines the electron density.
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\[ n_e = n_m \exp \left( \frac{eV}{T_e} \right) \]

Here \( T_e \) is the electron temperature in electron volt, \( n_m \) is the electron density far away from the charge, \( e \) is the electron charge and \( V \) is the perturbing potential from the charge. The potential \( V \) is given by Poisson’s equation. Solving the said equation in one dimensions gives (Hutchinson, 1995):

\[ V(x) = \exp \left( \pm \frac{x}{\lambda_D} \right) \]

where \( \lambda_D \) is the shielding length, called the Debye length given by:

\[ \lambda_D = \left( \frac{e_T e}{\epsilon_0 n_m} \right)^{1/2} \]

where \( \epsilon_0 \) is the vacuum permittivity. As obvious from the equation, the perturbing effect from a charge in plasma only penetrates an order of the Debye length into the plasma.

Here, we consider a non-magnetised plasma and assume that the plasma is terminated by a perfectly absorbing probe held at a negative potential, \( V_{pr} \). We assume that the probe radius \( a \) is much larger than the Debye length. Since the sheath thickness \( s \) is typically few times Debye length, we also assume that \( a \gg s \). With this approximation, it is reasonable to assume a plane probe geometry and to use a collision-less sheath theory. Further, we consider a time-independent model for the potential in a planar plasma sheath \( V(x) \), as a function of \( x \). We choose the position of the probe surface to be \( x = 0 \). Far from the probe, there is field free and neutral plasma. At position \( x = s \), where \( s \) is the sheath thickness, there is a transition from the non-neutral sheath to the neutral bulk plasma. Initially, it was assumed (Langmuir and Mott-Smith, 1924) that this transition was well-defined by a sheath edge, i.e. the entire drop in potential between the ionized gas and the collector is concentrated within this sheath. Later it has been shown that the bulk plasma is gradually converted to sheath in a region usually referred as pre-sheath. The pre-sheath is assumed to be approximately neutral, but is not strictly field free. From this point onwards, the pre-sheath and sheath interface would be referred as the sheath edge and the
subscript \( s \) would denote values at this position. The sheath potential \( V \) must satisfy Poisson's equation.

\[
\nabla^2 V = -\frac{e}{\varepsilon_o} (n_i - n_e)
\]

The electrons are assumed to be in thermal equilibrium, so that they follow Boltzmann relation. The ion density must be calculated from the equation of motion. The potential is chosen to be zero at the sheath edge, so that \( V_s = 0 \). The sheath is assumed to be collisionless and ionization free so that the energy conservation can be assumed.

\[
\frac{1}{2} m_{i} v_{i}^2 = \frac{1}{2} m_{e} v_{e}^2 + eV
\]

Solving for the ion velocity in the sheath, \( v_i \), we get:

\[
v_i = v_{i0} \left( 1 - \frac{2eV}{m_{i} v_{i0}^2} \right)^{1/2}
\]

The assumption that no ionization occurs in the sheath gives the ion continuity equation. \( \nabla \cdot (n_i v_i) = 0 \). Thus, the particle current density in the sheath and sheath-edge are related by:

\[
J_{is} = n_i v_{is} = j_i = n_i v_i.
\]

Using above three relations, we get:

\[
n_i = n_{i0} \left( 1 - \frac{2eV}{m_{i} v_{i0}^2} \right)^{1/2}
\]

If the value of \( n_i \) and \( n_e \) (from Boltzmann relation) is substituted in Poisson’s equation, we get:

\[
\frac{d^2 V}{dx^2} = \frac{en_i}{\varepsilon_o} \left[ \exp \left( \frac{eV}{T_e} \right) - \left( 1 - \frac{2eV}{m_{i} v_{i0}^2} \right)^{1/2} \right]
\]

where \( n_s = n_{i0} = n_{e0} \) is the density at the sheath edge. To solve this equation we use that \( V_s = 0 \) and that the electric field is zero at the sheath edge so that \( (dV/dx)_{s} = 0 \).

Multiplying above equation by \( dV/dx \) and on integration we get:

\[
\frac{1}{2} \left( \frac{dV}{dx} \right)^2 = \frac{en_i}{\varepsilon_o} \left[ m_{i} v_{i0}^2 \left( 1 - \frac{2eV}{m_{i} v_{i0}^2} \right)^{1/2} - 1 \right] + \frac{T_e}{e} \left[ \exp \left( \frac{eV}{T_e} \right) - 1 \right]
\]
If we now consider the area close to the sheath edge, where \(|eV| < T_e\) and \(|eV| < (m_e v_u^2)/2\), the right hand side of above equation can be simplified by Taylor expansion, giving:

$$\frac{1}{2} \left( \frac{dV}{dx} \right)^2 = \frac{1}{2} \frac{V^2}{\lambda_d^2} \left( 1 - \frac{T_e}{m_e v_u^2} \right)$$

Here, it should be noted that the right hand side should be positive, which implies that,

$$v_u \geq \left( \frac{T_e}{m_i} \right)^{1/2} = C_s$$

This inequality is called the Bohm sheath criterion where \(C_s\) is the ion acoustic velocity or the Bohm velocity \(v_{B}\). The criterion shows the need for a pre-sheath between the bulk plasma and the sheath region where the ions are accelerated up to the required velocity. Thus, for the first time, it was showed that a pre-sheath is needed to accelerate the ions (Bohm, et. al., 1949). To evaluate the sheath thickness, it is convenient to use another reference point for potential. Let us now define the zero potential in the bulk plasma so that \(V = 0\) in that region. Thus \(V_s = 0\) in this case. We assume that the electron density is negligible in the sheath region so that \(n_e = 0\) in Poisson’s equation. Following the above prescription again, we get in this case, as follows:

$$\frac{1}{2} \left( \frac{dV}{dx} \right)^2 = \frac{1}{2} \frac{V^2}{\varepsilon_s} \left( 1 - \frac{2eV}{m_e v_u^2} \right)^{1/2}$$

Taking the square root and integrating it again, we get a relation between the position and the potential in the sheath as:

$$s = \frac{\sqrt{2}}{3} \lambda_d \left( \frac{2eV_s}{T_e} \right)^{1/4}$$

This is known as the Child law sheath (Lieberman and Lichtenberg, 1994). In our case the electron temperature is typically \(\sim 5\) eV and if we assume the sheath potential of \(\sim 30V\), then the sheath thickness is of the order of a few Debye lengths (typically \(s \sim 3\lambda_D\)). For our kind of plasma with \(n_e \sim 10^{11} \text{ cm}^{-3}\), \(\lambda_D\) is about \(\sim 0.05\) mm, which results in a sheath thickness of about \(\sim 0.1\) mm. Thus, the sheath thickness would be small in extension compared with the dimension of the plasma or the probe.
**Langmuir probe characteristics**

A Langmuir probe consists of a small conductor. The probe is made of thin non-thoriated tungsten wire. The probe bias is swept from negative to positive values. The bias affects the current to the probe. Thus, a probe characteristic is obtained by changing the bias voltage. A typical characteristic curve for the Langmuir probe is shown in Figure 2-6.

![Figure 2-6 A typical characteristics of a Langmuir probe.](image)

If the probe bias is at the plasma potential, $V_p$, no sheath is formed between the probe and the plasma. Thus, assuming that $T_i \leq T_e$, the probe current will mainly consist of electrons only at this potential, $I \sim I_e = e\Gamma e$, where $\Gamma = n\sqrt{2/m_e}$ is the particle current density. If the voltage is increased above the plasma potential, $V_{pr} > V_p$ the electron current cannot increase any further, since all the electrons are collected at the plasma potential. The ion
current, $I_e$, decreases due to the repulsion of the ions, but this is already much less than the electron current, so the probe current, $I_p$, is approximately constant. This region is known as the electron saturation region and here the $I$ is equal to the electron saturation current. If the probe potential decreases such that $V_{pr} < V_p$, then the probe is negative with respect to the surrounding plasma. An increasing fraction of the electrons are reflected from the negative potential. When the potential is sufficiently negative the electron current would be only a small fraction of the electron saturation value. The current drawn by the probe is zero when the electron and ion current equal and it occurs at floating potential, $V_f$. If the potential is decreased further, only ions are collected, and the probe reaches the ion saturation region.

From the current to voltage ($I$-$V$) characteristics, it is possible to determine the plasma parameter such as density, electron temperature and plasma potential. Here, we present a simple analysis of a probe characteristic. Assuming that the Boltzmann relation gives the electron current density to the probe:

$$j_e = \frac{1}{4} n_e v_e = \frac{1}{4} n_e v_e \exp\left(\frac{eV_{pr}}{T_e}\right)$$

where $v_e = 2(2T_e/m_e)^{1/2}$ is the mean Maxwellian electron velocity. Accelerating the ions up to the ion-acoustic velocity requires a potential drop given as:

$$V_{drop} = -\frac{m_i C_a^2}{2e} = -\frac{T_i}{2e}$$

relative to the plasma potential. The ion current to the probe is then given by Boltzmann condition as:

$$j_i = n_i \left(\frac{T_e}{m_i}\right)^{1/2} \exp\left(-\frac{1}{2}\right)$$

The total current, which comprises the electron, and the ion current may be written as:

$$I = n_e eA_{pr}\left(\frac{T_e}{m_i}\right)^{1/2} \left[\frac{1}{2} \left(\frac{m_i}{m_e}\right) \exp\left(\frac{eV_{pr}}{T_e}\right) \frac{A_i}{A_{pr}} \exp\left(-\frac{1}{2}\right)\right]$$

Here, $A_i$ and $A_{pr}$ is the sheath size and the probe size, respectively and $V_{pr}$ is the probe potential. This equation is only valid when $T_i < T_e$, $\lambda_D \ll a$ and $A_s \sim A_{pr}$. When the probe
potential, $V_{pr}$ is at floating potential $V_f$, no current is collected by the probe, so setting $I=0$ gives $V_f$ as:

$$V_f = \frac{T_e}{2e} \left[ \ln \left( \frac{2\pi n_e}{m_i} \right) - 1 \right]$$

The problem is usually to find the plasma potential $V_p$, which is the reference point for voltage and the voltage where electron saturation is reached. To find $T_e$ the slope of the characteristic at the knee can be used as follows:

$$\frac{dl}{dV_{pr}} = \frac{e}{T_e} (I - I_{sat}) + \frac{dl_{sat}}{dV_{pr}}$$

where $I_{sat} = -eA_{pl}$ is the ion saturation current. Neglecting the last term in above equation we get,

$$T_e = \frac{e(I - I_{sat})}{dl}$$

Note that the current $I$ collected by the probe is exponential (see expression for total probe current above). Taking the logarithm of $(1 - I_{sat})$ and differentiating with respect to potential $V_{pr}$, we get:

$$\frac{d}{dV_{pr}} \left( \ln |I - I_{sat}| \right) = \frac{e}{T_e}$$

which shows that $T_e$ can be obtained numerically from the characteristic by fitting a line to $\ln |I - I_{sat}|$ versus $V_{pr}$.

When $T_e$ is found the plasma density may be determined by solving $I_{sat} = -eA_{pl}$, where $j_i$ is already defined earlier. Thus we get:

$$n_0 = \frac{I_{sat}}{eA_{pl} \left( \frac{m_i}{T_e} \right)^{1/2}} \exp \left( -\frac{1}{2} \right)$$

In all the discussions above, it was assumed that no magnetic field is present. However, presence of magnetic field introduces an anisotropy, which makes the problem at least two-dimensional. The electron and ion diffusion coefficient parallel and perpendicular to the magnetic field $D_\parallel$ and $D_\perp$, respectively are estimated as (Goldston and Rutherford, 1995),
\[ D_\parallel \sim v_\text{ne}^2 \text{ and } D_\perp \sim v_\text{in}^2 \]

where \( v \) is the electron-neutral or ion-neutral collision frequency, \( \lambda_{\text{ne}} \) is the collision mean free path along the magnetic field and \( r_L = mv/eB \) is the Larmor radius for the electron or ion. Thus, the effective mean free path across the field is of the order of \( r_L \), so the particles can travel only this far without making a collision. Since the Larmor radius is quite small for electrons even at moderate fields, there is essentially no collisionless theory in such a case (Chen, 1965).

When the probe is large compared to \( \lambda_{\text{ne}} \), it will collect so many electrons that the surrounding area will be drained faster than the electrons diffuse into the area. Hence, the electrons are collected to the probe from regions quite distant along the field, but not far away in the direction normal to the field. The number of electrons collected by the positively biased probe is therefore reduced in the presence of the magnetic field. With these arguments in mind it is claimed that the electron current is not of very much use in the probe analysis (Bohm et al., 1949).

The Larmor radius for positive ions is large compared to the probe radius, consequently the magnetic field can be neglected in the ion saturation region. It has been claimed that only ion saturation region is unaffected by the magnetic field and therefore this region should be used for analysis (Chen, 1965).

In practical matter, we can use a non-magnetized theory with small modifications for magnetized plasma (Hutchinson, 1995). The argument is that ions flowing to the sheath edge have a velocity equal to the ion acoustic velocity, but in a strong field this sonic flow can occur only along the field and not across it. Therefore it can be assumed that the field-free theory can be applied except that the effective collection area is not the total probe area, but the projection of the probe surface in the direction of the magnetic field.
Density measurement
Density is determined by measuring ion saturation current by biasing a Langmuir probe. A power supply with a large negative voltage ($V_{\text{bias}} > 3T_e$) bias the probe and voltage drop across the resistor $R$ is measured. The schematic is shown in Figure 2-7.

![Figure 2-7 Typical layout for measuring the ion saturation current for determining the plasma density. When the Langmuir probe starts conducting, a potential develops across the measuring resistance, proportional to plasma density, which gives a measure of plasma density.](image)

Care must be taken to choose the measuring resistor, i.e. it should be small for current measurement and large for potential measurement. As discussed earlier, for voltages, less than floating potential, the sheath repels most of the electrons. The presheath field accelerates the ions so that they enter with a velocity, $U_i$, in a plasma in which electron temperature is greater than ion temperature. Let the ion current change by $I_I$ for a change in voltage of the order of $T_e$ at $V = V_f$, then the effective output impedance $R_o$ at $V_f$ would be:

$$R_o = \frac{T_e}{I_I}$$

In our experiment, $T_e = 5 \text{ eV}$ and $I_I = 0.25 \text{ mA}$, hence we get $R_o = 20k\Omega$. The operating point is the intersection between the probe characteristics and a load line of current $R << R_o$, so that the operating point gives current independent of applied voltage. In our experiment we have used $R$-value as $1 \text{ k}\Omega$ and $5 \text{ k}\Omega$ depending on the signals level obtained in the experiment.
In measuring the ion saturation current we have used the circuit as shown in Figure 2-8. It consists of three parts. The first part provides a negative bias to the probe and acts as a current to voltage converter. The voltage that appears across the resistor (when the probe is conducting) is fed to the second part, which consists of two differential amplifiers. Here a part of the probe signal (~10%) is fed to the OPAMP CPA37A. The output from these OPAMPS are fed to the third stage, where a unity gain buffer, acting as a difference amplifier (OPA3550K), is used to subtract the bias voltage and only probe signal is obtained across output resistor (1kΩ - 5kΩ).

**Floating potential measurement**

For measuring floating potential, the load resistance, R, should be much larger than the effective output impedance; \( R_0 \) (i.e. \( R > R_0 \)) so that operating point is close to floating...
potential. Hence in measuring the floating potential we have used \( R = 1 \, \text{M} \Omega \). This resistance with the cable capacitance (100 pF/meter) reduces the frequency response of the probe (Roth and Krawczonek, 1971, Chen, 1985). The frequency response of the probe for a typical cable length of 10 meters in the experiment would be about 1 kHz.

\[
\text{For our experiment we need a maximum frequency response of about 25 kHz. In order to increase the frequency response and to use } R = 1 \, \text{M} \Omega, \text{we have used voltage follower (high input impedance and low output impedance), which is mounted near the probe shaft. The circuit diagram for the same is shown in Figure 2-9.}
\]

**Electron temperature measurement**

Plasma electron temperature is measured with the help of a Langmuir probe. It is measured by applying ramp voltage to cylindrical Langmuir probe. The electronic circuit used is similar to that used for density measurement. The ramp voltage is applied instead of a uniform bias voltage. A Waveteck oscillator is used to generate a ramp of \( \pm 1.5 \) V amplitude and slow sweep time of about 500 Hz. This is further amplified by a factor of 10-20 and then applied to the Langmuir probe. The ramp voltage and the probe current are recorded to determine the probe characteristic for obtaining plasma temperature. A typical data obtained is shown in Figure 2-10.
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Figure 2-10 A typical ramp voltage and probe current obtained while measuring electron temperature of the plasma.

Figure 2-11 A typical probe characteristic obtained while measuring electron temperature of the plasma.
In this figure, we have shown only one ramp data point (typically 10,000 points), however in a single shot we obtain 25,000 data points. The first graph shows the applied ramp voltage whereas the second graph shows the probe current signals. Each data point is sampled at every 20 µ-second. The probe characteristic is obtained by the sweeping the probe with the help of a ramp circuit. The DC offset and the period of the ramp is adjustable. The ramp frequency is set to 5 Hz. and the DC offset is kept at 20 V. A typical probe characteristic obtained in our experiment is shown in Figure 2-11.

During the experiment, we have used probes of different shapes for different purposes. A radially movable cylindrical Langmuir multi-probe system with exposed tip of 5 mm in length and 1 mm in diameter, is used to measure the equilibrium plasma density, electron temperature and floating potential at different radial and vertical locations. Equilibrium plasma density and floating potential are measured by an array of probes at a fixed vertical location but at different radial locations. This procedure is again repeated at different vertical locations to generate the contours at different toroidal magnetic field values. Two probes separated by a distance of 1 cm are used to measure the radial and azimuthal wave-number of the waves. The probe array is placed toroidally 135° away from the filament position or microwave launching port. The probes are cleaned periodically by applying a high positive voltage to it. Fluctuating components of density and floating potential have also been measured at these locations. Two probes separated by a distance of 1 cm are used to measure the density and the floating potential simultaneously. The signals are band passed with a pass band of about 500 Hz. to 25 kHz. to avoid artefacts. The data are analyzed using a standard fast Fourier technique (FFT) (Smith, et. al., 1974) to obtain the frequency, wave-number, coherence and the power spectrum of the fluctuations.

All the measurements are taken when the plasma is fully grown and well formed. For this reason, a circuitry is used to generate a number of trigger pulse with preset delays. The circuitry used for this purpose is shown in Figure 2-12. It operates in external trigger mode, as well as in manual triggering mode, with the help of a switch. A +5V signal drives IC74221, which generates a reference pulse. This pulse is used as an input to
various channels. For the sake of simplicity we have just shown two channels in the circuit diagram. Each channel is designed to give an output after a delay, with respect to the input reference signal. The delay is determined by a proper combination of resistor and capacitor used for biasing the IC. Finally, the transistor drives the output of the circuit.

In our case, the first signal triggers the toroidal magnetic field and filament plasma is formed. The second channel triggers the digital oscilloscope externally, after a delay of 500msec. This delay is enough for the plasma to fully grow into a steady state and the data is acquired thereafter. When the plasma is formed with the μ-wave, a third channel provides a trigger pulse to initiate μ-wave transmission, after a delay of 500 msec., after the toroidal magnetic field is triggered. In this case the oscilloscope is triggered after a delay of 500 msec., after launching of μ-wave power.
4 Microwave instruments and measurements

Before microwave power is launched into the plasma, one needs to calibrate each and every microwave components that are used in the system. We have used HP 8753E VNA to characterize different microwave components, which we have used in our experiment. Once the microwave launching system is installed on BETA machine, we have measured the launched frequency and power of the microwave, using HP 8562B spectrum analyzer. The detail of the instruments and the results obtained from the measurements are discussed in details in the following subsections.

VNA measurements
Vector Network Analyzer (VNA) is used to calibrate various microwave components that are used in the experiments. HP 8753E Network analyzer from Hewlett Packard is employed to characterize components like rectangular waveguide (WR340), waveguide to coaxial adapter, directional couplers, loads, etc. This VNA measures the reflection and transmission characteristics of devices and networks in the frequency range from 30 kHz to 6 GHz. The VNA consists of the following modules:

- Source
- Signal-separation devices
- Receiver and
- Display

The analyzer applies a signal that is transmitted through the test device, or reflected from its input, and then compares it with the incident signal generated by the swept RF source. The signals are then applied to a receiver for measurement, signal processing and display.

The HP 8753E VNA integrates a high resolution synthesized RF source, test set, and a dual channel three-input receiver to measure and display magnitude, phase and group delay of transmitted and reflected power. The analyzer has the additional capability of transforming measured data from the frequency domain to the time domain. A simplified block diagram of the VNA is shown in Figure 2-13.
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Figure 2-13 A block diagram showing the schematic of HP 8753E Vector Network Analyzer (VNA).

It consists of a built-in synthesized source, which produces a swept RF signal or continuous wave (CW) signal in the range of 30 kHz to 6 GHz. To achieve frequency accuracy and phase measuring capability, the analyzer is phase locked to highly stable crystal oscillator. For this purpose, a portion of the transmitted signal is routed to the R channel input of the receiver, where it is sampled by the phase detection loop and fed back to the source. The VNA features a built-in test set that provides connections to the test device, as well as to the signal-separation devices. The signal separation devices are used to separate the incident signal from the transmitted and reflected signals. The incident signal is applied to the R channel input through a jumper cable on the front panel. Meanwhile the transmitted and reflected signals are internally routed from the test port couplers to the inputs of A and B sampler/mixers in the receiver. Port 1 is connected to the A input and port 2 is connected to the B input. The test set contains the hardware required to make simultaneous transmission and reflection measurements in both the forward and reverse directions. An RF path switch in the test set allows reverse measurements to be made without changing the connections to the test device. The receiver block contains three sampler/mixers for the R, A and B inputs. The signals are
sampled, and mixed to produce a 4 kHz intermediate frequency (IF). A multiplexer sequentially directs each of the three signals to the analog to digital converter (ADC) where it is converted from an analog to a digital signal. The signals are then measured and processed for viewing on the display. Both amplitude and phase information are measured simultaneously, regardless of what is displayed on the analyzer. A microprocessor takes the raw data and performs all the required error calculation, trace math, formatting, scaling, averaging and marker operations according to the instructions from the front panel or over HP-IB. The formatted data is then displayed.

Calibration of the VNA.
Before making any measurements using VNA, it has to be properly calibrated using standard calibration kits. We have used rectangular WR340 waveguide components and hence waveguide calibration kit for WR340 waveguide is used to calibrate our components. The cable is connected to port1 and the other end is connected to waveguide to coaxial adapter (WR340). Similarly another cable is connected to port2 and the other end is connected to another waveguide to coaxial adapter (WR340). The waveguide to coaxial adapters are connected back to back for calibration. This defines the calibration plane. Later the device under test (DUT) is introduced in the calibration plane and measurements are made for the DUT. Here we list some of the main steps, which are followed for the full calibration of this two-port device:

- The calibration data is loaded on the VNA from the supplied floppy.

- Internal disc is chosen by selecting disc type from save recall menu.

- The menu button is pressed to select number of points to 201 and power level of 0 dBm is selected.

- The frequency range is selected by selecting the center frequency and span.

- Calibration button is pressed and interpolation button is switched on.
From the calibration menu, full two-port button is pressed.

Calibration menu for Reflection, Transmission and Isolation appears.

Standard short, lambda length, open and through (SLOT) connections are made for each of the port and readings are stored.

Isolation measurement is omitted.

Once the calibration is over, 'done two port' button is pressed to store the calibration data on the internal disc.

Now one can start making measurements by putting DUT at the calibration plane.

VNA results
The various components that are used in our experiment are measured for its microwave characteristics. We have used the VNA for calibrating various μ-wave components like standard waveguide (WR340), directional couplers, waveguide to coaxial adapter, cables, etc. Here, we present a typical result obtained during the measurement of WR340 rectangular waveguide. The waveguide is made out of copper and is characterized for its microwave performance. The waveguide is about 500 mm in length and has a cross section of 86mm x 43mm. Typical VNA output is shown in Figure 2-14. Measurement shows that the waveguide offers a return loss of about -35 dB at our frequency of interest. The x-axis of the figure represents frequency span, where start frequency is 2.4 GHz, and the stop frequency is 2.5 GHz. The y-axis of the figure shows a reference cursor at 0 dB and each division represent 10 dB.
The VNA measurement of insertion loss for the WR340 rectangular waveguide is shown in Figure 2-15. The measurements show that the waveguide offers an insertion loss (or $S_{12}$) of about -0.03 dB at our frequency of interest. The x-axis of the figure represents frequency span, where start frequency is 2.4 GHz and the stop frequency is 2.5 GHz. The y-axis of the figure shows a reference cursor at 0 dB and each division represent 10 dB.
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Figure 2-15 The figure shows $S_{12}$ or insertion loss for the standard WR340 waveguide measured using HP 8753E VNA. An insertion loss of about -0.03 dB is obtained at 2.45 GHz. The x-axis of the figure represents frequency span, where start frequency is 2.4 GHz and the stop frequency is 2.5 GHz. The y-axis of the figure shows a reference cursor at 0 dB and each division represent 10 dB.

Measurement also shows that the WR340 waveguide offers SWR of about 1.04 at our frequency of interest. The VNA measurement of SWR for the WR340 rectangular waveguide is shown in Figure 2-16. The x-axis of the figure represents frequency span, where start frequency is 2.4 GHz and the stop frequency is 2.5 GHz. The y-axis of the figure shows a reference cursor at 1.0 and each division represent 1.0.
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Figure 2-16 The figure shows SWR for the standard WR340 waveguide measured using HP 8753E VNA. A value of about 1.04 is obtained at 2.45 GHz. The x-axis of the figure represents frequency span, where start frequency is 2.4 GHz and the stop frequency is 2.5 GHz. The y-axis of the figure shows a reference cursor at 1.0 and each division represent 1.0.

The best way to represent the microwave measurement is to see its smith chart. The smith chart for the WR340 waveguide is shown in Figure 2-17. The x-axis of the cursor represents frequency span, where start frequency is 2.4 GHz and the stop frequency is 2.5 GHz. The marker is positioned on the horizontal line, showing almost no reactive impedance and lies on a impedance circle of value 1.0, showing perfect matching of the component.
Figure 2-17 The figure shows Smith chart for the standard WR340 waveguide measured using HP 8753E VNA. The measurement line around the cursor represents the response over the frequency span, where start frequency is 2.4 GHz and the stop frequency is 2.5 GHz. The marker is positioned on the horizontal line at the frequency of our interest, 2.45 GHz, showing almost no reactive impedance and lies on an impedance circle of value 1.0, showing perfect matching of the component.

Representative results of some of the microwave components that are obtained during the measurements are shown in Table 2-7.
Table 2-7 Typical characteristics of different microwave components.

<table>
<thead>
<tr>
<th>Components</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>WR340 waveguide</td>
<td>$S_{11} = -35 , \text{dB (SWR = 1.04)}$</td>
</tr>
<tr>
<td></td>
<td>$S_{12} = -0.03 , \text{dB}$</td>
</tr>
<tr>
<td>Circulator (Main arm)</td>
<td>Insertion loss = 0.2 , \text{dB}</td>
</tr>
<tr>
<td></td>
<td>SWR = 1.04</td>
</tr>
<tr>
<td></td>
<td>Power handling = 1.5 , \text{kW}</td>
</tr>
<tr>
<td></td>
<td>Isolation = -22.5 , \text{dB}</td>
</tr>
<tr>
<td>Directional coupler</td>
<td>$S_{11} = -35 , \text{dB} \quad S_{12} = -0.03 , \text{dB}$</td>
</tr>
<tr>
<td>Waveguide coupler</td>
<td>SWR = 1.09</td>
</tr>
<tr>
<td></td>
<td>Insertion loss = 0.2 , \text{dB}</td>
</tr>
<tr>
<td>Bends</td>
<td>$S_{11} = -35 , \text{dB (SWR = 1.04)} \quad S_{12} = -0.03 , \text{dB}$</td>
</tr>
</tbody>
</table>

**Spectrum analyzer**

Spectrum analyzer, HP 8562B, is used to measure the frequency of the rf source. Before making any measurements, the analyzer is calibrated. A short BNC cable is connected with the front panel's CAL OUT connectors and INPUT 50 $\Omega$. The center frequency, span and the reference level is set. From the AMPLITUDE menu, REF LVL CAL button is pressed. The peak of the signal is adjusted to the reference level, using the front panel knob. Finally, the calibration is stored by pressing STORE REF LVL button. Once the calibration is done, the source is connected to the experimental system, with a dual directional coupler in the transmission line. The dual directional coupler has one forward coupling port (with forward coupling coefficient of 40 dB) and one backward or reflected coupling port with same coupling coefficient. The coupling port of the directional coupler
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has a N-type output connector. The signal is obtained from the forward coupling port of the directional coupler and is connected to the input of the spectrum analyzer, through 20dB fixed attenuator. The front panel of the analyzer is set. The center frequency (2.45 GHz.), span (900 MHz.) and amplitude (10 dB/level) are the fundamental functions, which are set, using the front panels key, for our measurements.

![Figure 2-18 Typical output from a spectrum analyzer.](image)

The typical output obtained in our measurement is shown in Figure 2-18. The frequency of the source is around 2.47 GHz. with a bandwidth of 0.03 GHz. The amplitude of the signal is around 57 dBm, which corresponds to an input power of about 600 Watts.
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5 Design and analysis of different μ-wave components using HFSS

Ansoft High-Frequency Structure Simulator (HFSS), version 7, is an interactive software package that allows one to characterize full wave and radiative effects for passive high frequency transmission structures. It runs on SUN SOLARIS, Unix workstations running X windows. Using finite element based solvers, HFSS allows one to compute and view various quantities like basic electromagnetic field quantities, antenna parameters, radiated fields for open boundary problems, characteristics port impedances, propagation constants, generalized and renormalized S-parameters, etc. One is expected to draw the structure, specify material characteristics for each object, identify ports, sources, and special surface characteristics. The system then generates the necessary field solutions. HFSS allows one to specify whether to solve the problem at one specific frequency or at several frequencies within a range. Once the geometric model is defined, it is automatically divided into a large number of tetrahedral, where a single tetrahedron is basically four-sided pyramid. This collection of tetrahedral is referred as the finite element mesh. Dividing a structure into thousands of smaller regions (elements) allows the system to compute the field solution separately in each element. The smaller the system makes the elements, the more accurate is the final computed solution.

To highlight the capabilities of the software, the design and analysis of standard WR340 waveguide is presented here. A project is made in HFSS software, where the waveguide structure is drawn with appropriate dimension. Two rectangular blocks are drawn of length 500mm. The outer box has a thickness of 5mm and represents the copper waveguide whereas the inner box is assigned the property of air. Each output end of the waveguide is assigned as the input and output port to make it two-port device. The solution setup is appropriately set for the frequency span of 2.4 to 2.5 GHz and the problem is solved. Later, post processor is invoked for plotting various parameter obtained from the solution. The post processor has facility to generate field profiles, current profiles, etc. on any desired surface or geometry.
Figure 2-19 Typical result of HFSS analysis for WR340 waveguide is shown. The first graph shows a return loss of better than -50 dB over the frequency range lying between 2.4 GHz and 2.5 GHz. The second graph shows a low insertion loss, which is better than -0.01 dB and the last graph shows the VSWR of less than 1.02.

A typical result obtained for this set up is shown in Figure 2-19. The first graph shows a return loss of better than -50 dB over the entire frequency range starting from 2.4 GHz to 2.5 GHz. The second graph shows a low insertion loss, better than -0.01 dB, for the WR340 waveguide. The last graph shows the VSWR for the waveguide, which is less than 1.02 over the entire frequency band of our measurement. It is interesting to compare
these calculated results, with the measured results obtained from the VNA for the WR340 waveguide. Due to experimental limitations the instrument could not measure different S-parameter with that accuracy, with which it is predicted computationally. However we get the best results that one could obtain with the VNA instrument and are close to predicted value. Also some mismatch obtained in our measurement may be attributed to surface finish of the inner surface of the waveguide, fabrication tolerances, radius of curvature at corners, misalignment of the flanges, etc.

![Electric field profile in WR340 waveguide structure as obtained from HFSS analysis. The cutplane is generated midway across broad dimension of the waveguide. As expected the contours are vertical showing transverse electric field pattern. The formation of nodes and antinodes are also seen along the axial length of the waveguide, the direction in which the wave propagates.](image)

As mentioned earlier, HFSS is capable of generating different field profiles on any given surface or structure. One has to define a line, surface or volume in which the entity is to be plotted. We have generated the electric field profile within the WR340 rectangular waveguide. A flat surface is defined at the mid plane, such that it cuts the waveguide along the broad side of the waveguide. On this surface, we have generated the electric field profile, using HFSS post processor. The obtained result is shown in Figure 2-20. As
expected the contours are vertical as observed in TE$_{10}$ wave propagation mode. Similarly we have generated the electric field profile within the WR340 rectangular waveguide, along a flat surface, which cuts the waveguide along the narrow side of the waveguide. On this surface, we have generated the electric field profiles, using HFSS post processor. The obtained result is shown in Figure 2-21. The contours show the propagation of TE$_{10}$ mode and displays formation of nodes and antinodes along the direction of propagation.

![Figure 2-21 Electric field profile in WR340 waveguide structure as obtained from HFSS analysis. The cutplane is generated midway across the narrow dimension of the waveguide. The contours display propagation of TE$_{10}$ mode and clearly shows formation of nodes and antinodes along the axial length of the waveguide, the direction in which the wave propagates.](image)

We also used HFSS to design and fabricate waveguide to co-axial adapter. In this project, we optimized the waveguide to co-axial adapter for the mounting of magnetron on the WR340 waveguide. One end of the waveguide was closed with a metallic plate (short end of the coupler) and the other side was connected to the flange. To mount the magnetron on this adapter, a hole of diameter 20mm was required on the broad-side wall of the waveguide. For efficient mounting of the magnetron on the adapter, we required two parameters, first the coordinate of the hole (say 'x' and 'y') and second the depth upto,
which the exciter of the magnetron would be inserted inside the waveguide (let us call it ‘h’).

Figure 2-22 Typical result of HFSS analysis for the optimised waveguide coupler is shown. The first graph shows a return loss of around -25 dB at the frequency of our interest, i.e. 2.45 GHz. The second graph shows a low insertion loss, which is better than -0.015 dB at our frequency and the last graph shows the VSWR of less than 1.12.

For TE10 mode excitation, the exciter needs to be positioned at the center of the broad side dimension, i.e. $x = 43$ mm. Thus, our requirement reduces to optimize two
parameters, the axial distance or 'y' (from the short end of the adapter) and the depth or 'h'. We modeled the geometry and ran several runs by changing the above two parameters. Here, macro files were used for creating the object for efficient modeling of the geometry. The optimized values for the two parameters were obtained as y = 25 mm and h = 24 mm. Typical result of HFSS analysis for the optimized coupler is shown in Figure 2-22. The first graph shows a return loss of around -25 dB at the frequency of our interest, i.e. 2.45 GHz. The second graph shows a low insertion loss, which is better than -0.015 dB at our frequency and the last graph shows the VSWR of less than 1.12.

![Electric field profile in waveguide adapter as obtained from HFSS analysis.](image)

**Figure 2-23** Electric field profile in waveguide adapter as obtained from HFSS analysis. The cutplane is generated midway across the narrow dimension of the waveguide. The contours display propagation of TE_{10} mode and clearly show formation of nodes and antinodes along the axial length of the waveguide, the direction in which the wave propagates.

As shown earlier, in this case also we have generated the electric field profile within the rectangular waveguide coupler. A flat surface is defined at the mid plane, such that it cuts
the waveguide across the narrow side of the waveguide. On this surface, we have generated the electric field profile, using HFSS post processor. The obtained result is shown in Figure 2-23. The contours show the propagation of $TE_{10}$ mode and displays formation of nodes and antinodes along the direction of propagation.

![Electric field profile in waveguide adapter as obtained from HFSS analysis.](image)

**Figure 2-24 Electric field profile in waveguide adapter as obtained from HFSS analysis.** The cutplane is generated midway across broad dimension of the waveguide. As expected the contours are vertical showing transverse electric field pattern. The formation of nodes and antinodes are also seen along the axial length of the waveguide, the direction in which the wave propagates.

Similarly we have generated the electric field profile within the WR340 rectangular waveguide adapter, along a flat surface, which cuts the waveguide across the broad side of the waveguide. On this surface, we have generated the electric field profiles, using HFSS post processor. The obtained result is shown in Figure 2-24. As expected the contours are vertical as observed in $TE_{10}$ wave propagation mode.
Data analysis technique.

The instabilities present in the plasma manifest itself in the form of waves. Therefore, to study the instabilities, various signatures of the waves should be identified. A wave may be mathematically written as:
\[ y = y_0 \exp(i(kx - \omega t)) \]
and to identify the wave characteristics, identification of various signatures include measuring of amplitude \(y_0\), frequency \(\omega\) and wave-number \(k\). There are different methods to study wave phenomenon, of which correlation technique and digital spectral analysis technique are more popular and accepted methods. The correlation technique is extensively used when single wave dominates the system. However, in the studies of plasma physics wave phenomenon, there are cases, where more than one wave exists simultaneously. In this case, the correlation function does not give direct information about the waves but its Fourier transform leads to spectral power density in frequency domain. Fast Fourier Transform (FFT) technique is employed to carry out spectral analysis, where data is directly Fourier transformed. The FFT based spectral analysis technique is standard and has been described in detail elsewhere (Smith, 1974). Here, for the sake of completeness, we present some key features of this analysis. Let us assume that a physical quantity, \(y(r,t)\), is a fluctuating quantity and is being monitored at position \(r\). The same quantity may be written as:
\[ y(r,t) = \int Y_0(\omega)\exp[i(\omega t - k(\omega)r)]d\omega \]
where \(Y_0(\omega)\) represents the line profile and is a unknown quantity which would be determined later. Let us take two fluctuating signals of our interest \(y_1(t)\) and \(y_2(t)\) (say density fluctuations and/or floating potential fluctuations), which are monitored at two different spatial locations. After the signals are digitized, taking into account aliasing effect, it is Fourier transformed using FFT technique. Let us represent the obtained fourier transforms as \(Y_1(\omega)\) and \(Y_2(\omega)\), given by:
\[ Y_1(\omega) = Y_0 \exp[i(k(\omega)r_1)] \]
\[ Y_2(\omega) = Y_0 \exp[i(k(\omega)r_2)] \]
In experimental system, the acquired data is of finite length and it contributes to the side lobes in the power spectrum estimation. It is recommended to use ‘window’ during the estimation of power spectrum. Let us see how one may improve the estimates of the power spectrum by using windows. The finite data length such as those acquired during the experiment \( y_{\text{exp}} \) can be represented by a infinite data length \( y_{\text{true}} \) and a unit amplitude pulse of duration T. It can be written as:

\[
y_{\text{exp}}(t) = y_{\text{true}}(t)w_o(t)
\]

and the fourier transform of \( y_{\text{exp}} \) is given as:

\[
y_{\text{exp}}(\omega) = Y_{\text{true}}(\omega)W_o(\omega)
\]

where \( Y_{\text{true}}(\omega) \) and \( W_o(\omega) \) are Fourier transforms of infinite data length and window function respectively. There are several window functions like rectangular, Hanning, Barlett and Tukey window. The relative merits and demerits of different window functions are discussed by several authors (Harries, 1978). If one uses Hanning window, the refined estimates of power spectrum, obtained from the limited data length can be written as:

\[
f'(\omega) = f_{\text{true,}\omega,\omega} \left[ 1 - \cos \omega T \right]
\]

The superscript \( r \) represents the refined estimate of the power spectrum. The second multiplier term on the right hand side of the above equation represents Hanning window function. Corresponding fourier transform is given as:

\[
F'(\omega) = F_{\text{true,}\omega,\omega} W_o(\omega) \left[ \frac{1}{2} \delta \omega - \frac{1}{2} \delta \left( \omega + \frac{1}{T} \right) - \frac{1}{4} \delta \left( \omega - \frac{1}{T} \right) \right]
\]

where \( \delta \omega \) represents a unit pulse. Since \( F_{\text{exp}}(\omega) = F_{\text{true}}(\omega) \cdot W_o(\omega) \), we can rewrite the above equation in terms of fourier transform of experimental data as:

\[
F'(\omega) = \frac{1}{2} F_{\text{exp}}(\omega) - \frac{1}{4} F_{\text{exp}}(\omega + \Delta \omega) - \frac{1}{4} F_{\text{exp}}(\omega - \Delta \omega)
\]

Here, we have used \( \Delta \omega = 1/T \) in the above equation. The above analysis clearly shows that the refined spectral estimates of the fourier transform at a given frequency is expressed in terms of weighted average of the unrefined estimates at that frequency and the two adjacent frequencies. This is how the refined spectral estimates \( F_1'(\omega) \) and \( F_2'(\omega) \)
have been calculated from the unrefined estimates in our further calculations. We would drop out the subscript in further discussion for the sake of brevity.

Now, it is very simple to obtain different parameter of interest for data analysis. The auto-power, cross-power and phase spectrum are obtained from the spectral estimates and are as follows:

\[ P_{12}(\omega) = F_1^*(\omega)F_2(\omega) \]

\[ P_{11}(\omega) = F_1^*(\omega)F_1(\omega) = |F_1(\omega)|^2 \]

\[ P_{22}(\omega) = F_2^*(\omega)F_2(\omega) = |F_2(\omega)|^2 \]

\[ \delta_{12}(\omega) = k(\omega)\Delta r \]

Here * represents complex conjugates and \( \Delta r = r_1 - r_2 \) and \( \theta_{12} \) denotes phase spectrum. In cylindrical geometry, the azimuthal wave-number, \( k_\theta(\omega) \) and azimuthal mode number \( m(\omega) \) are related by \( k_\theta(\omega) = m(\omega)/r \), where \( r \) is the radial location at which the fluctuations are monitored. One monitors the fluctuations at two points separated by an angle \( \Delta \theta \) at same radial location to get azimuthal mode number. The azimuthal mode number is given by:

\[ m(\omega) = \delta_{12}(\omega)/\Delta \theta \]

The azimuthal phase velocity is given by:

\[ V_{\varphi} = \frac{\omega}{k_\theta} = \frac{2\pi r \Delta \theta}{\theta_{12}(\omega)} \]

The direction of propagation is determined by the direction of \( k \). The ability of determining frequency and wave-number simultaneously makes this analysis a powerful tool in the study of plasma physics data analysis. Another important quantity is coherence which is defined in terms of auto-power and cross-power parameter. It is the degree of cross co-relation between two time series \( f_1(t) \) and \( f_2(t) \) at each frequency and is defined as follows:

\[ \gamma_{12}(\omega) = \frac{|P_{12}(\omega)|}{(P_{11}(\omega)P_{22}(\omega))^{1/2}} \]
Chapter 2 Experimental setup, diagnostics and data analysis.

Here, one should note that when $|\gamma_{11}(\omega)|$ is zero at a particular frequency then the two signals are incoherent at that frequency. Similarly when $|\gamma_{11}(\omega)|$ is one at a particular frequency then the two signals are coherent at that frequency. If $0 < |\gamma_{11}(\omega)| < 1$ at a particular frequency then the two signals are partially coherent at that frequency. This also provides a measure of signal to noise ratio. The presence of background noise reduces the coherence to less than one.

In computing the power spectra digitally, $N$ discrete components of the power spectra will be separated by the elementary bandwidth $\Delta f = 1/T$, where $T = N \Delta t$; $\Delta t$ being the total time duration of sampled data with a sampling time of $\Delta t$. Under such circumstances, statistical variance on quantities is very large and it comes out in the form of jitter in the computer-generated plots of power spectrum. In order to reduce the error to an acceptable level, the spectra are smoothed or averaged over $M$ adjacent elementary frequency bands. In doing so, the frequency resolution decreases by $M\Delta t$, which results in $N$ frequency points. However, too much smoothening results in reduction of frequency resolution as well as coherence. While smoothing one has to consider the tradeoffs between the reduction in statistical variance and frequency resolution. The variance on the estimation of the cross-power spectra, phase spectra and coherence spectra is given by:

$$\text{var}[P_{12}(\omega)] = \frac{1}{2M} |P_{12}(\omega)|^2 \left( 1 + \frac{1}{|\gamma_{12}(\omega)|^2} \right)$$

$$\text{var}[\theta_{12}(\omega)] = \frac{1}{2M} \left( \frac{1}{|\gamma_{12}(\omega)|^2} - 1 \right)$$

$$\text{var}[\gamma_{12}(\omega)] = \frac{1}{2M} \left( 1 - |\gamma_{12}(\omega)|^2 \right)^2$$

It is obvious that all the above quantities decrease with increase in $M$ and coherence.

While acquiring data, the signals are passed through band-pass filter with a pass-band of 800 Hz to 20 kHz in order to avoid aliasing effects. The data is acquired on a dual
channel Lecroy 9640 digital oscilloscope with a simultaneous sampling time of 20 $\mu$-seconds, which can acquire upto 32 kbyte of samples, recorded in each channel. Data are then processed on VAX 8810 and Sun Solaris computer. The data is analyzed using above-mentioned technique to obtain the frequency, wave-number, coherence and the power spectrum of the fluctuations.